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TRANSVERSE MODE-COUPLING EXPERIMENT IN DCI

M.P. Level, P.C. Marin, E.M. Sommer, B. Zotter*), H. Zyngier

Laboratoire pour l'Utilisation du Rayonnement Electromagnétique
91405 Orsay Cédex, FRANCE

Abstract

Measurements of several head-tail modes as a function of the current in DCI are reported showing clearly the existence of negative mode-numbers. The comparison with the transverse mode-coupling theory has been done including bunch lengthening by combined potential well and longitudinal turbulence.

1. Experimental Observations of Head-Tail Modes

The experiments were mainly done at .8 and .7 GeV and with RF cavity voltages of 250, 350 and 500 kV. The RF frequency is 25,35 MHz and the energy loss per turn at 1 GeV is 28 keV.

We excite the beam transversally by radio frequency and we detect the signal on an electrode followed by a spectrum analyser. For weak current and weak chromaticity we observe only one betatron frequency. When the chromaticity is higher, satellites appear on each side of the initial frequency, they are separated by approximately the synchrotron frequency.

As the current increases, new satellites can appear and the amplitude of the first frequency decreases, some satellites may become more important than the initial frequency. The values of these frequencies vary and for large current two of them are very close to each other and cannot be distinguished. No instability was observed up to 300 mA.

The measurements are shown Fig. 1 for E = .8 GeV and V_{RF} = 350 kV. As a result, it became clear that these frequencies are head-tail modes and that positive but also negative mode numbers exist.

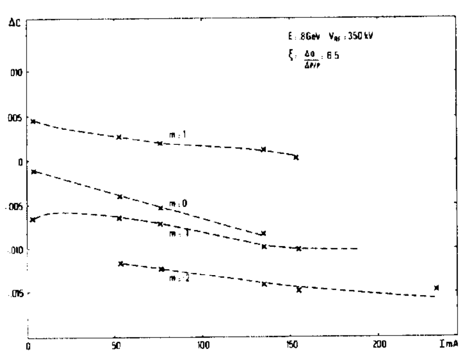

Fig. 1 : Head-tail modes frequencies versus current.

2. Theory of Head-Tail Modes [1] [2]

The basic theory is well developed in the literature. We give here only the most important points.

A single bunch can oscillate with different bunch shape modes m which depend on the relative phase with which particles along the bunch oscillate. For vanishing chromaticity the lowest such mode = 0 corresponds to a motion where all parts of the bunch are in phase. For the mode m = 1 the head of the bunch oscillates with a phase opposite to the one of the tail etc.

The power spectrum of these modes consists of lines $\omega_p = \omega_o(p + Q + mQ_s)$ and magnitude $h_m(\omega_p)$ with $-\infty < p < +\infty$, Q betatron tune, Q_s synchrotron tune and ω_o revolution frequency.

For electron bunches with Gaussian longitudinal distribution (rms length σ) the envelope $h_m(\omega)$ is well approximated by hermitian modes.

$$h_m(y) = \frac{1}{\Gamma(|m| + \frac{1}{2})} \, y^{2|m|} \, e^{-y^2} \; ; \; y = \frac{\omega\sigma}{c} \tag{1}$$

However, for a finite chromaticity ξ

$$\xi = \frac{dQ_y}{dp} \, \frac{p}{Q_y} \, ,$$

the whole mode spectrum is shifted by the chromatic frequency

$$\omega_\xi = \frac{\xi Q}{\alpha} \, \omega_o$$

(α momentum compaction) and becomes

$$h_m(\omega - \omega_\xi) = \frac{1}{\Gamma(|m| + \frac{1}{2})} \, (y - y_\xi)^{2|m|} e^{-(y - y_\xi)^2} \tag{2}$$

with

$$y_\xi = \frac{\omega_\xi \sigma}{c}$$

The part played by the beam surrounding in its interaction with the beam itself can be described by a frequency dependent impedance containing a resistive (real) and a reactive (imaginary) part.

*) CERN, LEP Division, CH-1211 GENEVA 23.

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The longitudinal impedance is defined as

$$Z_L = \frac{\int_0^{2\pi R} E_L \, ds}{I_o}$$

where I_o is the beam current and E_L the longitudinal component of the electric field,

and the transverse impedance as

$$Z_T = j \frac{\int_0^{2\pi R} (E + v\times B)_T}{I_o \Delta y} \, ds$$

where Δy is the displacement from the axis of the beam.

The fields and the current are taken at a particular frequency ω.

For elements with nearly circular symmetry, there exists an approximate relation between longitudinal and transverse impedance

$$Z_T \simeq \frac{2c}{b^2} \frac{Z_L(\omega)}{\omega} \tag{3}$$

where b is the effective radius of the vacuum chamber. Fig. 2 shows the power spectrum of a Gaussian bunch σ = 500 ps and the impedances with the following parameters Q = 1 and $\omega_r/2\pi$ = 1.3 GHz.

