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STUDY OF RANDOM AND SYSTEMATIC MULTIPOLES IN THE SSC LATTICE

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Abstract

The expression for the tune shift and an orbit distortion is derived by the first order canonical transformation. The program based on these formulas is described and used for numerical comparison of the non-linear effects in the SSC lattices.

The results are compared with analytic estimations. The choice of the main parameters of the SSC lattices is discussed.

The Formulas

The method of canonical transformation is a reliable method to derive accurate formulas for analytic estimations of nonlinear effects and for an efecient program for numerical analysis of such effects. This approach seems reasonable for first screening of parameters and can fill a gap between time-consuming tracking programs and too crude analytical estimations. With magnet field of a form

$$B = 1/2\ B_o\ b_n\ (x + iy)^n + c.c.$$

the following formulas have been obtained for the tune shift and the distortion $\Delta\varepsilon/\varepsilon$ with accuracy $o(b_n^{\ 2})$:

$$\Delta Q_x = -\ R/\rho\ 2m_x/\varepsilon_x\ C_m\ b_{n-1}\sigma_x^{2m_1}\sigma_y^{2m_2}x\ x_c^{n-2m_1-2m_2} \quad (1)$$

$$(\Delta\varepsilon_x/\varepsilon_x)_k = \frac{2\pi\ (m_1 - m_2)}{N_s\ \varepsilon_x\ \mathrm{Sin}\ (\alpha/2)}\ C\ (m_1,m_2,m_3,m_4). \quad (2)$$

$$< R/\rho\quad b_n^{-1}\ \sigma_x^{m_1+\ m_2}\ \sigma_y^{m_3+\ m_4}\ X_c^{n-\ m}\quad \mathrm{Cos}\ (\beta+K\alpha)>$$

where

$$\sigma_x^{\ 2} = \varepsilon_x/2\ \beta_x(s);\quad X_c = (\Delta p/p)\quad \eta_x$$

$$\alpha = 2\pi/N_s\ \Big((m_1 - m_2)\ Q_x + (m_3 - m_4)\ Q_y\Big)$$

$$\beta(s) = (m_1-m_2)\ [\mu_x+\phi_y-\pi/2-\pi Q_x/N_s\ ]+$$

$$(m_3-m_4)\ [\mu_y+\phi_y-\pi Q_y/N_s];$$

$$C\ (m_1m_2m_3m_4) = \frac{(N-1)!\quad (-)^{m3} + (-)m4}{4\ (N-\ m)!\ m_1!\ m_2!\ m_3!\ m_4!}$$

$C_m = C(m_1,m_1,m_2,m_2,)$;
m_1, m_2, m_3, m_4 are integers.

The emittances ε and initial phases ϕ are related to the coordinates of the particle as following

$$x = (\Delta p/p)\quad \eta_x(s) + \sqrt{2\varepsilon_x\beta_x(s)}\ \mathrm{Sin}\ (\phi_x+\mu_x+0(b_y)$$

In the linear approximation the emittances ε are constant along the lattrice. Formula (2) gives the distortion at the exit of the k-th period, N_s is the number of periods per ring. The brackets <> means an averaging along a period.

The Beta-functions $\beta_{x,y}(s)$, phase advances $\mu_{x,y}(s)$ and $\eta_x(s)$ are determined for a linear lattice and given $\Delta p/p$.

Analysis

The factor Sin(/s) in f. (2) describes the resonance enhancement of the distortion. For decapole errors there is a coupling resonance (m_1,m_2,m_3,m_4) = (2,0,0,2), (0,2,2,0) which gives a big distortion near the diagonal $Q_x \sim Q_y$. The resonance is narrow and goes down 10 times with $|Q_x-Q_y| \sim 0.05$. The similar

enhancement of the distortion exists also for sextupole errors in the second order in b_2.

