

RESONANCE CROSSING IN THE PRESENCE OF SPACE CHARGE

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In circular accelerators the tune of a particle may cross a resonance line due to space charge. A self-consistent treatment requires taking into account the time-dependent space charge force in addition to the driving field errors. This is investigated by means of computer simulation for linear and non-linear resonances. The new features as compared with resonance crossing in the absence of space charge are discussed. In the case of a dipole resonance space charge is found to have a stabilizing effect, whereas it can lead to undesirable tune locking for non-linear resonances.

1. Introduction

The stability of orbits in circular accelerators has been studied extensively under the assumption of no space charge effects. The importance of space charge for crossing of linear or nonlinear resonances is evident in synchrotrons or storage rings with high beam intensity and low energy due to the decrease of space charge effects as $1/\gamma^3$. Space charge modifies resonance crossing in several ways: both the single particle tune and the tune of coherent oscillations are shifted by space charge; a nonlinear space charge force has a dynamical effect on the development of nonlinear resonances, which can be stabilizing due to detuning of betatron frequencies. This has been shown by analytical work assuming a stationary distribution of space charge during resonance crossing¹. This approach is thus limited to the onset of resonance. The question can be raised, how the resonance crossing is affected beyond the early behaviour by giving up the constraint of a stationary distribution of space charge, i.e. making the problem self-consistent. We have studied this by means of computer simulation with the particle-in-cell code SCOP-2. This approach is strictly self-consistent, but on the expense of relatively large computer time due to the direct orbit integration of some 10^3 particles. We are thus limited in the simulation to a few thousands of revolutions, hence consider relatively large perturbing terms in the percent range to enhance the effect of resonance crossing. Such perturbations can be expected from beam beam interaction rather than magnetic field errors, which are usually much smaller.

In section 2 we describe the features of the numerical procedure. In section 3 we present results for integer resonance crossing with a dipole error. The difference between self-consistent and non self-consistent treatment is most obvious here, since we observe that crossing of the (space charge shifted) tune over the integer has no effect on the beam. In section 4 we treat an octupole perturbation as example of a nonlinear resonance.

2. Numerical Method

For simplicity we consider a ring with constant focusing providing a tune $\nu_{0,x,z}$ (in the absence of space charge) and a magnetic field perturbation of harmonic m . Throughout these calculations a coasting beam with no momentum spread is assumed. Space charge gives rise to an electric field $E_{x,z}$, which is calculated self-consistently from the actual density distribution by solving Poisson's equation every time step. $E_{x,z}$ thus gives rise to an incoherent tune shift $\Delta\nu_{x,z}$ (due to the linear part of $E_{x,z}$), a spread in tunes due to the nonlinear part of $E_{x,z}$ and a time-dependent coherent

force as a result of beam excitation during resonance crossing. The latter is an essential part of our work, since it makes the problem dynamically self-consistent. We thus trace particles according to

$$(1) \quad x'' + \nu_{0x}^2 x - q/(M\gamma^3 v^2) E_x(x, z, \theta) = \epsilon h_{p-1,x}(x, z) \sin m\theta$$

(similar in z), where θ is the angle around the machine, h a polynomial of order $p-1$ ($p=1$ dipole; $p=2$ quadrupole etc.) to describe a resonance term of order $p-1$ and single harmonic m . ϵ gives the strength of the resonance term. We have traced 8×10^3 particles to obtain a good resolution for space charge calculation and assumed a spatially uniform distribution at start.

3. Dipole Resonance ($p=1$)

Here we assume $h_0 \equiv 1$ in x -direction and $m=2$. In the absence of space charge the equilibrium orbit is displaced by² ($\epsilon = R^2 \delta B_m / B_0$)

$$(2) \quad \delta x = \epsilon / (\nu_0^2 - m^2) \sin m\theta$$

whereas crossing of the resonance at $\nu_0 = m$ yields

$$(3) \quad \delta x = 1/2 \epsilon / m (\pi/a)^{1/2}$$

with $a = dv_0/d\theta$ the speed of crossing.

This obvious result is confirmed by the simulation shown in Fig. 1a, where ν_0 is slowly shifted from 2.1 to 1.9 at constant speed ($m=2$). As a next step we assumed $\nu_0 = 2.4$ and raised the current such that the space charge depressed tune ν is shifted from 2.4 to 1.6 at the same rate (Fig. 1b). No displacement of the beam is found during this integer crossing of ν . This is explained by the fact that ν applies to the single particle oscillation within the stationary beam, whereas the coherent dipole oscillation is only affected by ν_0 , which is far above the integer.

The obvious conclusion is that in a synchrotron dipole errors are not of concern, if ν crosses an integer as a result of high space charge at the bunch center. This also removes the concerns about integer crossing that were expressed in the context of fast bunch compression in heavy ion fusion storage rings^{3,4}. Basically the same result was found for a (transversely) parabolic initial density distribution. The effect of a spread in ν due to finite $\delta p/p$ has not been explored numerically; theoretical work gives evidence for a small residual effect^{5,6}.

