A NEW FFAG ORBIT CODE*

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Introduction

A new digital computer code has been written for studying motion in scaling fixed-field alternating-gradient synchrotrons (FFAG). The 2600 line code, written in Pascal, has been run on several computers, from the lap-size Sharp PC-5000 to a VAX 11/780. The azimuthal field profile over one sector is fit with a periodic array of up to 400 cubic splines. An eighth-order Runge Kutta integration method, suggested by R. Servranckx, is used. Resonant growth, or lack thereof, is shown by means of real time transverse phase space plots.

Motivation

The FFAG accelerator, which was so thoroughly studied at MURA two decades ago, now shows great promise as a moderate cost means of achieving great intensities of spallation-bred neutrons. For this reason, it is being studied both at Argonne National Laboratory and at the Kernforshungsanlage, Juelich. Although a great many isochronous cyclotron versions of the FFAG principle exist, no ion synchrotron versions have been built.

The MURA codes designed to study FFAG accelerators were written in machine language for the IBM 704 Computer and are thus not usable today. The code here described was written to fill the gap. Following the author's current passion, it was written in Pascal in order to be easily followed and operable on a great variety of computers. Most of the development was done on the author's Western Digital "Pascal Microengine", which is able to compile the entire code in two minutes.

Method

FFAG synchrotrons have much stronger nonlinearities than the usual alternating-gradient synchrotron. In particular, an adequate study of the stability limits in the vertical plane requires the inclusion of some nonlinear terms--at least those that yield tune shifts with amplitude in the vertical plane.

The methods used are basically those of the Oak Ridge National Laboratory cyclotron codes (e.g., #1482) as described by Welton [1]. These have been extended to include third-order terms in the axial direction in order to be able to determine the axial acceptance. The ORNL methods are very fast and yield the exact motion in the median plane.

In this FFAG code, six simultaneous equations are integrated to locate the closed orbit. Using the radial transfer matrix thus derived about a trial orbit, corrections are made and the process repeated until convergence is achieved. The error in closing is generally reduced by one to two orders of magnitude per iteration. Once the closed orbit is determined, four more equations are added in order to obtain the linear properties in the axial plane also. The extreme values of the Courant and Snyder amplitude functions are also determined.

Having the closed orbit, nonlinearities may be analyzed in several ways. A great number of orbits may be run through one sector to provide fitting data for a canonical transformation program [2]. Single orbits may be run in the radial plane (two equations) or both planes (four equations) with the axial contributions either linearized or extended to include third-order terms. One can thus determine (1) the absolute radial stability limit, (2) the radial limit for which small-amplitude motion in the axial plane remains small, and (3) the vertical stability limit. Unstable fixed points can be located directly.

Magnetic Field Representation

The code treats scaling fields that are free of error. The magnetic field in the median plane is completely determined by giving the field index, k, the spiral angle, $\mathcal{L}$, and the azimuthal profile of the magnetic field over one sector. The azimuthal profile is normally given as an irregular mesh. This mesh is fit with a series of (up to 400) cubic spline functions such that the profile and its first two azimuthal derivatives are continuous at each mesh point, and the splines are periodic. With the periodic condition dropped, the splines are determined by a Gaussian reduction algorithm that operates on a diagonally-dominant matrix whose non-zero elements can always be stored in an array of size 3n, where n is the number of mesh points [3]. The solution with the periodic condition imposed is determined by an iterative method that uses the reduced matrix to quickly converge on the desired solution. The azimuthal profile may also be given by means of Fourier harmonics; however, the spline representation is far faster.

