IEEE Transactions on Nuclear Science, Vol. NS-32, No. 5, October 1985

EFFECT OF LONG RANGE BEAM-BEAM INTERACTION ON THE STABILITY OF COHERENT DIPOLE MOTION*

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The coherent effect of beam-beam forces on the stability of the motion of beams as rigid bunches in a collider is studied by means of simulation. The number of bunches per beam is taken to be large, with many bunches colliding simultaneously within each interaction region. It is also assumed that they populate the beams uniformly, and that they are equally spaced. The interaction regions are all identical and evenly spaced. The collision forces are assumed to be one-dimensional linear kicks. Results for stability limits are presented, as a function of tune, for various beam configurations and several values of the crossing angle.

Introduction

The SSC has one feature that may have an important effect on its design: the fact that the beams have several thousand bunches makes the interaction between beams significant, so that beam stability is potentially affected.[1] This, in turn imposes a restriction on the choice of tune, and may weaken the conclusions of tracking studies which ignore the beam-beam interaction. Basically, the effect arises from the fact that there are many bunches in a given interaction region simultaneously, so that they interact several times (with different strengths, which depend on the value of the crossing angle) before leaving. The induced transverse motion gets quickly compounded, and this has a potential effect on the stability and the choices of tune and crossing angle.

Ideally, one would want to include, in beam-beam interaction studies, the effect of the forces on each particle produced by the electromagnetic fields of the other particles within its own bunch and of those with which it collides, in addition to the forces produced by the magnets and the walls of the beam pipe. This is a formidable task from the programming point of view, and, in any case, no computer exists now nor will exist within the SSC design time scale which would be able to produce significant results from such a program.

In this note we present first results on the effect that the beam-beam interaction has on the stability of the beams. While we make (so far) many simplifying assumptions, the key ingredients of many bunches per beam and multiple, simultaneous, bunch-bunch interactions within all interaction regions are kept. We present results for several beam configurations and crossing angles. While these results are preliminary, they do suggest an important effect on beam stability.

Analysis

Assumptions

We consider here only the motion of the center of charge of the bunches.[2, 3, 4, 5] In practice, this amounts to treating them as rigid, disk-like objects. In this sense the effect is "coherent", since all the particles within a bunch move together. This dipole approximation to the charge distribution may be taken as the starting point of a more realistic calculation in which higher multipoles are treated.[6, 7, 8, 9, 10] Furthermore, we study the motion in one dimension only, so that each bunch is fully described by its transverse coordinate x and its slope x'. The bunches are evenly spaced and populate the beams uniformly. Both beams are identical. All the interaction regions (IR's) are also identical, as are the arcs between them. Thus the superperiod of the machine equals the number of IR's.

The configuration of the machine is therefore described by the number of IR's, the number of bunches within an IR, and the number of bunches within an arc. Thus we have

$$N_b = N_{ir}(m+m') \tag{1}$$

where N_b is the number of bunches per beam, N_{ir} is the number of IR's, m is the maximum number of bunches that can fit simultaneously within an IR and m' is the minimum number of bunches that can fit simultaneously within an arc.

The bunches are assumed to interact only within the IRs. A given bunch interacts every time it moves a distance L/2, where L is the interbunch distance. Within the IR, the bunch is drifted between interactions by a simple 2x2 drift matrix D(L/2). In the arcs, it is transported by a phase advance matrix T from the end of an IR to the beginning of the next one. The matrix T satisfies the relation

$$D\left(\frac{(m-1)L}{2}\right)\ T\ D\left(\frac{(m-1)L}{2}\right) = \begin{bmatrix} \cos(\pi\mu^*) & \beta^*\sin(\pi\mu^*) \\ \frac{-\sin(\pi\mu^*)}{\beta^*} & \cos(\pi\mu^*) \end{bmatrix} \tag{2}$$

where μ^* is the tune between the centers of two neighboring IRs and β^* is the value of the beta function at the center of the IR.

There is one "head-on" collision at the center of the IR plus several "long-range" interactions away from the center. We assume these interactions between bunches to be kicks which can be linearized as follows: consider two opposing bunches, one from each beam, whose coordinates are (x_1, x'_1) and (x_2, x'_2). The xs and x's are measured relative to the respective design trajectories. They "collide" at a point where the distance between the design trajectories is d. If the distribution is Gaussian, the slopes change according to $x'_1 \to x'_1 + \Delta x'$, and $x'_2 \to x'_2 - \Delta x'$, where

$$\begin{aligned} \Delta x' &= \frac{1}{f}\left[E(d+\Delta x,\sigma)-E(d,\sigma)\right] \\ E(x,\sigma) &= \frac{1-\exp(-x^2/2\sigma^2)}{x/2\sigma^2} \\ \Delta x &= x_1 - x_2 \end{aligned} \tag{3}$$

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0018-9499/85/1000-2297$01.00

The parameter σ^* is the effective transverse size of the beam at the center of the IR. The effective "focal length" f determines the strength of the kick. For the head-on collision, d=0, $\sigma = \sigma^*$ and $f=f^*$. For the long-range interactions, $\sigma^2 = \sigma^{*2}(1+s^2/\beta^{*2})$ where s is the distance from the collision point to the center of the IR; f is related to f* by $f=f^*(\sigma^2/\sigma^{*2})$, (see eq. (5)).

