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Limiting effects of the parasitic sextupole component in the CERN Large Hadron Collider.

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Introduction

A tracking program, written ad hoc, has been used to compute the dynamical aperture of the Large Hadron Collider in the LEP tunnel.

As expected, the most relevant limitation has been found to arise from the sextupolar component of the field error in the superconducting dipoles, essentially due to the persisting magnetisation current flowing through the superconducting wires, to the random variation of the diameter of the wires and to the mechanical misalignment of the coils.

The basic assumptions coded in our tracking program are the following:

1. The field error in each dipole is supposed to be purely sextupolar.
2. Two thin sextupoles, located at the centre of each half dipole, are used to represent the sextupolar deviation of the bending field.
3. The particles are tracked with no energy deviation.

These hypotheses simplify enormously the calculations, at the cost of a partial loss of generality. Indeed:

i) the problem of the simplecticity is solved

ii) the synchrotron motion can be neglected

iii) the description of the dispersion suppressor can be disregarded

iv) the matching of the interaction regions to the regular arcs can be avoided

v) the interaction region can be represented by a simple linear matrix.

Parameters of the model

The following parameters are assumed: [1]

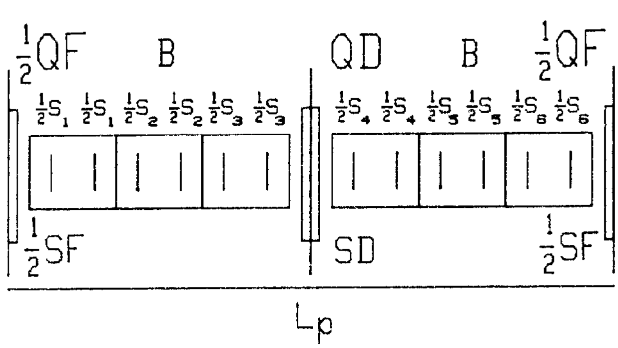

Fig. 1 Schematic representation of the periodic cell in the arcs
Si are the parasitic sextupoles in the dipoles in thin lens approximation.

TABLE 1:
Parameter of the tracking program

	1st Option	2nd Option
Period Length (L_p)	79 m	158 m
Phase advance per period (μ_p)	60°	90°
Number of dipoles per period	6	6
Bending radius	2700 m	2700 m
Maximum β	136 m	269 m
Minimum β	46 m	56 m
Number of insertions	8	8
Number of periods per octant	36	18

They correspond to the two main options of the LHC.

The basic period of the lattice is shown in Fig. 1. The usual sextupoles have been located in the middle of each quadrupole and have been represented by thin lenses.

The value of the tunes was fixed by choosing an appropriate value of the phase advance of the matrices representing the interaction regions.

Chromaticity and Parasitic Sextupole Strength

Table 2 shows the relative importance of the three different phenomena which have to be compensated by the sextupoles. SF and SD.

i) Chromaticity of the octants (low energy, detuned β).

ii) Chromaticity of the octants and of the low-β (high-energy, low-β).

iii) Chromaticity produced by the parasitic sextupoles in the dipoles of the LHC with an inner coil diameter of 35 mm.

TABLE 2

Sextupole strength in a period of the regular arc of the LHC

Effect to be compensated	Strength integrated in a period $\int_0^{L_p} \frac{B''}{B\rho}\,ds$	
	μ_p = 60°	μ_p = 90°
Chromaticity of the octants		
Horizontal	-0.01193 m^{-2}	-0.00385 m^{-2}
Vertical	$+0.01975$ m^{-2}	$+0.008034$ m^{-2}
Chromaticity of the octants and the insertions		
Horizontal	-0.03851 m^{-2}	-0.009564 m^{-2}
Vertical	$+0.05708$ m^{-2}	$+0.02349$ m^{-2}
Chromaticity produced by the parasitic sextupole in the bending		
Horizontal	-2.3106 m^{-2}	-1.9756 m^{-2}
Vertical	-3.3093 m^{-2}	-3.1813 m^{-2}

The usual chromaticity of the lattice and the chromatic effect of the parasitic sextupoles in the bending magnets were compensated by the appropriate excitation of the lumped sextupoles incorporated in the quadrupoles in the periods of the octant. The corrections were computed using MAD [2].

