TRANSVERSE INSTABILITIES DUE TO WALL IMPEDANCES IN STORAGE RINGS

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Abstract

RF voltages are induced by charged particle beams in the impedances of surrounding metallic or dielectric structures. These voltages may reinforce perturbations of the charge distribution and cause instabilities of a beam. Transverse oscillations are particularly dangerous, as the usually small transverse apertures of the vacuum chamber often lead to loss of part or all of the particle beam. In this review, we limit ourselves to a transverse instability of single bunches which has been observed rather recently in large electron positron storage rings and which has become known as transverse turbulence, fast head-tail effect or transverse mode-coupling instability.

1. Introduction

In particle storage rings the beam current - and hence the luminosity - is often limited by coherent instabilities. These instabilities can occur either in the longitudinal or in a transverse direction, but the latter are usually more dangerous since they may lead to instantaneous particle loss, while longitudinal ones often only cause bunch lengthening or an increase of the loss-rate. Here we shall restrict ourselves to transverse instabilities of bunched beams since coasting beams have been studied extensively in the past [1]. Furthermore, we shall exclude coupled bunch oscillations of rigid bunches [2] and the head-tail effect [3], for which the description has been unified [4] and become established knowledge by now.

The latest generation of large electron-positron storage rings has been plagued by another transverse instability of single bunches which does not depend strongly on chromaticity as the familiar head-tail effect. It has been observed first in SPEAR [5], but it became really troublesome only in the much bigger storage ring PETRA where it limited the injection current severely [6]. Although PEP is a storage ring of approximately the same size, at first the instability did not occur there, probably due to the higher injection energy available at SLAC. However, the effect appeared when the betafunctions at the RF cavities were increased involuntarily during the installation of a "mini-beta" scheme, and it disappeared when they were again reduced by changing the beam optics. The somewhat smaller storage ring CESR so far has not been limited by this instability.

It thus seems that the beam current limitation becomes more severe the bigger the storage ring. It is therefore of particular concern for LEP, which will have 12 times the circumference of PEP or PETRA. It has also been studied in detail for TRISTAN, which will be the largest electron storage ring of the near future. However, it has so far never been observed in proton rings. It is not expected to present a current limitation for LHC, the proposed proton storage ring in the LEP tunnel - or even the much larger SSC - if reasonable care is taken in the design. This is due essentially to a completely different set of parameters such as a very large number of long and weak bunches, a much smaller number of RF cavities and a betatron frequency spread which covers several synchrotron frequency sidebands.

2. Frequency-Domain Analysis

Most instabilities can be analysed best in the frequency domain. In the presence of betatron oscillations, the synchrotron motion of the particles in a bunched beam leads to coherent modes with frequencies at the sidebands of the betatron frequency $\omega_\beta = \nu\omega_0$

$$\omega_m = \omega_\beta + m\omega_s = (\nu + m\,\nu_s)\,\omega_0 \qquad (1)$$

where ω_0 is the revolution frequency, ν the betatron tune, ν_s the synchrotron tune and m can be any positive or negative integer including zero. Due to the voltages induced in the transverse impedance of the beam surroundings (RF cavities, tanks, bellows and other cross-section variations of the vacuum chamber), the mode frequencies change with beam current. The frequency of the m=0 mode (or "coherent betatron oscillation") changes most rapidly and invariably decreases with increasing current since wall impedances are large and inductive at low frequencies where the spectrum of the m=0 mode has its maximum. It thus approaches the frequency of the m=-1 mode which changes much less with current and may even increase for very short bunches whose spectrum lies in the range where the impedance becomes capacitive (see fig.1).