Only the median field is needed to obtain the exact motion in the median plane, and only the first radial and first azimuthal derivatives are needed to obtain the linearized motion in both planes about the closed orbit. To include higher order terms in the axial motion, we expand the axial component of the magnetic field as a power series in z; however, because of the symmetry condition, only even terms can exist in this expansion:

$$B_z(r,\theta,z) = \sum_{j=0}^{\infty} z^{2j} B_{2j}(r,\theta).$$

Ignoring the current in the beam, the curl of the field vanishes, and thus we can obtain the radial and azimuthal components from the axial field:

$$B_\theta = \frac{1}{r}\sum_{j=0}^{\infty} \frac{1}{2j+1} z^{2j+1} \frac{\partial B_{2j}(r,\theta)}{\partial \theta};\quad B_r = \sum_{j=1}^{\infty} \frac{1}{2j+1} z^{2j+1} \frac{\partial B_{2j}(r,\theta)}{\partial r}$$

The vanishing of the divergence provides us with a recursion relationship sufficient to obtain all terms from the median-plane field:

$$B_{2j+2} = -\frac{1}{(2j+2)(2j+1)}\left\{\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right\} B_{2j}(r,\theta).$$

The median plane field for a scaling FFAG is given by: $B_z(r,\theta,0) = r^k f(\Psi)$,

where $\Psi = N\{\theta - \tan\zeta \ln r/r_0\}$, with $\mathcal{L}$ being the spiral angle. Note that we can write

$$B_z(r,\theta,z) = r^k \sum_{j=0}^{\infty} (z/r)^{2j}\, b_{2j}[\Psi(r,\theta)],$$

which shows the convergence of the series for the FFAG where z<<r. The first two terms are: $b_0 = f(\Psi)$;

$$b_2 = -\tfrac{1}{2}\{k(k-1)f(\Psi) + N\tan\zeta f'(\Psi) + N^2\sec^2\zeta\, f''(\Psi)\}.$$

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We do not need any more for motion including third-order terms of the axial motion (but still exact for the radial contributions). We see that we will need the first three azimuthal derivatives of the median plane field. The third derivative of the spline representation is not continuous, but we can make the error sufficiently small by choosing an adequately dense mesh.

Integration Method

The code uses an eighth-order Runge-Kutta Integration method by Shanks [4], suggested by R. Servranckx [5]. Although the method requires ten evaluations of the derivatives per integration step, the resulting eighth-order-polynomial fit is sufficient to allow very large steps for a given accuracy, and thus it is significantly faster than the usual fourth-order method. Recently, a variable step size has been provided. Also a transformation may be invoked in lieu of the integration for steps where the field and its azimuthal derivative are sufficiently small.

Output

The code determines the closed orbit, betatron frequencies, Courant and Snyder functions, and extreme values of the beta function and their location. For stability runs, coordinates -- both actual and normalized, Courant and Snyder invariants, and maximum values for the invariants with location are available. Parameter surveys provide single-line summaries for each system. Non-cyclic orbits (e.g., injection, extraction, stripped heavy ions) can be tracked. All output is oriented for use with plotting programs such as the KFA SNQ-ABT's IGM [6].

The code is menu-driven, but the menu is dynamically adjusted to contain only operations legal at the time.

Accuracy

The closed orbit and linear motion have been checked against simple machines where analytic solutions are available. The determinants are output; with double precision on the VAX 11/780, these differ from 1.0 by 1E-9 to as little as 1E-12; the error is less than 1E-6 with the Microengine. The accuracy of radial-plane aberrations was tested by following the locus of a circle displaced in both distance and angle relative to the closed orbit; this is an excellent test as the linear approximation of such a trajectory is poor indeed. In every case, the error was less than 1E-6 with single precision arithmetic. Published calculations and measurements for the MURA Mark V spiral model [7] were accurately duplicated.

Application to a Spiral-Sector FFAG

Extensive calculations have been done on an FFAG as a possible component in the German spallation neutron source (SNQ). The design is based on the ANL ASPUN design [8]. The azimuthal profile used was derived from a relaxation solution in the MURA transformation coordinates [9]; it is shown in Fig. 1. With this 20 sector, low flutter field, it was necessary to make the spiral angle 70.52 degrees with k=14.6 in order to obtain the desired tunes of 4.25 (r) and 3.25. The courant and Snyder amplitude functions are shown in Fig. 2.