For the purposes of our simulation, we linearize the above expressions about $\Delta x=0$, and assume the following values for the parameters: $\sigma^*=7$ µm, $\beta^*=1$ m, L=15 m. Then the expressions for the kicks are well approximated by the following:

$$\text{head on:} \quad \Delta x' = \frac{1}{f^*}\Delta x$$
$$\text{long-range:} \quad \Delta x' = -\frac{1}{f^*}p\,\Delta x \qquad (4)$$

where the parameter $p = 2(\sigma^*/d)^2$. The distance d between the design trajectories at the collision point is determined by the beam configuration, crossing angle and interbunch distance.

The difference in sign between the head-on and long-range kicks arises from the shape of the force function E, eq.(3): close to the origin, the slope is positive, but it is negative out in the tail. In the linearized form of the kicks, it is obvious that $\Delta x'$ is proportional to this slope.

Instead of the parameter f*, we use a dimensionless one,

$$\xi = \frac{\beta^*}{4\pi f^*} = \frac{\beta^* N r_0}{4\pi\gamma\sigma^{*2}} \qquad (5)$$

where N is the number of particles per bunch, r_0 is the classical radius of the particle and γ is the usual relativistic factor.

Method

In practice we start the simulation by assigning values to μ^*, ξ, β^*, L, σ^* and the crossing angle α. Then we assign random values, within a certain range, to x and x' for all the bunches. We go around the ring bunch by bunch either kicking it, drifting it within the IR's, or transporting it through the arcs; this constitutes one step. At the end of this step, all bunches have moved a distance L/2, and we repeat the process until a full turn is completed. After each turn we evaluate the maximum amplitude x for all the bunches at the center of the IRs; if this exceeds a certain value, we call the motion unstable. If no instability is found after a large number of turns, we call the motion stable. In this way we can find a stability boundary in the (ξ,μ^*)-plane for a given beam configuration.

For linear kicks it is also possible to study the stability of the beams by finding the full transfer matrix for one turn, and diagonalizing it. If there is an eigenvalue greater than one in absolute value, the motion is unstable; otherwise it is stable (the simplecticity of the matrix ensures that it is not possible to have all eigenvalues less than one in absolute value, so there can not be damping). We have used this method as a check for one beam configuration only, as explained below.

Results

We present here results for only three beam configurations. The simpler one has $N_{ir}=2$, m=2 and m'=0. Thus there are 4 bunches per beam. Each bunch interacts three times within each IR: there is one head-on collision at the center of the IR, and two long-range interactions at either side of the center. Figure 1 shows the stability limits for various crossing angles. We plot the maximum value of ξ for which the beam is stable vs. the <u>tune ν of the entire machine</u>. The periodicity of the graph is one unit of tune, so we plot only two cycles. For 50 µrad crossing angle the limit is given by the curve ACE. The parameter p takes on the value 6.97×10^{-4} for the long-range interaction closest to the center of the IR. The region above the curve is unstable, below it is stable. For 10 µrad the corresponding curve is ABE. In this case $p=1.74\times10^{-2}$ for the long-range kick closest to the center. As the crossing angle is increased, the right side of the curve becomes steeper, until it becomes vertical. All this means in our calculation is that, in this limit, the long-range interactions have zero strength, and only the head-on collision is present. Thus the curve AD is also the stability limit for the 2-bunch per beam configuration $N_{ir}=2$, m=1, m'=0. In this case there is a simple analytic expression for the curve [2,3], namely, $\tan(\pi\nu/2)/4\pi$, with which our simulation agrees. The right side of the curves (BE and CE), with negative slope, is caused by the destabilizing effect of the <u>attractive</u> long-range interactions, while the rising edge (AB, AC, AD) is due to the <u>repulsive</u> head-on kicks (for beams with oppositely charged particles the curves are reversed, i.e., points A and E are interchanged).

Figure 2 shows the corresponding results for the 36-bunch per beam configuration $N_{ir}=6$, m=m'=3. In this case each bunch interacts a total of 5 times within each IR. In this case the periodicity of the graph is 3 units of tune, and we show only one cycle (the tune is, again, that of the entire ring). For 50 µrad, the stability curve is ADA'C'A"C"E. For 10 µrad, the attractive long-range interactions have a stronger destabilizing effect, and the stability curve is, in this case, ADA'D'A"D"E. In the limit of infinite crossing angle there is only a head-on kick per IR, so the stability curve corresponds to the 6-bunch per beam configuration $N_{ir}=6$, m=1, m'=0, which is given by ABA'B'A"B". In this case there is also a simple analytic expression [11] for the stability curve, with which our result agrees.