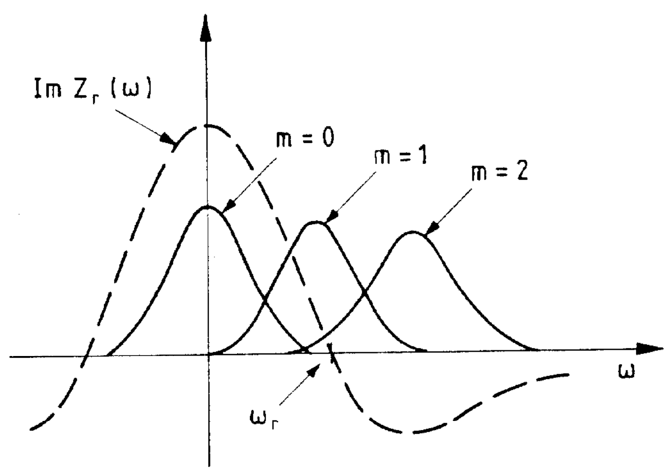

Fig.1 - Spectrum of the lowest modes with imaginary part of transverse broad-band impedance.

The two mode frequencies will coincide when the frequency shift of the m=0 mode is about equal to - or even somewhat less than - the synchrotron frequency:

$$\Delta\nu \approx \nu_s \sim I_0\, Z_{eff}\langle\beta\rangle/E \qquad (2)$$

Above this threshold current, the mode frequencies become complex-conjugate and the transverse oscillation amplitudes increase exponentially at such a fast rate that conventional feedback systems cannot suppress the blow-up. The instability is therefore sometimes called "fast head-tail effect", but this name does not stress the important difference in the dependance on chromaticity. From Eq.(2) one sees that the threshold current will be proportional to the synchrotron frequency times the beam energy, and inversely to the (average) betafunction and the effective transverse impedance in the plane of oscillation.

The mathematical analysis of this effect is usually based on the Vlasov equation in 4 dimensional phase-space [7,8,9], neglecting quantum excitation and radiation damping which are weak at the lowest operation energy where the effect is strongest. The resulting integral equation for the modes can be solved

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by expansion in orthogonal functions, leading to an (infinite) eigenvalue problem for the mode frequencies, at least if the frequency dependence of the transverse impedance is slow. The "broad-band resonator model" is usually a good approximation for short bunches, where sharp resonances cannot be resolved in the limited time of a single bunch passage. Originally a distributed impedance was assumed for convenience, but it is more realistic to study localized impedances [10] since most obstacles in the vacuum chamber are short compared to a betatron wavelength. The results of this refinement are rather important and will be discussed below. Quantum excitation and radiation damping can be included in the analysis by using a Fokker-Planck equation [11]. However, the resulting differences are small, in particular at low energies where damping is weak.

3. Synchro-Betatron Resonances

While the frequency-domain analysis leads to insight into the physical mechanism and to the parameter dependence in general, quantitative predictions of threshold currents are best obtained by computer simulation which is most easily performed in the time domain. Both longitudinal and transverse effects can be included there simultaneously, as well as quantum excitation and radiation damping or non-linearities of the RF voltage and/or the focussing elements. We shall not discuss it here since a detailed review of this technique has been presented in the last US Accelerator Conference [12].

Continuing work has shown that the threshold currents are sharply reduced when the non-integer part of the betatron frequency is near a multiple of the synchrotron frequency [13]. These "synchro-betatron resonances" - SBR's for short - appear even in the absence of longitudinal wakefields, and in particular of the m=1 component which depends on transverse displacement. On the other hand, the transverse kick on a particle does in general depend on its longitudinal position in the bunch through the wakefield. There is