Even though the high current to be accelerated requires a very large beam emittance, the radial stability limit as shown in Fig. 3, is an order of magnitude larger than that needed. In Fig. 4, a similar plot for a sharper fringing field (smaller spiral angle) is shown. The threshold in radial amplitude for vertical growth, obtained with one linearized axial equation (fast!), is almost exactly the same as that derived from the graphs of Parzen and Morton [10].

We show the effects of coupling in Fig. 5 (including non linear terms) and Fig. 6 (linearized vertical motion). The motion shown is unstable (the radial emittance is about 12 times that required for the SNQ), but it is interesting to see that the essentials of the motion are contained within the linearized vertical motion. This particular ring does not have an adequate vertical stability limit; however, a slight reduction in spiral angle, which can be achieved with a small increase in flutter, does provide an ample margin.

These studies to date are based on an error-free field and thus show only the effects of essential resonances. Further details on both the program and the FFAG studies are contained in a series of KFA SNQ-ABT (Abteilung Beschleuniger Technoligie) notes.

Future Extensions

Depending upon the availability of funding, it is planned to extend the code to include error fields, to extend to a 2-D mesh for the study of nonscaling machines, and to incorporate measured fields in three planes [11] when such become available.

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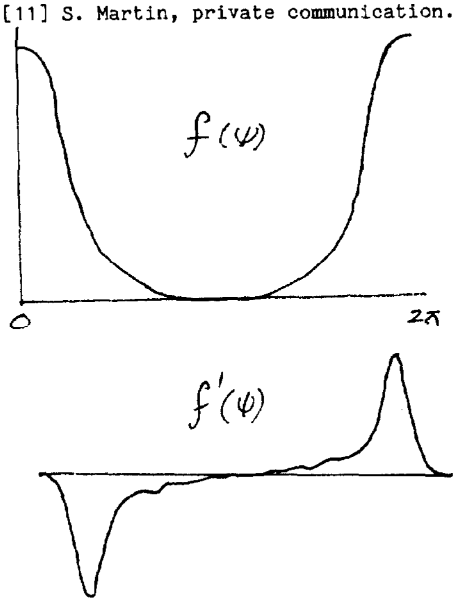

Fig. 1. Azimuthal field profile and azimuthal derivative.

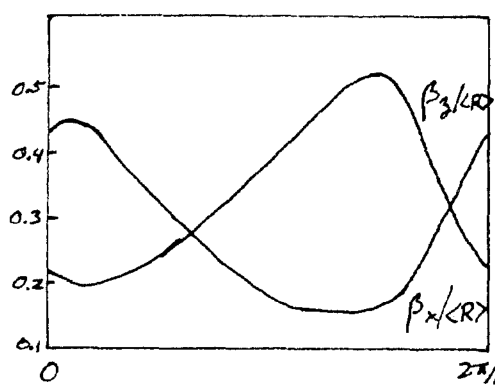

Fig. 2. Courant and Snyder amplitude function over one sector of spiral FFAG.

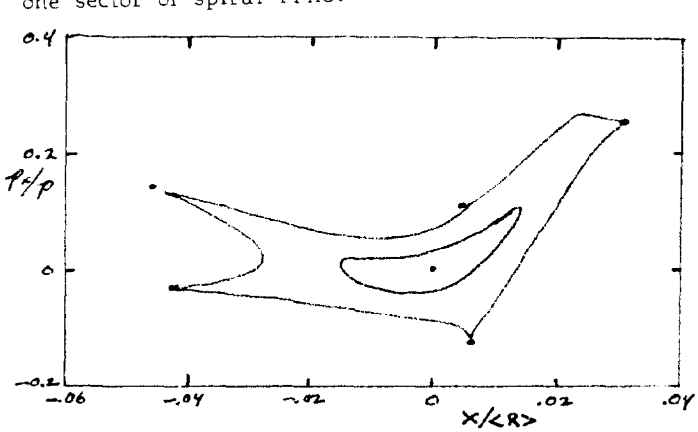

Fig. 3. Phase space contours showing radial stability limit for FFAG with field of Fig. 1.