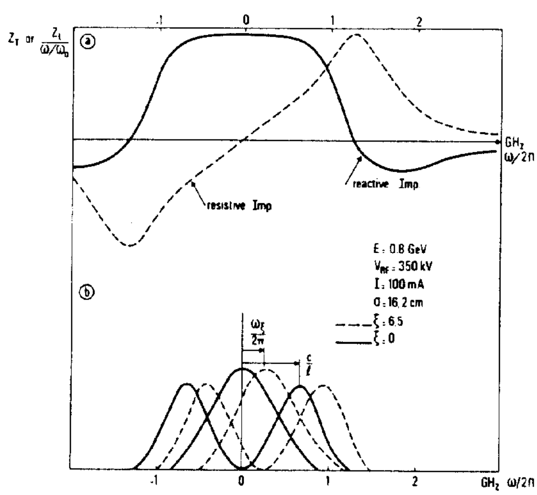

Fig. 2 : a) Impedance of a resonator Q = 1, $\omega/2\pi$ = 1.3 GHz

b) Envelope of the power spectrum $h_m(\omega)$

The result of the interaction between a mode of bunch oscillation and the beam surrounding can be expressed as a shift $\Delta\omega_m$ of the oscillation frequency from its undisturbed value.

$$I_m(t) = \hat{I}_m e^{j\omega_p t} = \hat{I} e^{j(\omega_{po} + \Delta\omega_m)t}$$

A frequency shift $\Delta\omega_m$ which is real leads only to a change of the oscillation frequency while a negative imaginary value of $\Delta\omega_m$ leads to an exponentially growing oscillation with growth rate

$$\frac{1}{\tau} = j\Delta\omega_m$$

For rather low current the different modes are uncoupled. In this case, the broad band impedance does not lead to instability if the chromaticity is $\geqslant 0$. The real frequency shifts of the head-tail modes are given by [1] :

$$\Delta\omega = \frac{1}{|m| + 1} \; \frac{e I_o \beta_y c}{2\ell E} \; \frac{\sum_p \mathcal{I}\{Z_T(\omega_p) \, h_m(\omega_p - \omega_\xi)\}}{\sum_p h_m(\omega_p - \omega_\xi)} \tag{4}$$

where β_y is the vertical β function at the impedance location and ℓ the bunch length with $\ell = 4\sigma$.

When the current increases, adjacent mode frequencies are shifted towards each other until they merge and become complex leading to an unstable oscillation.

Following the theory of mode coupling [3] we consider two bunch shape mode numbers m and m' which might be coupled. Then the coherent frequencies are determined by the infinite determinant

$$\det\{\omega - (Q + mQ_s)\omega_o - M_{mm'}\} = 0 \tag{5}$$

where

$$M_{mm'} = \frac{j e \omega_o I_o \beta_y}{(|m| + 1)4\pi E} \sum_p Z_T(\omega_p) \, h_{mm'}(\omega_p - \omega_\xi)$$

with $h_{mm'}$ being the normalized cross power density

$$h_{mm'} = j^{|m|-|m'|} \frac{2\sqrt{2\pi}}{3\sqrt{\Gamma(|m|+\frac{1}{2}) \, \Gamma(|m'|+\frac{1}{2})}} \, y^{|m|+|m'|} \, e^{-y^2}$$

3. Comparison of the Experimental Results with Theory

3.1. Excitation by an external radio frequency

The response to an external excitation depends on the amplitude of the mode at this frequency and on the integral $\sum_p \mathcal{R}\{Z(\omega_p) \, h_m(\omega_p - \omega_\xi)\}$.

Let us consider Fig.2 : when the chromaticity is very weak the amplitude of mode 0 at f $\simeq$. 6 MHz is large while that of mode 1 is nearly zero. To obtain a large response on the mode 1 and test the model we have shifted the modes by changing the chromaticity.

Fig. 3 shows the power spectrum for different chromaticities and different intensities of the beam for the conditions E = .7 GeV and V_{RF} = 100 kV.

For I = 1 mA (σ = 9 cm) and $\xi \leqslant 1$ the response is zero for m = ± 1. When ξ = 6.5 the response to m = ± 1 mode increases slightly but remains small.

For I = 300 mA (σ = 35.5 cm) and ξ = 6.5 the response is very large for the m = ± 1 mode and almost zero for the m = 0 mode.

These considerations explain the experimental observations described in the introduction and the behavior shown Fig. 4.