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Table 3 gives the gradient of the parasitic sextupolar deviation of the bending field expected in the dipoles of the LHC.

The following definitions are used:

$$h_{B3} = \frac{B''}{B_o \rho} \quad , \quad b_2 = \frac{1}{2} \frac{B''}{B_o} x^2$$

TABLE 3

Gradient of the parasitic sextupole in the dipoles of the LHC

Origin of the error	Φ_{coil} (mm)	Constant Sextupole component		Random sextupole component (r.m.s.)	
		h_{B_3} [m^{-3}]	b_2 at x = 1 cm	h_{B_3} [m^{-3}]	b_2 at x = 1 cm
Persisting currents at injection, ie B_o = 0.5 T	35	$1.8\ 10^{-2}$	$2.4\ 10^{-3}$	$9\ 10^{-4}$	$1.2\ 10^{-4}$
	50	$6.6\ 10^{-3}$	$0.9\ 10^{-3}$	$3.3\ 10^{-4}$	$0.45\ 10^{-4}$
Fabrication tolerances	35	-	-	$9\ 10^{-4}$	$1.2\ 10^{-4}$
	50	-	-	$3.3\ 10^{-4}$	$0.45\ 10^{-4}$

Results of the tracking computations

The initial conditions of the tracked particles have always been chosen along one unique direction in the four dimensional phase space of the transverse motion, viz.

$$x_o = (\beta_x \epsilon)^{1/2}, \quad x'_o = 0, \; z_o = (\beta_z \epsilon)^{1/2} \; , \; z'_o = 0$$

where ϵ is the emittance.

The starting emittance has then been varied from zero up to the crossing with the boundary of an assumed central island of stability.

The motion was supposed to be stable if the trajectory of the particle stayed inside the pipe for at least 100 revolutions.

In Fig. 2 we analysed many initial conditions corresponding to 100 different distributions of the random component of the parasitic sextupole.

Both options of the lattice period, i.e. μ_p = 60° and μ_p = 90°, have been considered for each value of the inner diameter of the bending coils, i.e. Φ_{coil} = 35 mm and Φ_{coil} = 50 mm.

The constant component h_{B3} and the r.m.s. deviation of the random components h_{B3} (random) of the parasitic sextupole in the dipoles were fixed to their expected values of Table 3. The r.m.s. value of the two random contributions have been composed quadratically.

The case of a local compensation of the constant sextupole in each bending, i.e. h_{B3} = 0, has also been considered.

All these results prove clearly the limiting effect of the random parasitic sextupole on the dynamical aperture. Indeed, the local compensation of the constant sextupole seems not very helpful in this respect, at least for the on-energy particles. However, other studies[1] proved the mandatory need of the local compensation to reduce the tune-spread of the off-energy particles, at least when Φ_{coil} = 35 mm.

Figure 3 shows results at high energy where the effect of the parasitic currents disappears, whereas the usual sextupoles are strengthened to compensate also the chromaticity of the low-ß insertion.

As a comparison with the results shown in Figs. 2 and 3, we have indicated in Table 4 the aperture limit for a machine with no parasitic sextupole, as determined by the chromatic correctors only.

TABLE 4

Φ_{coil}	Pipe radius (mm)	Compensation of the chromatic correctors	X_{LIM} (mm) for μ_p = 60°	X_{LIM} (mm) for μ_p 90°
35	15	Natural chromaticity	14.70	14.80
35	15	Natural chromaticity + low-ß insertion aberration	13.70	14.40
50	20	Natural chromaticity	19.47	19.60
50	20	Natural chromaticity + low-ß insertion aberration	17.20	18.90

It is interesting to know if a big statistical fluctuation in the strength of the random sextupole can strongly affect the aperture of the LHC. In fact due to the high number of dipoles in the LHC, about 2000, it is highly probable that some of them have a random component of the diffuse sextupole far apart from the average value, in the tails of the Gaussian distribution. These magnets could in practice be eliminated after magnetic measurements. However, this would hardly improve the situation: we have made simulations using a 2 * r.m.s. truncated Gaussian but the results were not different from those of Figs. 2 and 3.