Figure 3 is similar to the previous one except that it corresponds to the beam configuration $N_{ir}=6$, m=5, m'=1. The larger number of bunches within the IR's have a stronger destabilizing effect, which manifests itself in downslope curves farther away from the vertical line. This effect will be even more pronounced for the SSC, which is expected to operate at m=26.

Remarks

As mentioned above, we have also studied the beam stability by finding the full transfer matrix and diagonalizing it. If there is at least one eigenvalue greater than unity in absolute value, the beam is unstable; otherwise it is stable. This is not a convenient method of programming because every beam configuration requires a different matrix. Besides, the matrices, of dimension $4N_b \times 4N_b$, become too large in most cases of interest. However, we have used this method to verify the simulation results for the particular beam configuration $N_{ir}=2$, $m=3$, $m'=0$. In this case, the results of both methods agree within the precision of the computer (these results are not shown here).

We have also done another "experiment" on our simulation program: we have replaced the drifts within the IR's by appropriate phase advance matrices corresponding to the motion between collisions. In this case there is no clear distinction between arcs and IR's, so the stability pattern should reflect the configuration of a beam with many more IR's. This is indeed what we observed: the sawtooth shape of the curve remains, but the periodicity is increased. For the configuration $N_{ir}=2$, $m=2$, $m'=0$, the pattern takes on a periodicity of 2 units in tune, while for the configuration $N_{ir}=6$, $m=m'=3$ the periodicity becomes 36 units of tune.

Conclusions

Even though we have made many simplifying assumptions, our results show the effect of the long-range coherent beam-beam interaction on the stability of the beam. Generally the stop-band width is increased significantly for small crossing angle, and therefore has a potential effect on the choice of tune.

Admittedly, the parameters used here are not realistic for the SSC. For instance, the SSC is expected to operate at a value of $\xi=0.005$, and therefore our simulation, if taken at face value, does not restrict the choice of tune significantly except near integers. We have taken into account only the dipole motion of the bunches. However, each higher multipole approximation to the motion is expected to introduce its own stop-band. The non-linear character of the beam-beam force is not expected to change significantly the stability of the dipole motion, but it will excite higher order multipole motion. If the effect we have observed and described here persists to higher order multipoles, it may seriously restrict the working point of the SSC. We are presently extending our simulation to include these effects, and the results will be presented elsewhere in the near future.

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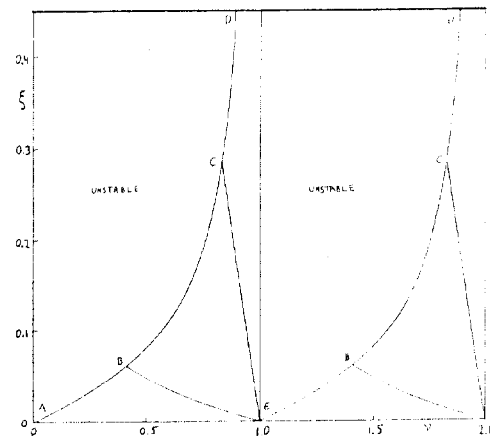

Figure 1

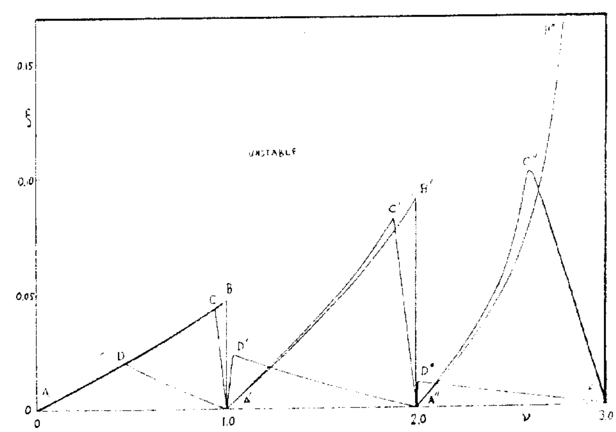

Figure 2

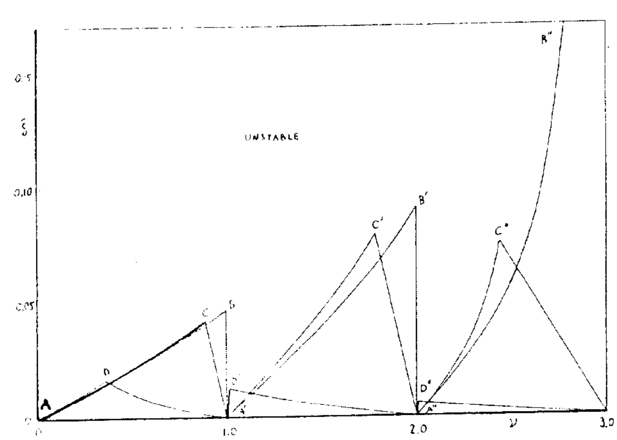

Figure 3