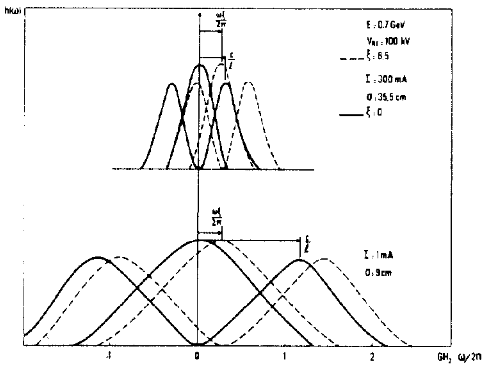

Fig. 3 : Power spectrum for different intensities and chromaticities at E = .7 GeV.

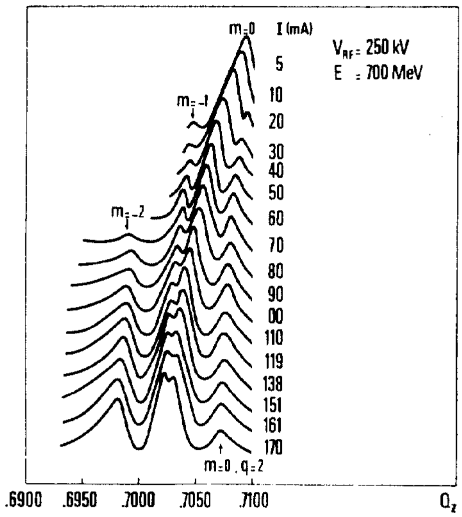

Fig. 4 : Relative amplitudes of the response to the excitation versus current (spectrum analyser).

Unfortunately the lifetime becomes very bad for high current and high chromaticity, especially at low energy, due to a high order non linear term of sextupolar fields. For this reason, at E : .7 GeV, $\xi = \frac{\Delta P}{\Delta p/p}$ was chosen equal to 2.6, a rather low value which explains that at low current (below 20 mA) one single peak appears (Fig. 4).

On the other hand, when one increases the chromaticity all the frequencies vary by translation (Q depends on sextupolar fields) but the frequency difference beetween the adjacent modes does not change.

This is also explained by the diagram in Fig.2, it can be seen that the reactive impedance remains constant over the frequency range.

3.2 Comparison between calculated and measured frequency shifts

To make this comparison we have to take into account the bunch lengthening with current.

Measurements of the dependance of energy spread and bunch length on beam current have been checked [4] against the combined potential well and turbulent lengthening models.

With the broad band resonator model, Q = 1 and $\omega_r/2\pi$ = 1.3 GHz, the results lead to a longitudinal impedance (zero frequency) value $|Z/\omega/\omega_o|_o$ of 7.3 Ω which fits very well with the measurements at different energies and RF voltage values.

From measurements of the first head-tail mode for long bunches we otherwise deduced a value of transversal impedance $Z_T = 4 \times 10^5\ \Omega/m$*)

We can now introduce these parameters into equation (5) to calculate the different head-tail modes and the threshold of the transverse instability due to mode coupling.

The calculations were made by the computer program BBI. It appears that the instability is predicted for 200 mA by a coupling of modes - 2 and - 3.

It is approximately the maximum current we can store with a good lifetime at low energy and we checked the calculation and theory on Fig.5 .

The agreement is not too bad but, on the one hand, the modes 0 and - 1 are closer than the prediction and, on the other hand, there is no sign of instability on the machine up to 300 mA.

A new formulation especially done to explain the behavior of long bunches in SPS [5] shows that two modes can be coupled at a certain intensity and uncoupled at slightly higher intensity. In such cases the growth rate is small and the instability is cancelled by radiation damping.

These results describe qualitatively very well the behavior of the beam in DCI and we plan to check them numerically very soon.

In conclusion, the experiences on DCI show clearly the existence of negative mode-numbers and we expect that the agreement with the transverse mode coupling theory can be still improved.

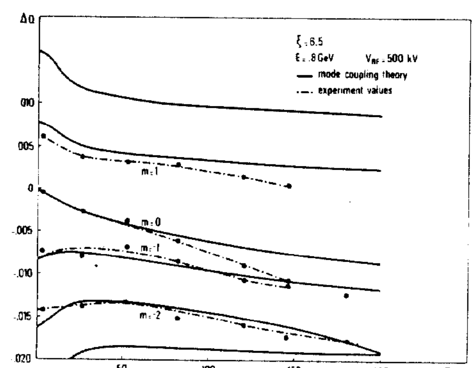

Fig. 5 :Head-tail frequencies versus current E = .8 GeV and V_{RF} = 500 kV.

References

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[5] Yong-Ho CHIN, CERN/SPS/85-2 (DI-MST)

*) In reference [4] Z_T = 2.8. This discrepancy is due to a different form factor $\ell = 2\sqrt{2}\,\sigma$ instead of $\ell = 4\,\sigma$.