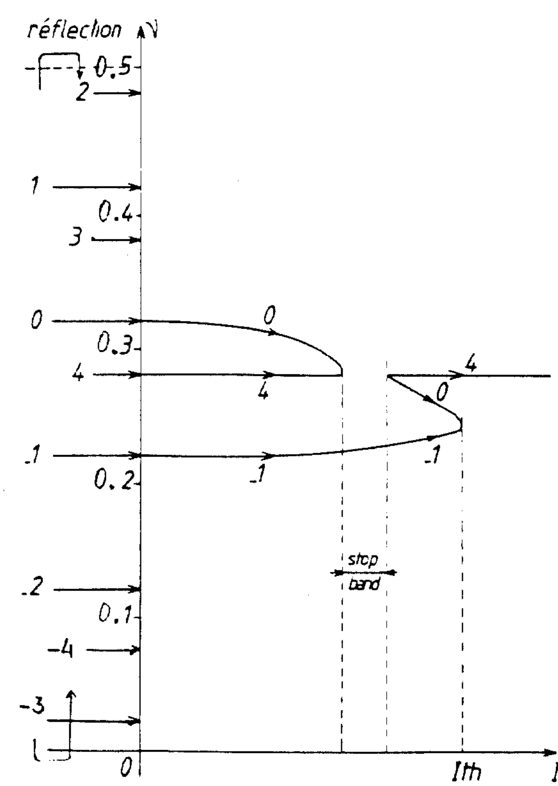

Fig.2 - Mode frequencies versus bunch current from Vlasov equation with localized impedance.

thus no closed loop causing the instability, but these SBR's are driven resonances similar to those of displaced beams in cavities [14]. However, in the simulation they occur even for centered beams in rotationally symmetric cavities - although the growth starts from random displacements of the superparticles in a beam of finite dimensions. Similar resonances have been found in the Vlasov-equation analysis when the transverse impedance is localized [10], and where no transverse displacement of the particles is necessary, as well as in the two-particle model [15] to be discussed below. These SBR's occur even when only short-range wakefields are included, and no high-Q resonances with long memories are needed to drive them.

The analytical results also offer an explanation for these SBR's. The mode-frequencies are determined as eigenvalues of an (infinite) matrix whose elements depend only on the cosine of 2π times the coherent tune and the sines and cosines of 2π times the sideband frequencies. As a result, higher modes appear to be reflected from the tune-values ν_z = 0 and 1/2 (see Fig. 2). When the non-integer part of the betatron tune is near a multiple of the synchrotron tune, these higher modes can cross the m=0 or m=-1 mode at rather low currents and cause unstable stopbands (see Fig.3).

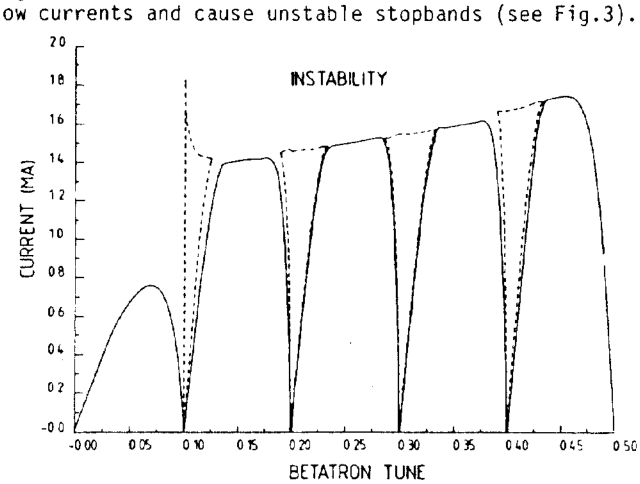

Fig.3 - Threshold current as function of betatron tune from Vlasov equation with localized impedance.

The higher the order of the resonance, the narrower the stopband and the smaller the growth-rate, and thus radiation or Landau damping may suppress these resonances above a certain order. This coupling of higher modes has also been found by a Fourier-analysis of the particle motion obtained from the "several particle model" discussed in the next section, but so far no experimental observations have been reported which confirm these theoretical results.

4. Two-or-More Particle Models

Soon after the mode-coupling instability was analyzed using the Vlasov equation, it was realized that its essential features can be obtained also from a much simpler two-particle model [16,17]. For a step-function wake which is zero in front and constant behind the exciting particle, the coupled differential equations for the oscillations of the two particles can be solved explicitly. They yield a time evolution of the center-of-mass motion in excellent agreement with measurements at PEP [18]. On the other hand, no center-of-mass oscillations have been observed at PETRA [6] where the blow-up of the beam-height alone appears to cause the current limitation. In these models a distributed impedance had been assumed tacitly.

