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NU SHIFTS IN BETATRON OSCILLATIONS FROM UNIFORM PERTURBATIONS IN THE PRESENCE OF NON-LINEAR MAGNETIC GUIDE FIELDS*

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Abstract

Uniform magnetic field perturbations cause a closed orbit distortion in a circular accelerator. If the magnetic guide field is non-linear these perturbations can also cause a Nu shift in the betatron oscillations. Such a shift in radial Nu values has been observed in the Bevalac while studying the low energy resonant extraction system. In the Bevalac, the radial perturbation comes from the quadrants being magnetically about 0.8% longer than 90°. The normal effect of this type of perturbation is a radial closed orbit shift and orbit distortion. The nu shift, associated with this type of perturbation in the presence of a non-linear guide field, is discussed in this paper. A method of handling the non-linear n values is discussed as well as the mechanism for the associated Nu shift. Computer calculations are compared to measurements.

Observations From Extraction Studies

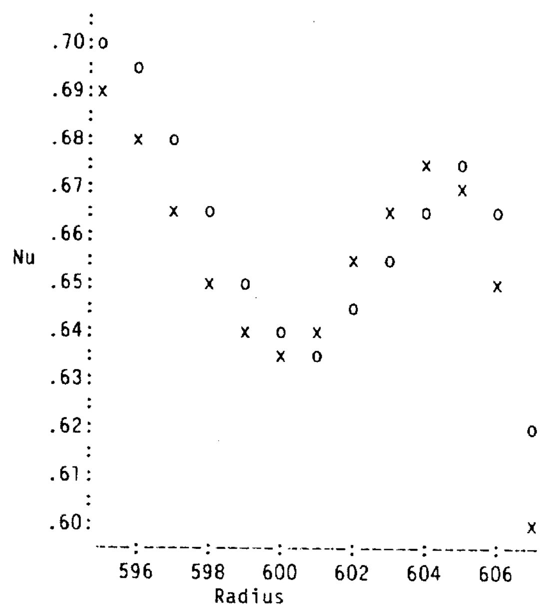
A new resonant extraction magnet was installed at the Bevalac to allow more efficient extraction at lower energies. Because of less damping of radial betatron oscillations at lower energies, the new magnet had a larger radial aperture than the original magnet. This caused some changes in the resonant extraction system.[1] The results reported here are part of the study to find a better model to calculate the extraction parameters. One of the observations was an apparent Nu shift (Fig. 1) from what had been calculated from original magnetic field data.[2] This magnetic field data was meager, so a check was made using a current in one of the pole face windings (PFW). These windings are a set of windings on the upper and lower pole tips used to make small changes in the n value across the gap. The field index n is defined in the equation

$$B= B0*R0^n/R^n \qquad (1)$$

where n is a constant.

The Nu value is measured by flipping up a U shaped target that is centered on the beam. The rf accelerating voltage is turned off, the frequency shifted from rotational frequency to near the radial betatron oscillation frequency, and the rf voltage is then turned on again. By slowly sweeping the frequency, the frequency at which beam loss starts is noted. This is the betatron oscillation frequency for that radial position and the maximum amplitude particle just contained within the U target. The ratio of the betatron oscillation frequency to the rotational frequency measured just before the frequency is switched is the Nu value. These measurements are made on flattop where there is no energy gain necessary for the beam to remain at constant radial position.

Two radial scans were made with this probe. One with the PFW currents on and one scan with the currents off. The difference between these two curves gives the Nu shape for the PFW current. This also showed a radial shift of Nu. This is shown in Fig. 2. The radial position of the probe was rechecked and shown to be correct. Until these calculations we had no explanation for the observed shift.



x Field Data
o Calculated Data

Fig. 1 Nu shift between calculated and magnetic field data.

Equation of Motion with a Non-Linear Force

The second order differential equation of motion describing radial betatron oscillations is given in Eq. 2.

$$d(mR)/dt = mv^2/R - evB \qquad (2)$$

It is more convenient to transform from a time to azimuthal coordinate θ, using the relationship $w= v/R = d\theta/dt$. Because the momentum is constant, we can substitute $mv=e*R0*B0$ and rearrange terms to yield Eq. 3.

$$d^2R/d\theta^2 = R(1- B*R/B0*R0) \qquad (3)$$

Using Eq. 1 and expanding around R with $R=R0+x$ yields the familiar relationship shown in Eq. 4.

$$d^2x/d\theta^2 = -(1-n)x \qquad (4)$$

If n varies as a function of R, then the magnetic field value B at R is no longer defined by simply stating the radial position and the field index n. The derivation of Eq. 4 from Eq. 3 taking $n=n(R)$ would yield Eq. 4 with $n=n(R)$. This is a non-linear equation and doesn't have the familiar sinusoidal solution of the linear equation. In addition, n at

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any radial position in the field is usually determined by measuring the field gradient DB over a radial difference DR and defining $n=-(dB/B)(R/dR)$. The relation $n=n(R)$ would not define the same n value as that measured. The method of measuring Nu with the U target as described above, also yields magnetic field values as defined by the gradient definition of n above.

