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THE ISR IMPEDANCE BETWEEN 40 KHZ AND 40 GHZ

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Abstract

Different current dependent effects have been measured in the ISR over the years and used to obtain information on the impedance of the beam surroundings over a large frequency range. The transverse impedance at low frequencies $f < 20$ MHz was obtained from the response of the beam to an excitation (beam transfer function) and at medium frequencies $30 < f < 200$ MHz from the growth rate of the head tail instability. Potential well bunch lengthening and synchrotron frequency shifts gave the longitudinal reactive impedance over low and medium frequencies while the observation of the microwave instability (turbulent bunch lengthening) gave its resistive part at high frequencies $f \sim 1$ GHz. The measured energy loss of an unbunched beam gave an integral over the resistive impedance up to frequencies of 7 to 70 GHz depending on the beam energy. To analyse all the measured data the approximate relation between transverse and longitudinal impedance was used and a model impedance was fitted which has the expected behavior at low, intermediate and very high frequencies where the impedance is dominated by the smooth resistive wall, the effect of higher frequency resonances and by diffraction respectively.

Introduction

Ideally the resistive and reactive longitudinal and transverse impedance is measured for a sufficient number of frequencies such that the impedance curve can be drawn over the range of interest. In praxis only one part (e.g. the resistive transverse impedance) can be measured at some frequency and another part at a different frequency. To relate these measurements and to obtain a picture of the whole impedance, some general properties have to be implied.

The impedance $Z(\omega)$ has a real (resistive) Z_R and an imaginary (reactive) Z_I part. It is the Fourier transform of the wake potential G(t) which is the longitudinal field integrated over one turn seen by a probe particle following a source particle at a time distance t. For the ultra relativistic case there is no field in front of the source particle, therefore G(t)=0 for $t < 0$. The impedance is

$$Z(\omega) = \int_0^\infty G(t)\, exp(-j\omega t)\, dt \qquad (1)$$

with $Z(-\omega) = Z^*(\omega)$. At $\omega = 0$ the reactive impedance vanishes due to the above symmetry relation and the resistive part vanishes because electrostatic fields cannot result in a net energy loss over a closed turn.

In a storage ring with a smooth wall with conductivity κ ($7.7\ 10^5\Omega^{-1}m^{-1}$ for the ISR) the impedance is given by the skin effect. For a storage ring with radius R having a circular vacuum pipe of inner radius b this impedance is

$$Z_{rw}(\omega) = (1+j)\frac{R}{b}\sqrt{\frac{\mu_o\omega}{2\kappa}} \qquad (2)$$

This expression is an approximation valid between the lower frequency for which the skin depth is about equal to the chamber thickness (~40 kHz for the ISR) and a certain upper frequency (~ 20 GHz for the ISR). The resistive wall impedance is usually dominant at low frequencies. At intermediate frequencies ($30 < f < 200$ Mhz) the impedance is dominated by the low frequency tail of parasitic resonances due to aperture changes, bellows and accidental cavities. These elements have resonances at high frequencies (0.5 to few GHz) which usually can not be resolved by measurements. Here the situation can be approximated by a broad maximum. Each resonance is characterized by a resonant frequency ω_r, a shunt impedance R_s and a quality factor Q and has an impedance:

$$Z(\omega) = \frac{R_s}{Q^2}\,\frac{\omega^2\omega_r^2 - j\,Q\omega\omega_r(\omega^2-\omega_r^2)}{\omega^4 - 2\omega^2\omega_r^2(1-1/(2Q^2)) + \omega_r^4} \qquad (3)$$

At very low frequency this impedance can be approximated by $Z(\omega) \sim R_s\left((\omega/\omega_r)^2 + j\,Q\omega/\omega_r\right)$ In many current dependent effects the impedance divided by the mode number $n = \omega/\omega_o$ ($\omega_o = revolution frequency$) is a more relevant parameter $Z(\omega)/n$. At very high frequencies where wave propagation in the vacuum chamber becomes possible, the impedance is determined by diffraction on the aperture changes. It can be described by the Sessler-Vainshtein[1,2,3] model used here in a simplified form[4]. The real part of the very high frequency impedance is approximately

$$Z_R(\omega) \doteq R_{sv}\frac{\sqrt{\omega/\omega_{sv}}+1}{\left(\omega/\omega_{sv} + 2\sqrt{\omega/\omega_{sv}} + 2\right)^2} \qquad (4)$$

Here $\omega_{sv} = c\sqrt{gd}/(4\,b^2)$ where b is the inside radius of the pipe, g the typical length of the accidental cavities and d the distance between them, c is the speed of light. In the ISR the aperture changes are rather irregular and the above model can only be applied as an approximation; the expected values for $\omega_{sv}/(2\pi)$ lie between 5 and 12 GHz. The impedance factor is $R_{sv} = N\,650\,Ohm$ where N is the number of accidental cavities ($N \sim 100$ to 200 for the ISR). At very high frequencies this impedance goes like $\omega^{-3/2}$.