The crude estimation of the nonlinear tune shift and distortion can be done using f.(1), (2). It gives tune shifts due to systematic mulitipoles
$(\Delta Q)_s \sim (R/Q)\quad \Delta x^{n-2}\ b_n{-1}$

tune shift due to random multipoles
$(\Delta Q)_R \sim \langle b_n{-1}\rangle\ \ (\Delta x)^{n-2}\ /\sqrt{N_c}$

distortion due to systematic multipoles
$(\Delta\varepsilon/\varepsilon)_S \sim \pi/\mathrm{Sin}\ \alpha/2\ (\Delta Q/Q)_s$

distortion due to random multipoles
$(\Delta\varepsilon/\varepsilon)_R \sim (\Delta\varepsilon/\varepsilon)_s\quad Q/\sqrt{N_c}$

Here N_c is the number of cells per ring, Δx is the efficient beam size $\Delta x \sim X_c+\sigma_x{\div}\sigma_y$.

The last formula shows, that the distortion due to random multipoles can be comparable or even bigger than the distortion due to systematic multipoles.

The Program

The program was developed on the base of the formulas (1), (2). The program uses β_x,β_y,η_x functions as they are given by the program MAD for a linear lattice for a given $\Delta p/p$. The program can be used:

1. To calculate strength of two regular sextupole families in the lattice to cancel the linear chromaticity.
2. To calcualte the tune shift due to systematic or random multipoles of a given order. For random multipoles rms value of b_n has to be given.
3. To calculate the distortion. The result of this calculation can be plotted out on a phase plane or as distortion versus number of turns, or as variation of the distortion along a period to study the local distortion.

The program is rather fast. It takes 20 seconds of CPU time for tracking with sextupole errors instead od 16 minutes for tracking with MARYLIE under MAD for the same lattice on VAX 780. Of course, time increases with multipole order.

Results

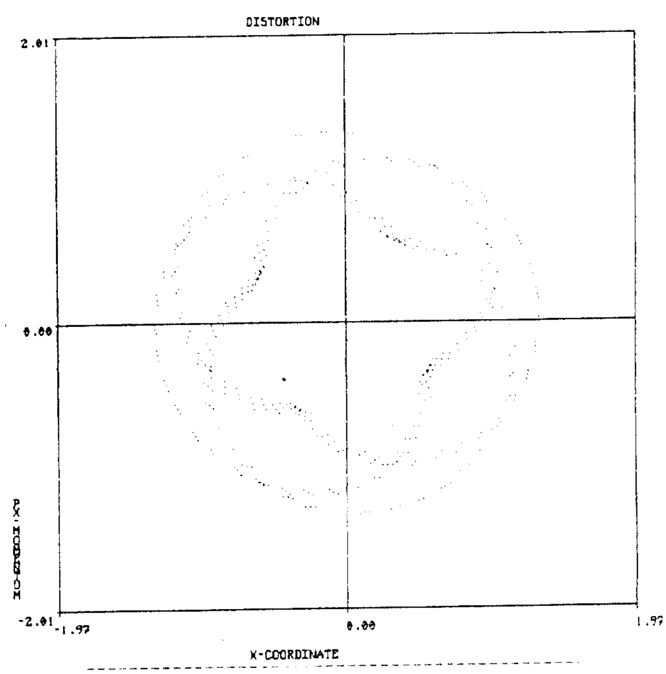
The program was used to calculate nonlinear effects for three SSC sixfold lattices with insertions. The lattices are described in the paper < >. They were the same in all respects (including tune shift per period) except the half cell length and phase advance per cell. They were equal to (140m,60^o), (140m,90^o). There were 81.5 cells per arc and 4 kicks per dipole in the first two cases and 110.5 cells and 3 kicks per dipole in the lase case. Fractional part of the tune per ring was close to 0.3 and $\beta^* = 1m$ in all cases. The results of the calculations are given in table I. In all cases b_n in dipoles was assumed to be equal too units (in units 10^{-4} cm^{-n}).