The wealth of theory in the literature and the ease of visualizing the motion in the sinusoidal solution make it desirable to retain that form. I propose then to maintain the form of Eq. 4 but redefine n as an effective n that will give the correct value of magnetic field B at position R and hence the same restoring force. The field index n is then defined as $neff = \log(B/BO)/\log(RO/R)$. Note that neff defined this way is no longer uniquely defined at a radial position but at a radial position relative to the closed orbit values RO and BO. This technique allows the handling of non-linear magnetic guide fields while still retaining the form of the linear case. However, if you change the radius of the closed orbit you must calculate a new set of n values.

Field Perturbations

The Bevalac is a weak focusing synchrotron with four 90° quadrants and four straight sections. The quadrant magnetic fields are about 0.8% longer than 90° at about 5 to 6 KG. The normal betatron oscillation calculation assumes that the magnetic field will bend the particle through 360° in going once around the machine on the closed orbit. Any excess or shortage of guide field must then be handled as a radial perturbation. This results in an orbital offset of about 33cm in the Bevalac (radius of curvature = 15.24 m) and a scallop of 3.5 cm as shown in Fig. 3.

Computer Model

Combining this field perturbation with the non-linear n values in a computer calculation of the betatron oscillation yielded, among other results, a radial Nu shift about three times the amount observed. This was interesting as we had been unable to explain the observed shift by any other mechanism.

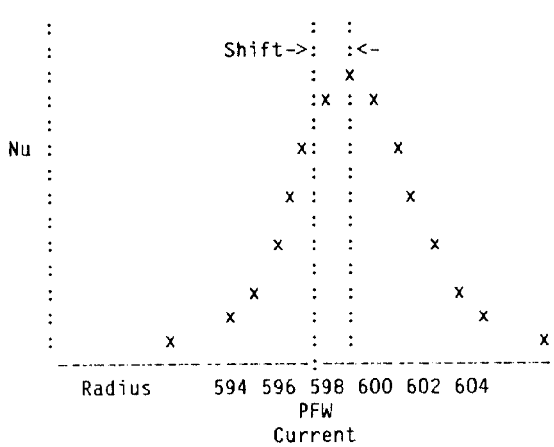
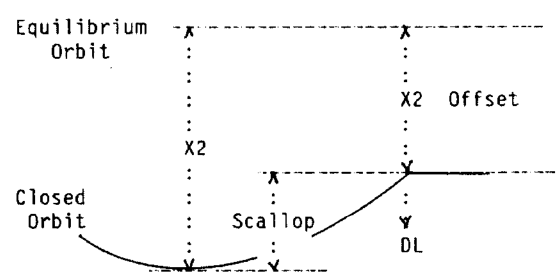


Fig. 2 Difference between Nu measurements with PFW current on and PFW current off. Peak is normally centered at radial location of current. Figure shows apparent radial shift. The calculation in this paper shows a similar shift.

A second result was that in a number of cases the motion appeared unstable. This was caused by the fact that in the region of the equilibrium orbit, 33cm outside the closed orbit, the n values were greater than 1. The calculation for neff is about the reference orbit, so these values greater than 1 effect the calculation. As the particles are never in that region of magnetic field,it cannot determine the details of motion. This difficulty was handled by chosing a nominal Nu value for the closed orbit calculation with the perturbation. A second calculation is then made using a simple betatron oscillation supperimposed on top of this distorted closed orbit using the neff values for the actual radial position of the particle relative to the calculated closed orbit. While this method was originally used to eliminate an instability from the $n>1$ region, physical consideration say that it must be handled this way as the particles are never in the region of the equilibrium orbit and cannot be effected by the n values there. This method reduces the radial Nu shift about a factor of three which agrees with the observed shift. Fig. 1. It also presents a rather simple physical explanation of the shift. The particle is really moving at a smaller radius than is observed in the staight section, where monitoring is done. It therefore has a Nu value associated with that smaller radial position of the closed orbit.