When the transverse impedances are localized at one or several positions around the ring, another two particle model can be formulated in terms of 4x4

transfer matrices [15]. Using normalized displacements and their derivatives for both particles as a state vector, the passage through a linear lattice is described simply by a rotation of 2π times the (non-integer part of the) tune. The effect of the transverse impedance is given by a kick, i.e. a change of slope of the rear particle, proportional to the displacement of the particle in front. This process repeats for half a synchrotron period, i.e. we have to take the n-th power of the transfer matrix for one revolution (or part of a revolution if there are several impedances) when the synchrotron tune is $\nu_s = 1/2n$. This procedure should again be repeated for the second half synchrotron period, where the other particle is in the rear and receives the kick. After this transformation the original situation is reestablished, i.e. the transfer matrix of a full period is obtained. Following a proposal by M. Bassetti [22], the description of this process can be simplified by renumbering the particles after half a synchrotron period, which is done in practice by multiplication with a permutation matrix.
This reestablishes the original situation if the two (super)particles are identical.

The transverse motion will be stable if all eigenvalues of the transfer matrix for one period have unit norm, i.e. if $\lambda = \exp(i\nu)$ with real ν. Due to the special properties of transfer matrices, the 4-th order characteristic equation for the eigenvalues is symmetric, i.e. the coefficients of the k-th and (4 - k)th terms are equal. It then has the form :

$$\lambda^4 + p\lambda^3 + q\lambda^2 + p\lambda + 1 = 0 \qquad (3)$$

and it can be reduced to a quadratic equation with the substitution $u = \lambda + 1/\lambda = 2\cos(\nu)$

$$u^2 + pu + q - 2 = 0 \qquad (4)$$

The conditions for stability then become simply $|u| < 2$ and u real. In terms of the coefficients of the characteristic equation these can be written :

$$2 + q + |p| > 0 \qquad (5)$$

and

$$2 - q + p^2/4 > 0 \qquad (6)$$

Evaluating the coefficients p and q, most easily with the help of algebraic computer routines, one finds that p is linear and q quadratic in the kickstrength α. This fact remains true for any synchrotron tune $\nu_s = 1/2n$ and for any number of impedances per turn, although the coefficients may become rather complicated for a large number of kicks per turn. In any case, one can solve explicitly for the stability limit, i. e. for the maximum permitted kickstrength (which is proportional to the current per bunch) as function of the tune. Since the coefficients contain only sines and cosines of 4π times the non-integer part of the tune, the thresholds are periodic with $\Delta\nu = 1/2$ and symmetric with $\Delta\nu = 1/4$ (the first property remains true even if more than two particles are considered, while the second one disappears). The results of these calculations for $\nu_s = 1/12$ are shown in Fig.4.

The most obvious feature of these results is the appearance of pairs of stopbands originating at every multiple of the synchrotron tune, similar to what had been observed first in simulation and later in the Vlasov equation analysis with localized impedances. In the analysis using distributed impedances these resonances had not appeared, but they could be seen when the localized nature of the impedances was introduced by considering only the non-integer part of the tune [19].

When several localized impedances are present, the mode-coupling threshold is reduced simply by the increase in total impedance independent of their position. However, this is not the case for the resonances whose strength depends on the phaseshift between the various impedances. In particular, the resonances may even vanish [20] if there are n equal impedances spaced exactly by $2\pi/n$. In general, however, the resonances will only be weakened and may remain dangerous, at least if they are stronger than the damping in the ring (see Fig.5).

While the stepfunction wake simplifies the analytic expressions, it also introduces spurious stability regions for very high kickstrengths. It is possible to include arbitrary wakefields with only slightly more work by making the kickstrength a function of distance. For a linear wake most of these spurious stability regions disappear as indicated in Fig.4.

Nevertheless, the two particle model should not be carried too far as it describes only a very simplified situation. More than two particles can be considered, but then the dimensions of the transfer matrix increase and the characteristic equation is of rather high order and with much more complicated coefficients. Nevertheless, numerical solutions can still be found without too many problems [21] for a not too large number of particles.