Conclusion

Bearing in mind that the results of our simulation may be optimistic, one can assess that the dynamical aperture is small but possibly sufficient for the short period, 60° option of the LHC, and seem only marginally sufficient for the long period, 90° option.

REFERENCES

1) Large Hadron Collider : a feasibility study of possible options by the CERN Machine Groups DIR/TECH/84-01, May 1984.

2) The MAD program by C. Iselin, CERN LEP/TH/83-30.

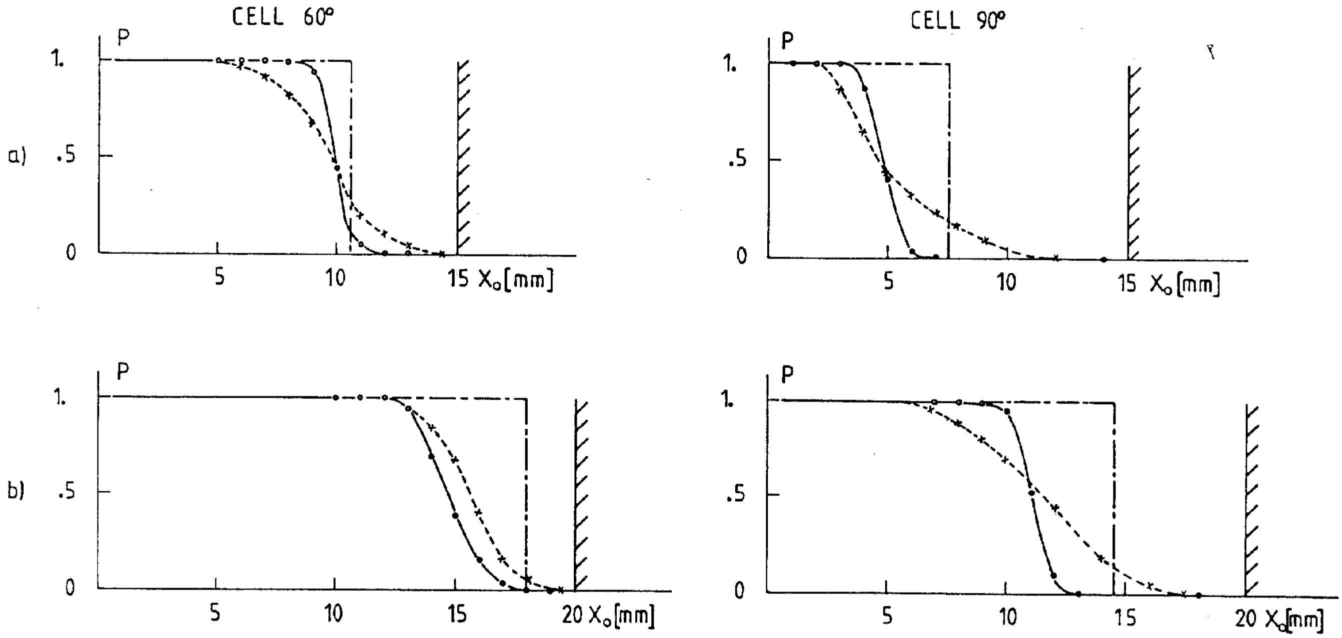

Fig.2 Probability P for a given particle of initial amplitude x_0 at β_{xmax} in the regular arc to survive 100 turns without hitting the vacuum chamber. (Statistics on 100 distributions)

a) Φ_{coil} = 35 mm

	h_{B3} [m^{-3}]	h_{B3}(random) [m^{-3}]
continuous line	1.8 10^{-2}	12.73 10^{-4}
dashed line	0	12.73 10^{-4}
dashed and dotted line	1.8 10^{-2}	0

b) Φ_{coil} = 50 mm

	h_{B3} [m^{-3}]	h_{B3}(random) [m^{-3}]
continuous line	6.6 10^{-3}	4.67 10^{-4}
dotted line	0	4.67 10^{-4}
dotted and dashed line	6.6 10^{-3}	0