Details

One question that must now be examined is what is the error associated with using a constant nominal value of Nu to calculate the closed orbit. Fig. 3 shows the trajectory through a quadrant and into the straight sections. If all four quadrants are the same, then by symmetry we can evaluate the scallop and offset. The scallop is given by X1-X2 and the maximum displacement XO is calculated by setting $X1'+DL=0$. The values of XO/DL and (X1-X2)/XO vs Nu are shown in Table I. The change is not large for a few tenths variation in Nu. Several iterations can be made if a more precise value is needed. This value of Nu is for the quadrant only ($Nu=\mathrm{SQRT}(1-n)$) not the quadrant plus drift spaces.



Where: $X=XO*\cos(Nu*\theta+PI)$
$X'=-Nu*XO*\sin(Nu*\theta+PI)$

Fig.3 Closed orbit shift and distortion from quadlength perturbation DL.

Table I Variation in orbit offsets vs Nu.

Nu	XO/DL	(X1-X2)/XO
.58	3.919	.102
.60	3.671	.109
.62	3.447	.116
.64	3.243	.124
.66	3.058	.131
.68	2.889	.139
.70	2.734	.147
.72	2.592	.156

The next point to be considered is phase advance per turn. Fig. 4 shows a typical phase plot for a betatron motion about the equilibrium orbit using the radial perturbation from the extra magnetic field. Notice that relative to the equilibrium orbit there is no continuous phase advance as in a normal betatron oscillation only an advance and then a retarding of the phase along with a change in radial position. A continuous phase advance and typical betatron oscillation only exist relative to a point marked as the closed orbit. In the computer calculation, this point is taken as the average radial position, at that azimuth, for 100 turns about the machine. In the calculation for the Bevalac, this point was calculated at the center of each straight section and for each 2.5° in the quadrants.

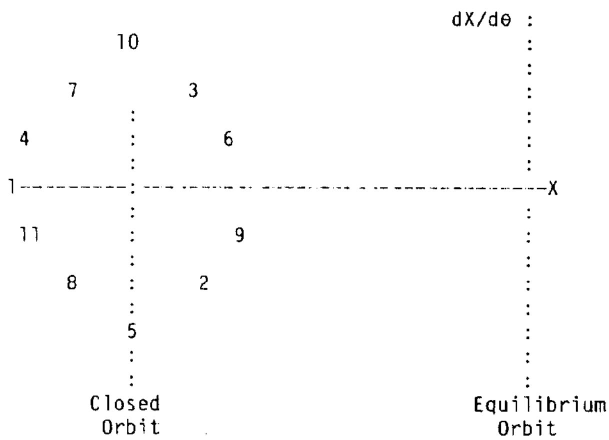


Fig. 4 Phase plot about equilibrium orbit. Numbers indicate position of successive turns.

To compare the phase advance determined by this method to the normally defined constant phase advance in the literature, we must understand what these quantities are. The phase advance calculated in this paper is the actual physical quantity. The constant phase advance from theoretical calculations is a mathematical parameter not the actual physical phase advance. Unfortunately, this fact is never mentioned in the literature and as a result some people treat it as the actual physical phase and get erroneous results when phase information is needed. The constant phase advance is the physical phase advance after j oscillations and k sectors, or turns depending on how the constant phase advance was determined, where j and k are integers. For many turns, the average value of the phase advance is equal to the constant phase advance. To compare the phase advance from the computer calculations to theory, I have taken the average value over 100 turns.

As it is more convenient to talk about Nu values than phase advance, the results are presented in terms of Nu values. As the Nu value is just the ratio of phase advance divided by machine degrees, the same arguments as to the nature of phase advance apply to Nu values. Nu values, as calculated from theory, are average values over many turns.

Conclusions

All accelerators must have some perturbation fields similar to the Bevalac associated with the fringe fields of the guide field magnets. If they also have some non-linearities in field shape, they may also experience some similar Nu shifts. In normal operation the orbital shifts produce a slight change in acceleration frequency, which is not always noticed as the absolute value of frequency expected is not always known. In the case of the Bevalac, the orbit change is enough to produce an energy shift of about 1.2%. The experimenters are now starting to want heavy ion energies known to better than 1%. So this type of effect must be included in energy correction calculations.

Acknowledgements

For cost considerations, these computations were done in the background of the ModComp computer used to control the Bevalac. I want to thank Jim Guggemos for his assistance in making these programs run within the idiosyncrasies of the compter system.

References

[1] K.C.Crebbin, "Characteristic Time Structure in Slow Resonant Extracted Beam", Proc. 12th International Conference on High-Energy Accelerators, August 11-16, 1983, 366.

[2] Marsh Tekawa, private communication.