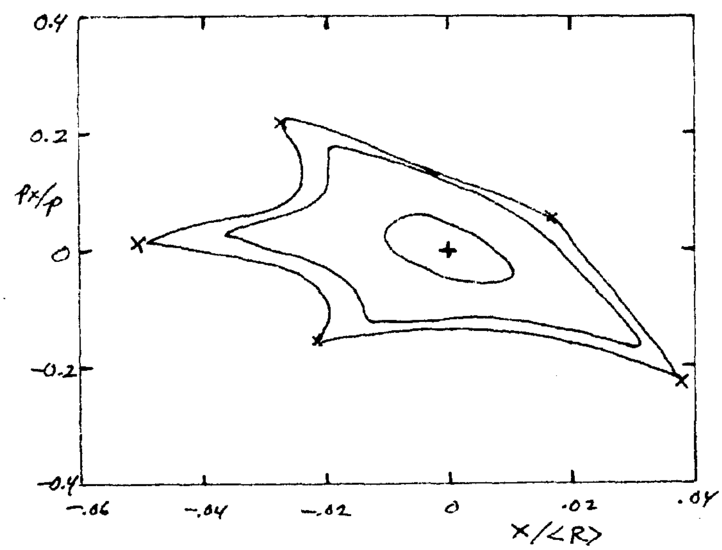

Fig. 4. Phase space contours showing radial stability limit for FFAG with sharper fringing fields.

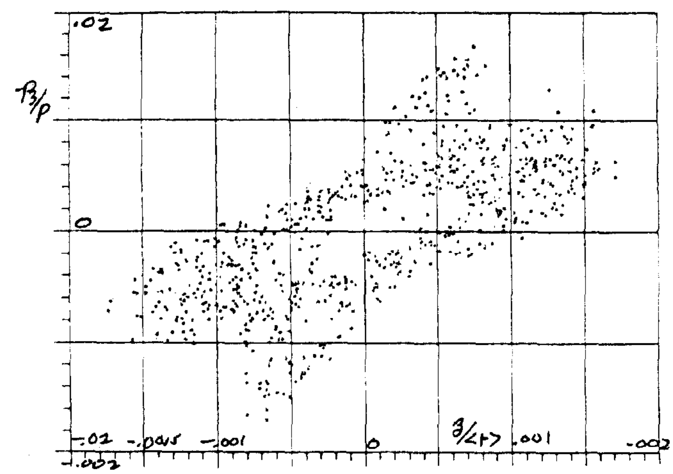

Fig. 5. Exact motion--Projection onto the axial plane. Plotted once per sector for 800+ sectors (motion is unstable).

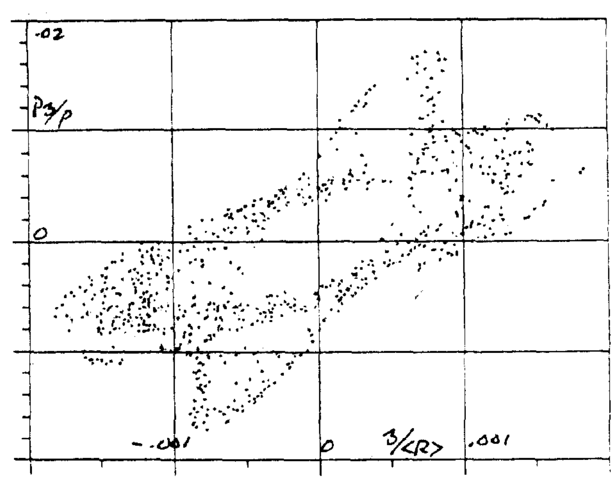

Fig. 6. Linearized axial motion--Projection onto the axial plane. Compare to Fig. 5.