The transverse impedance $Z_T(\omega)$ has an approximate relation to the longitudinal one

$$Z_T(\omega) = \frac{2R}{b^2}\frac{Z(\omega)}{n} \qquad (5)$$

which we will use to relate longitudinal and transverse measurements. Since this relation holds for circular chambers while the ISR has partly an elliptic chamber we have to estimate an effective radius b to use the above relation. From the geometry of the chamber we get $b \sim 32\,mm$.

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Measurements

Most impedance measurements have been reported before and only a short description of the different methods are given here.

Transv. low freq. impedance from the beam transfer function

In this experiment an unbunched beam is excited transversely with a frequency being swept through a betatron side band. The response of the beam is measured with a position monitor and compared in amplitude and phase with the excitation by a network analyzer. The result is the so-called beam transfer function. In the absence of an impedance a plot of the inverse beam response would give the stability diagram. If an impedance is present this diagram is shifted by a vector which is proportional to the impedance. The resistive part of this impedance can be obtained from a single experiment. However, since the position of the undisturbed stability diagram is in general not known, the reactive impedance can only be obtained from at least two measurements carried out with different intensities or different frequency spreads in the beam. The latter has been achieved for unbunched beams by changing the chromaticity[5,6].

Transv. medium freq. impedance from the head-tail effect

In this case a bunch is injected into the ISR having a negative chromaticity. This leads to head-tail instability of the lowest mode of which the growth rate is measured as a function of current. This rate is proportional to an integral over the resistive impedance times the bunch spectrum. The latter is directly observed during the development of the instability. The measurement gives the resistive transverse impedance averaged over this bunch spectrum. As a further measurement a positive chromaticity is used and the now stable head-tail mode is excited and its frequency measured as a function of intensity. This gives the reactive impedance again averaged over the bunch spectrum[7,8,9].

Long. medium freq. impedance from potential well effect

In the medium frequency range the longitudinal impedance is mainly inductive. The fields induced by the beam in the wall tend to reduce the slope of the external RF-voltage. This leads to reduction of the incoherent synchrotron frequency and in connection to an increase of the bunch length. The incoherent frequency cannot be observed directly but the inductive wall also reduces the quadrupole mode synchrotron frequency which can be measured quite accurately. The measurement gives the reactive impedance averaged over the quadrupole mode spectrum which peaks around 50 MHz. The measurement of the bunch length as a function of current gives an integral over the bunch spectrum which extends to $\sim$ 40 MHz. The observation of the inductive wall effect gives therefore the impedance somewhere between 0 and 60 MHz[10].

Long. high freq. impedance from turbulence

At larger bunch current turbulent bunch lengthening can be observed in the ISR. It is characterized by an increase of the momentum spread and with the occurrence of high frequency signals during its development. It is therefore also called micro wave instability. By measuring the bunch length for large bunch current and subtracting the inductive wall effect the turbulence can be quantified and observing the high frequency signals gives an estimate of the frequency range over which the impedance is relevant for this effect. Since there is no easy to use exact theory of turbulence the so-called coasting beam model has been used to analyse the data. From this we estimate an impedance which is mostly resistive and extends between 0.3 and 1.8 GHz with an average value of $Z/n \sim 14$ Ohm. This measurement has a large error in the impedance as well as in the frequency range which was limited by instrumentation at its upper limit[10,11].

High freq. impedance from the coasting beam energy loss

In the ISR it is observed that an unbunched (coasting) beam looses energy such that the orbit moves to the inside at a slow but clearly observable rate of 0.5 to 1 mm per day. The effect is due to the wall current induced by the Schottky noise of the beam. Since this is a statistical phenomenon the energy loss is independent of the beam intensity. There is also an energy loss due to synchrotron radiation which is calculated to be small and subtracted from the measurements. To analyse this parasitic mode energy loss of the coasting beam we consider the electric field of a single proton with Lorentz factor γ going through a circular vacuum chamber of radius b. The electric field is mainly perpendicular to the direction of motion with an opening angle of about $1/\gamma$. The form of the induced wall current pulse is calculated for the boundary condition of perpendicular field lines at the wall. The expression in time domain is relatively complicated but the rms. width of the pulse is simply $\sigma = b/(\sqrt{2}\gamma\beta c)$. The spectrum of the wall current is also relatively simple and can be expressed by the modified Bessel function $I_o(x)$[12]

$$\widetilde{I}_w(\omega) = \frac{-e}{\sqrt{2\pi}\, I_o(\frac{b\omega}{\gamma\beta c})} \tag{6}$$