The permutation technique [22] is particularly advantageous for the case of n particles and a synchrotron tune $\nu_s = 1/n$. Then the transfer matrix of one period becomes simply the product of a kick, a rotation

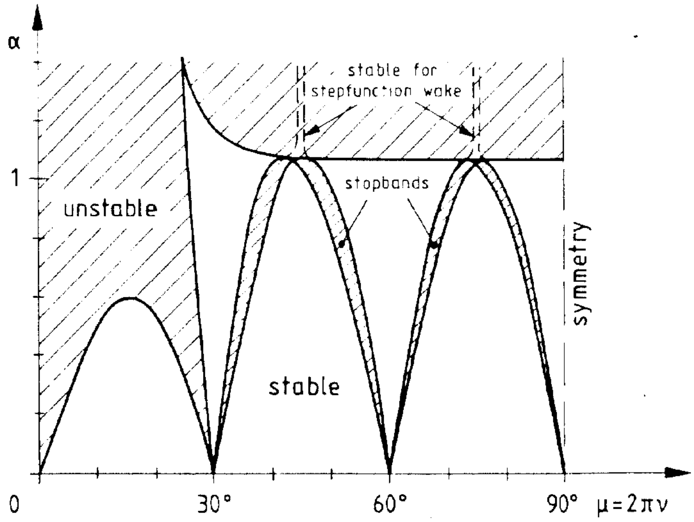

Fig.4 - Threshold kickstrength from two-particle model ($\nu_s = 1/12$).

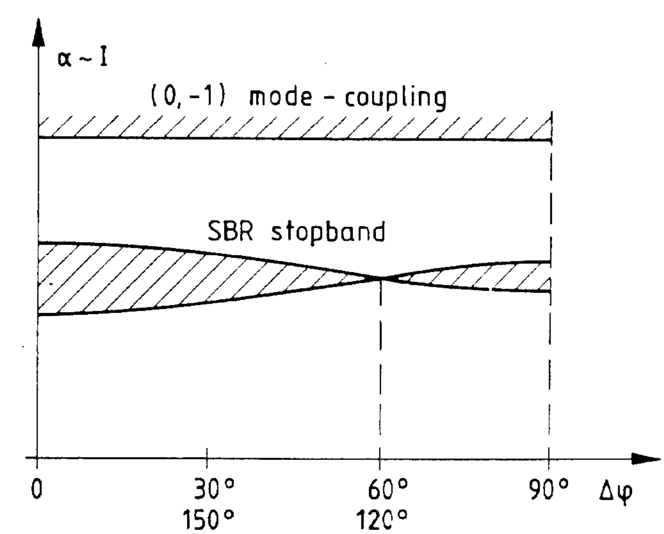

Fig.5 - Threshold versus phase-shift between 3 equally spaced cavities.

and a permutation, and one is not required to take powers of matrices. Models with ten or more particles can be treated easily in this manner, although in general their eigenvalues have to be found numerically.

On the other hand, for the case $\nu_s = 1/mn$ one can use a smaller number of particles n and take a low power m of the transfer matrix This still yields tractable analytic expressions for the stability limits as function of the betatron tune for 3 or 4 particles.

5. Transverse Impedances

The transverse mode-coupling instability of single bunches occurred only in the last generation of rather large electron-positron storage rings. The reason for this is essentially the fact that the transverse impedance, which causes this instability, increases linearly with the machine circumference as can be seen from the approximate relation :

$$Z_T \approx \frac{2R}{b^2} \frac{Z_L}{n} \tag{7}$$

where Z_L is the m=0 longitudinal impedance. The longitudinal threshold is proportional to the ratio of Z_L to the mode-number $n = \omega/\omega_0$ and is essentially independent of the machine circumference. For economic reasons, larger machines also tend to have smaller vacuum-chamber dimensions, in particular (half-)heights b. Hence an increase in the average machine radius R will shift the balance away from the usually benign longitudinal "microwave" instability (which often only causes "turbulent" bunch-lengthening since the rather strong longitudinal non-linearities of an RF bucket limit the growth). Since transverse non-linearities are usually much weaker, the transverse blow-up due to an exponential instability is often limited only by the vacuum chamber wall.