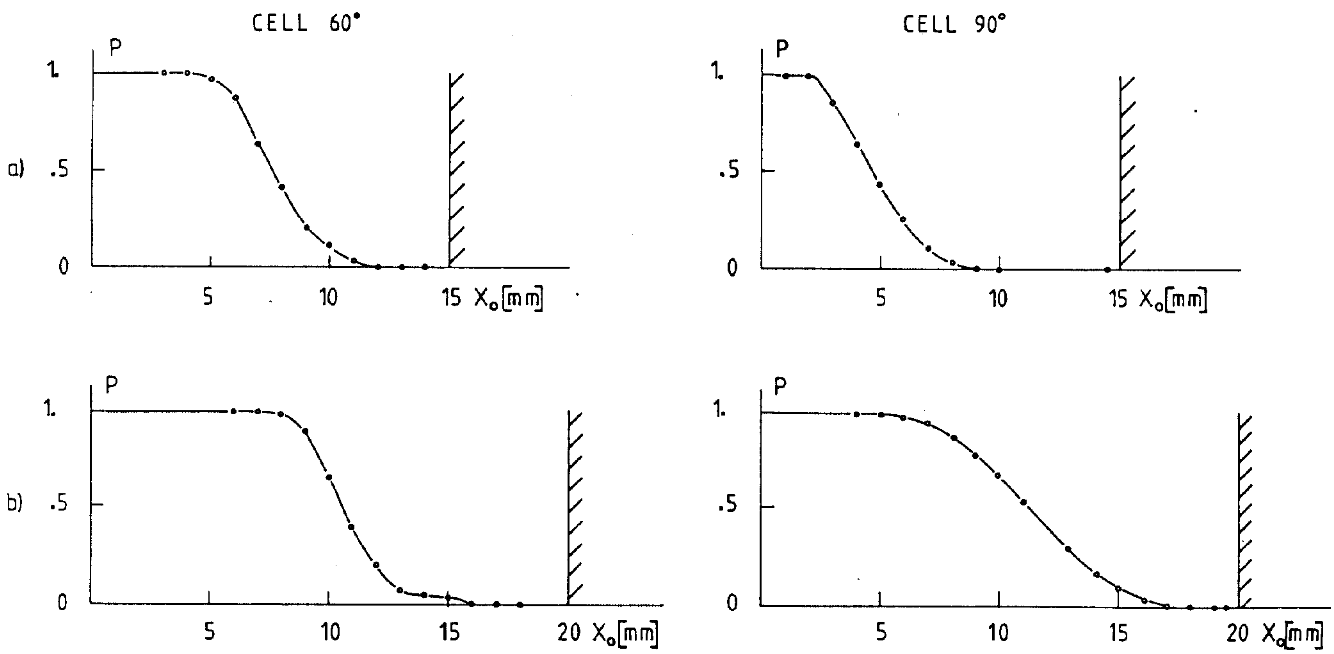

Fig.3. Probability P for a given particle of initial amplitude x_0 at β_{xmax} in the regular arc to survive 100 turns without hitting the vacuum chamber. (Statistics on 100 distributions).

h_{B3} = 0, compensation for natural + insertion chromaticity (high-energy simulation).

a) Φ_{coil} = 35 mm h_{B3}(random) = 12.73 $10^{-4} m^{-3}$

b) Φ_{coil} = 50 mm h_{B3}(random) 4.67 $10^{-4} m^{-3}$.