The energy loss U per unit charge e is usually called parasitic mode loss parameter $k = U/e^2$ and is therefore given by[13]

$$k = \frac{2}{e^2} \int_0^\infty \widetilde{I}_w^2(\omega)\, Z_R(\omega)\, d\omega \tag{7}$$

From the measured energy loss at different energies we obtain the parasitic mode loss factor k for different pulse length σ, fig.1. Each such measurement represents an integral over impedance times the square of the wall current spectrum. These spectra are identical at low frequencies but have different cut-off frequencies where they decay. The frequency where this power spectrum reaches the half value is $\omega_{1/2} \sim 1.23\gamma\beta c/b$. By subtracting two measurements we get approximately the average impedance between the two $\omega_{1/2}$ which gives two values of $Z_R = 12\,k\Omega$ at $\omega/(2\pi) \sim 18.5$ GHz and $Z_R = 5.1\,k\Omega$ at $\omega/(2\pi) \sim 51$ GHz.

Analysis of the Measurements

We start by analyzing the measured transverse impedances at low and medium frequencies plotted in fig.2. In this frequency range we expect the impedance to be dominated by the resistive wall effect (2) and by the low frequency tail of the resonances (3). Using the relation (5) between longitudinal and transverse impedance and taking only up to order 2 in ω of (3) we try to fit the resistive impedance measurement by $Z_{TR}(\omega) = A/\sqrt{\omega} + C\omega$ and the reactive one by $Z_{TI}(\omega) = A/\sqrt{(\omega)} + B$. From the resistive measurements, which have smaller errors, we determine the effective chamber radius b = 28.2 mm since all other parameters in (2) and (5) are known. This value is slightly lower than the expected value of about 32 mm; we assume from now on a value of b = 30 mm which lies in between measurement and expectation. With this value we make a fit through the reactive measurements and find for the parameter $B = 7\,M\Omega/m$. Translating this value into a longitudinal impedance with (5) we find for the medium frequency range $Z_I/n \sim 21\,\Omega$. From the inductive wall effect measurement we find for the corresponding impedance 26 Ohm. Taking the average between the two we get for the longitudinal reactive impedance at medium frequencies $Z_I/n = 23.5\,\Omega$.

From the measurement of the parasitic mode loss of a coasting beam we have two longitudinal resistive impedance points at very high frequencies. We fit the Sessler-Vainshtein impedance (4) through them and determine the two free parameters; we get $\omega_{sv}/2\pi = 7.78$ GHz and $R_{sv} = 318\,k\Omega$ which are not too far from the rather vague expectations for the ISR.

Impedance Model

The different groups of impedance measurements are rather widely spaced and we need an impedance model to connect them. We separate first the resistive wall impedance (2) for which we know now all the parameters. We write it in the convenient form

$$Z_{rw}(\omega) = (1+j)\, R_{rw}\sqrt{\omega/\omega_o} \tag{8}$$

with $R_{rw} = 6.38\,\Omega$ for our parameters in (2) and (5), R=150 m, b=30 mm and $\kappa = 7.69\,10^5\,\Omega^{-1}\,m^{-1}$. For the impedance due to aperture changes, bellows, accidental cavities ets. we expect the impedance to be proportional to ω^2 at low frequencies. For the very high frequencies we have the Sessler-Vainshtein model which eventually goes like $\omega^{-3/2}$; however this model goes to a finite value al low frequencies. We try therefore to create a model impedance which has the correct behavior at low and at very high frequency and has a broad maximum in between. It should also have a relatively simple analytical expressions for the real and imaginary part as well as for the wake potential. We come up with the following impedance

$$Z = R_m \left[\left(\sqrt{\frac{2}{1+(\frac{\omega}{\omega_2})^2}} \frac{1}{\sqrt{\sqrt{1+(\frac{\omega}{\omega_2})^2}+1}} - \frac{1}{1+(\frac{\omega}{\omega_1})^2} \right) - j \left(\frac{2}{(\frac{\omega}{\omega_2})} - \sqrt{\frac{2}{1+(\frac{\omega}{\omega_2})^2}} \frac{1}{\sqrt{\sqrt{1+(\frac{\omega}{\omega_2})^2}-1}} - \frac{\omega/\omega_1}{1+(\frac{\omega}{\omega_1})^2} \right) \right] \tag{9}$$

which has a wake potential:

$$G(t) = 2\, R_m \omega_2 \left(\mathrm{erfc}(\sqrt{\omega_2 t}) - \frac{\omega_1}{2\omega_2} \exp(-\omega_1 t) \right) \tag{10}$$

which contains the error function erfc(x).

The above impedance (9) with the resistive wall impedance (8) is fitted through all the measured data (fig.3) including the integrals (7) as shown on fig.1. We get for the parameters in the model impedance $R_m = 282\,k\Omega$, $\omega_1/(2\pi) = 1.91\ GHz$, $\omega_2 = 2.8\,GHz$.