Synchro-betatron resonances, on the other hand, cause only a linear growth of transverse dimensions and thus radiation damping - which is proportional to amplitude - will only lead to a larger equilibrium beam-size, corresponding to an increased emittance. This has been observed e.g. at CESR and was explained by the off-center passage of bunches through the RF cavities [14]. In PETRA, however, crossing of SBR's of not too high order leads to a loss of beam current. In computer simulations, the observed blow-up near the resonances is neither clearly linear nor exponential, and depends strongly on the relative strength of the longitudinal and transverse interaction. Whether emittance growth or particle loss occur first has to be computed numerically for each particular case.

In the design of new storage rings one tries to keep the transverse impedance as low as possible. However, it was not always realized that the strong dependence of Z_T on the chamber height - nearly b^{-3} - emphasizes the contributions of small (but often numerous) cross-section variations - such as bellows, or even bellows shielded by chambers with sliding contacts which introduce new cross-section variations. Up to quite recently, the transverse impedance of the RF cavities was usually considered dominant, since high-energy electron storage rings need a large number of them in order to replace the energy lost by synchrotron radiation. In addition, these cavities are normally designed for maximum shunt impedance in order to minimize the RF power requirements, which has the side-effect of also increasing the transverse impedance, in particular if the radius of the beam-holes is reduced. For super-conducting cavities [26], however, these considerations are no longer valid, and the beam-hole radius can be chosen quite large without unduly increasing the RF power as the shunt impedance remains large thanks to the high quality factors of such cavities. It is due to this fact that we hope to be able to install enough RF cavities in LEP phase II in order to increase the energy to over 80 GeV without limiting the beam current and thereby reducing the luminosity to unacceptably low values.

6. Proton Storage Rings

The stability of high-energy proton or antiproton bunches is of particular interest for the design of LHC, the Large Hadron Collider in the LEP tunnel, and for the even larger Superconducting Supercollider SSC. In these machines a number of parameters are sufficiently different from electron storage rings to make the mode-coupling instability less dangerous.

In all these designs already the injection energy is much higher than the top energy of even the largest electron storage ring LEP. Nevertheless, radiation losses of the much heavier protons remain very small and therefore a few RF cavities are sufficient for both beam acceleration and loss compensation. This fact will reduce the contribution of the RF cavities to the transverse impedance which is usually large in electron machines. In addition, the current per bunch is very small as the total beam current will be distributed over many thousand bunches in order to limit the number of events per collision.

All these factors tend to make proton bunches more stable. On the other hand, the transverse impedance of cross-section variations could become quite large due to the big machine radius R and the small vacuum-chamber radius b (see Eq.7). The vacuum-chambers inside the superconducting magnets operating in liquid helium will also be at very low temperatures. Their resistance will therefore be quite low, but a large number of bellows will be required to permit the contraction during cool-down which will contribute a large transverse impedance. However, the effective impedance is reduced due to the fact that the bunches are expected to be long compared to the dimensions of the bellows convolutions. The coupling of the modes m=0 and m=-1 then is strongly weakened and leads only to a small growth-rate over a limited current range (see Fig.6). At higher currents, coupling of higher (negative) mode-numbers will cause a limitation when the growth-rate exceeds damping [24].

Since radiation damping is very weak in proton machines even at highest energies, it is necessary to investigate the effects of Landau damping which were quite negligible in electron machines. The spread in synchrotron frequency due to the non-linearity of the RF voltage is usually quite small, in particular when the stable-phase angle is near 180° as is the case for protons with low radiation losses. However, it was found that it actually reduces the threshold current [23], contrary to what was expected ! This can be understood from the fact that the distance between mode-frequencies is reduced in the presence of spread as shown in Fig.7, where the natural case is S > 0 when $\omega_s = \omega_{sc} - S(r/\hat{r})^2$ (r amplitude of oscillation).

The synchrotron frequency spread may be increased by potential-well deformation due to space charge or by application of a higher harmonic RF voltage. The spread in betatron frequency will be even larger due to the small synchrotron tune in proton machines, and may easily extend over one or more synchrotron sidebands. Under these conditions, Landau damping becomes effective in stabilizing higher beam currents [25] as can be seen in Fig.8.