Table I

	Q_x	Q_y	ΔQ_x	ΔQ_y	$\Delta\varepsilon_x/\varepsilon_x$	$\Delta\varepsilon_y/\varepsilon_y$
140,60 Regular b_x	97.30335	97.29750	-----	----	+ 0.016	0.016
Syst. b_2	97.30335	-----	-0.0776	0.0741	0.090	0.101
Rand b_2	-----	-----	0.47 10^{-3}	-0.245 10^{-2}	0.160	0.186
Syst. b_4	96.9985	96.9084	0.0508	0.0648	0.1375	0.156
Rand b_4	97.3256	97.2995	-0.37 10^{-2}	-0.33 10^{-3}	0.284	0.280
Syst. b_6	96.7045	97.1331	0.0998	-0.0726	1.435	0.940
Rand b_6	97.3028	97.2595	0.94 10^{-4}	0.63 10^{-2}	0.397	0.437
140,90 Regular b_x	145.2979	145.2920	-----	----	0.022	0.022
Syst. b_2	-----	-----	-0.0340	0.0305	0.047	0.059
Rand. b_2	-----	-----	-0.26 10^{-3}	-0.1097 10^{-2}	0.089	0.071
Syst. b_4	-----	-----	0.0103	0.0164	----	----
Rand b_4	145.3089	145.2918	-0.18 10^{-2}	0.28 10^{-4}	0.1851	0.1972
Syst. b_6	145.1860	145.3011	0.018	-0.011	0.457	0.539
Rand b6	145.2963	145.2784	0.27 10^{-3}	0.23 10^{-2}	0.129	0.151
110,90 Regular b_x	187.3008	187.2961	-----	----	0.0278	0.0281
Syst. b_2	-----	-----	-0.0148	0.0131	0.0386	0.0364
Rand b_2	-----	-----	-0.60 10^{-4}	-0.53 10^{-3}	0.1093	0.1857
Syst. b_4	187.2817	187.2651	0.317 10^{-2}	0.517 10^{-2}	0.091	0.072
Rand b_4	187.3036	187.2949	-0.46 10^{-3}	0.201 10^{-3}	0.1026	0.082
Syst. b_6	187.2738	187.3125	0.45 10^{-2}	-0.27 10^{-2}	0.134	0.125
Rand b_6	187.2995	187.2936	0.212 10^{-3}	0.425 10^{-3}	0.041	0.045

Emmittances $\Delta p/p = 0.5\ 10^{-3}$ and $\varepsilon x = \varepsilon y = 10^{-7} m$ (X_o = = 0.44mm) were used. For random multipoles the Gaussian distribution was chopped off at 2σ. The table shows the tune shift per period ΔQ due to a given multipole error only (for sextupole it doesn't include tune shift due to regular sextupole families in the arc). The amplitude of distortion variation $\Delta\varepsilon/\varepsilon$ at IP is given in 256 periods.

Analysis of the data shows that the higher-order multipole shift and distortion significantly depend on the cell length and phase advance, making shorter latticfe favorable. The distortion due to random multipoles is of order of distortion due to the systematic multipoles. The effect of dodecapoles is comparable with the effect of decapole errors, for displacement X_o = 0.5 cm. There is a tendency to follow the analytic estimations given above, although there are significant difference, especially for dodecaples. Partially it can be explained by increase of number of different terms in the sum (1), (2) and interference between them, what is difficult to take into account in estimations. It shows, that analytic estimation are not very reliable for the system like SSC where there is no safety factor. So, the use of the simple program as described above seems very reasonable for preliminary screening of different parameters. In particular, the low-field SSC lattice has to use shorter cells, for example 115 m for half cell length.



CANONICAL TRANSFORMATION PROGRAM
LATTICE TAC14090
SYSTEMATIC SEXTUPOLES, B2=5 UNITS
X0=3.13 E-04, Y0=3.13 E-04