References:

1) A.M. Sessler, private communication.
2) L.A. Vainshtein, Sov. Phys. JETP (1963) p 703.
3) E. Keil, Nucl. Instr. and Methods 100 (1972) p 419.
4) D. Brandt, B. Zotter, CERN Lep note 388 (1982).
5) A. Hofmann, B. Zotter, IEEE Trans. on Nucl. Sci. NS 24-3 1487 (1977).
6) J. Borer et al. IEEE Trans. on Nucl. Sci. NS 26-3 3405 (1979).
7) J. Gareyte, F. Sacherer, Proc. 9th Int. Conf. High Energy Accel. , SLAC 1974 p 341.
8) G. Fischer, A. Hofmann, CERN ISR Perf. Rep. April 1975.
9) A. Hofmann, F. Sacherer, CERN ISR Perf. Rep. Feb. 1978.
10) S. Hansen et al., IEEE Trans. on Nucl. Sci. NS 22-3 1381 (1975).
11) B. Bramham et al., IEEE Trans. on Nucl. Sci. NS 24-3 1490 (1977).
12) A. Hofmann, T. Risselada, IEEE Trans. on Nucl Sci. NS 30-4 2400 (1983).
13) P.B. Wilson, J.B. Styles, K.L.F. Bane, IEEE Trans. on Nucl. Sci. NS 24-3 1496 (1977).

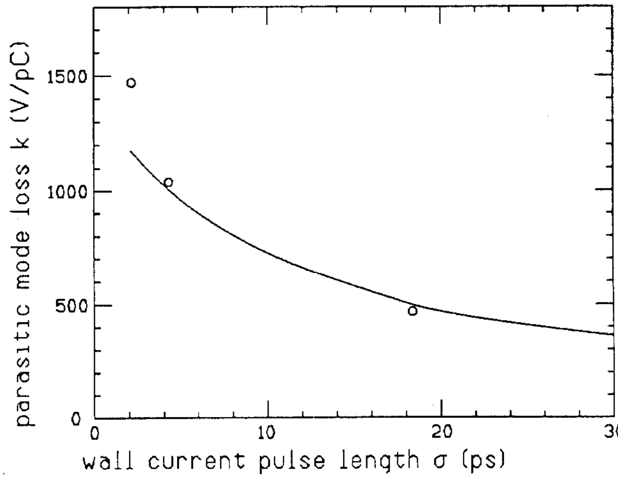

Fig.1: Parasitic mode loss factor k vs. pulse length σ.

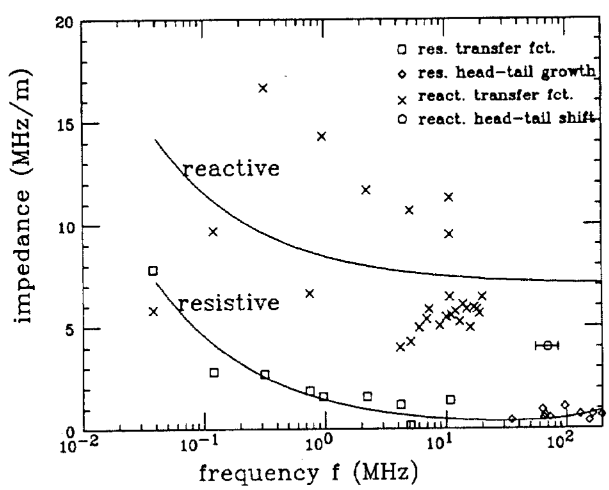

Fig. 2: Low and medium frequency data with fit.

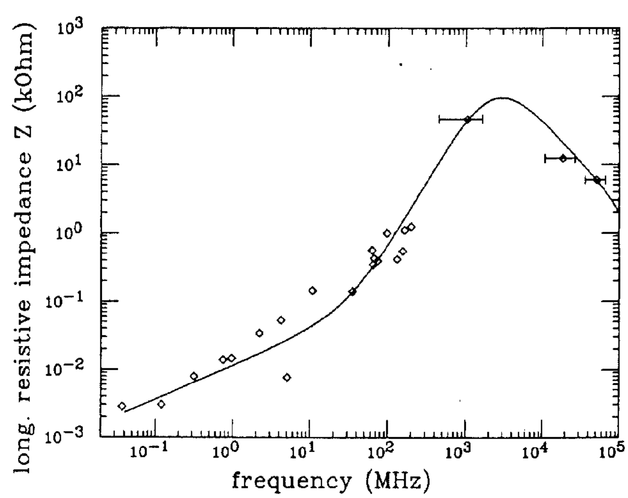

Fig. 3: Resitive model impedance fitted through the data.