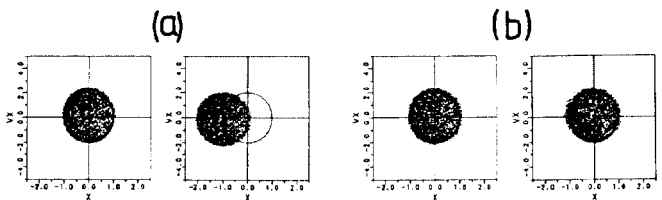


Fig. 1: Phase space projections for dipole error before and after integer crossing. (a) crossing of ν_0 ; (b) crossing of ν (ν_0 fixed).

4. Nonlinear Resonance ($p = 4$)

We assume an octupole term with $h_{3,x} = x^3 - 3x z^2$, $h_{3,z} = z^3 - 3z x^2$ and harmonic $m = 9$. A resonance is expected for $\nu_{0,x,z} = 2.25$, in the absence of space charge. There is a stopband width $\delta\nu$ depending on ϵ^7 . We consider perturbations large enough to cause significant emittance dilution for ν_0 inside the stopband ($\epsilon = 10^{-2} \dots 10^{-1}$ for an initial beam radius of 1 cm).

Crossing of the resonance at a rate $d\nu/d\theta = 3 \cdot 10^{-5}$ (the tune changing from 2.3 to 2.2 during 500 revolutions) leading to beam loss on the aperture (at ± 2.5 cm) is shown in Figures 2 and 3a, with $\epsilon = 0.1$ and a stopband width of $\delta\nu = \pm 10^{-2}$. As a measure for the effect of resonance we use in Fig. 2 the fraction of intensity contained within the original beam cross section.

As a next step we allow for uniform space charge, which results in a shifted linear tune $\nu = \nu_0 - \Delta\nu$. With resonance crossing we observe the following new phenomena:

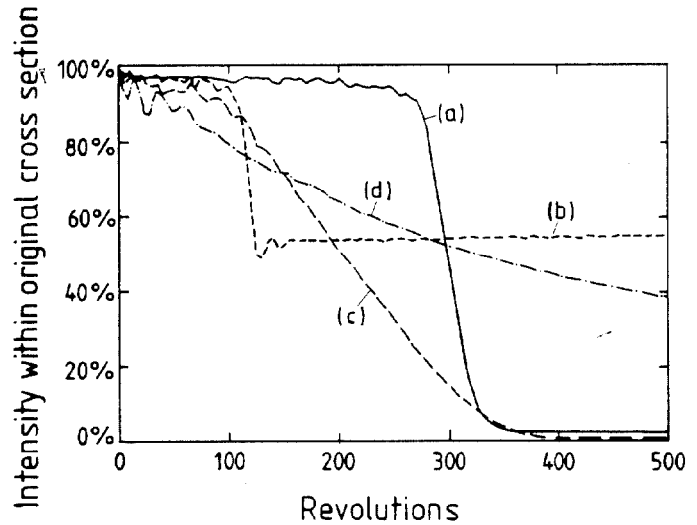


Fig. 2: 4-th order resonance crossing showing intensity within original beam cross section
 (a) no space charge
 (b) fixed current, ν_0 increasing
 (c) fixed current, ν_0 decreasing
 (d) fixed ν_0 , current increasing

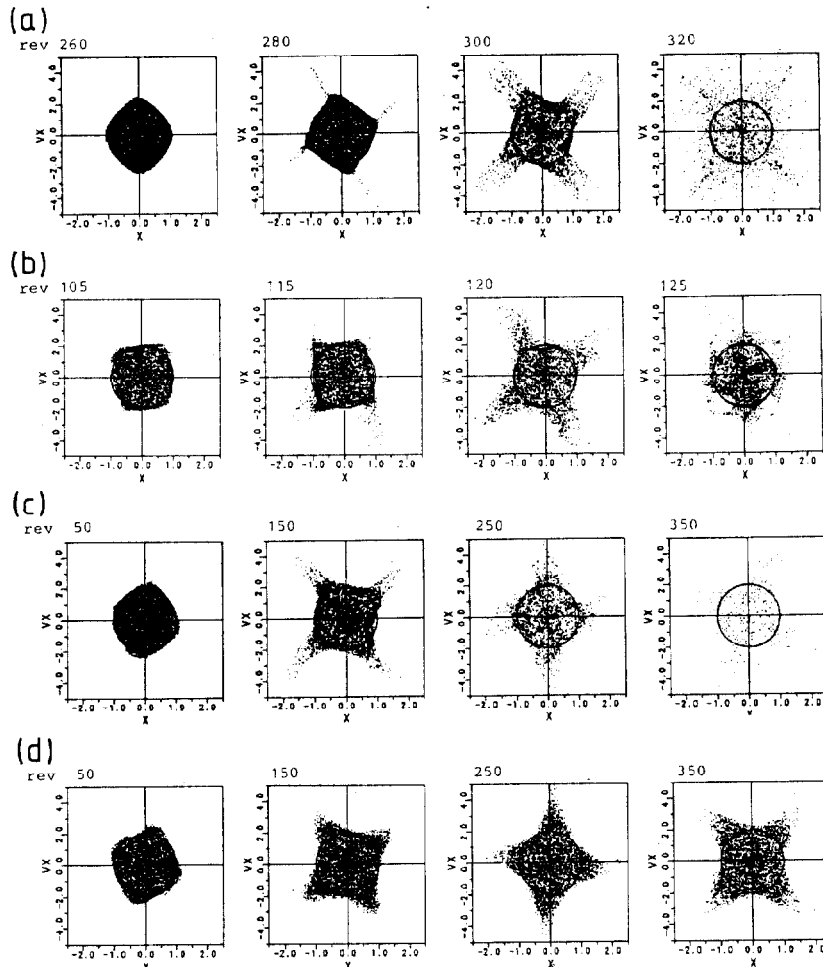


Fig. 3: Phase space projections for examples of Fig. 2 shown at different steps of revolutions.

A. Coherent Shift of Resonance

We choose $\nu_0 = 2.5$ (fixed) and allow for a linear decrease of the single particle tune ν by increasing the current. For ν crossing 2.25 we see no effect, but only for ν below 2.24. Hence, the upper edge of the stopband has been shifted to $\nu = 2.24$. This is explained by the fact - in analogy with the dipole mode - that it is the coherent frequency $\nu_{coh.}$ of multipole oscillations (determined by h_3), which must satisfy the resonance condition $4\nu_{coh.} = 9$. The difference $\nu_{coh.} - \nu$ is proportional to $\Delta\nu$; for the mode considered here it is about 10 % of $\Delta\nu$ (see Fig. 4).

B. Tune Locking at Resonance Crossing

In the next examples we choose $\Delta\nu = 0.05$ and cross the resonance by shifting ν_0 upwards or downwards, which gives a substantially different behaviour (in contrast with the crossing neglecting space charge, where there is no difference).

We first cross the resonance from below by shifting ν_0 from 2.25 to 2.35, hence ν approaches the stopband from below. The actual crossing shown in Fig. 2 and 3b is faster than in the spacechargeless case and considerably less harmful. We explain this by the observation that onset of some emittance growth pushes ν further upwards and thus resonance crossing is accelerated. For this to be effective it is important that

$$(4) \quad \Delta\nu \gg \delta\nu,$$

hence a relatively small increase of the emittance shifts ν above the narrow stopband $\delta\nu$, which suppresses further growth.

In the reverse case we shift ν_0 from 2.3 to 2.2 and thus allow ν to enter the stopband from above as shown in Fig. 2 and 3c. A small increase of emittance pushes ν upwards again. We have evaluated the actual ν (averaged) and find it is locked to the value at onset of resonance. As a consequence, all beam is lost as soon as ν_0 approaches 2.25. This locking of ν to the stopband is observed, if the working point ν_0 changes at a sufficiently slow rate determined by the parameter ε and the ratio $\delta\nu/\Delta\nu$.

Finally, we have also examined the case, where ν_0 is fixed ($= 2.3$) and ν is slowly depressed by increasing parametrically the charge carried per simulation particle to simulate the effect of slow bunching or - by analogy - of beam cooling (Fig. 2 and 3d). Again ν is locked to the stopband. The particle density decreases to the effect that the actual charge density remains constant, hence also $\Delta\nu$. In both cases the stopband (displaced by the coherent frequency shift) acts as a real barrier if approached from above (see Fig. 4).

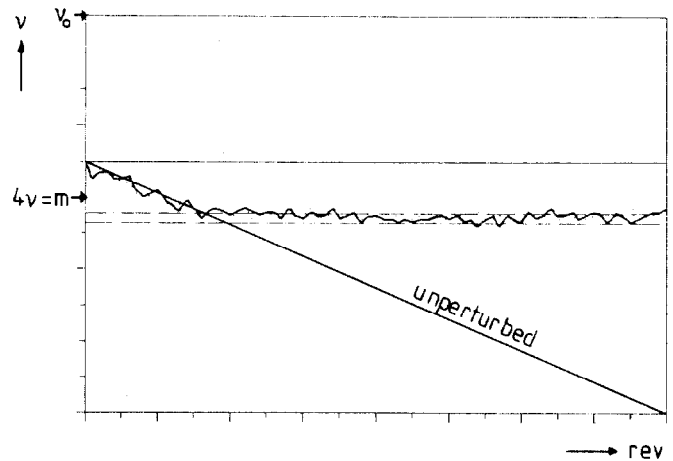


Fig. 4: Behaviour of ν (rms averaged) locked to stopband for ν_0 fixed and increasing current.

Conclusion

We have investigated self-consistent space charge effects for resonance crossing. Relatively strong perturbations, which would have a serious effect in the absence of space charge, have been considered. The observed tune locking at the stopband may be important for beam-beam nonlinear resonances in the presence of phase space cooling. It has to be examined in future work with much smaller perturbations, when the loss mechanism is dominated by trapping on outwards moving island structures. It is expected¹ that these structures are affected by nonlinear space charge. The code used here is probably not suitable for a large number of revolutions in view of the large CPU time it requires. It will be necessary also to ensure to what degree the code deviates from a strictly symplectic transformation, due to the errors involved in the orbit integration and space charge calculation.

References:

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