Considering all these arguments, single proton bunches in LHC and SSC have been found stable for normal p-p operation. However, this situation may change when one wants to obtain a high proton current in a few bunches as is desirable for e-p collisions. A very careful analysis of the transverse impedances would be required to obtain good estimates of the threshold current in such a machine.

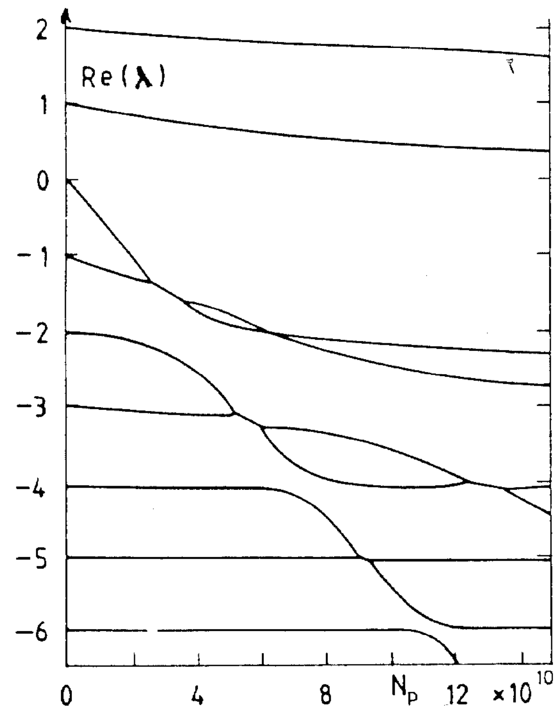

Fig.6 - Mode frequency versus current for the SPS at 315 GeV/c ($\lambda = \omega - \omega_\beta/\omega_s$).

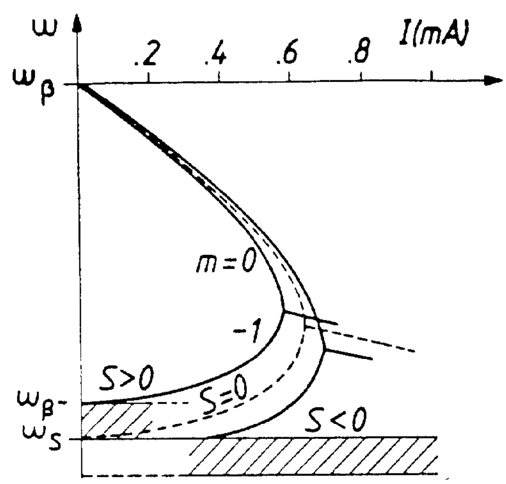

Fig.7 - Mode-coupling with synchrotron frequency spread.

7. Conclusions

We have reviewed here only one particular type of transverse instability which is caused by the coupling of adjacent synchrotron side-bands of the betatron frequency. This "mode-coupling" instability of single bunches is of primary concern in large, high energy electron or positron storage rings with only few short bunches. Coupled-bunch oscillations in these machines, if they occur at all, are usually slow enough to be rather easily cured by feedback systems. The single bunch "head-tail" effect, can be controlled by making the chromaticity zero or slightly positive. However, the much faster mode-coupling instability is quite insensitive to chromaticity and to conventional feedback. On the other hand, a "reactive" feedback system, which keeps the frequency of the coherent betatron oscillation constant, was expected to increase the threshold by quite large factors [21]. Computer simulation and analysis with "localized" two-particle models showed that the improvements were actually smaller than predicted but still not negligible. Experimental verification on an existing machine (PEP) is now being prepared.

Fortunately, the threshold currents can be influenced by quite a number of factors such as lower beta-functions, longer bunches, higher synchrotron tunes and if all else fails, higher injection energies. The most important method, however, remains the minimization of the transverse impedance of all components which can be seen by the beam and this task is pursued vigorously in the design of all new storage rings.

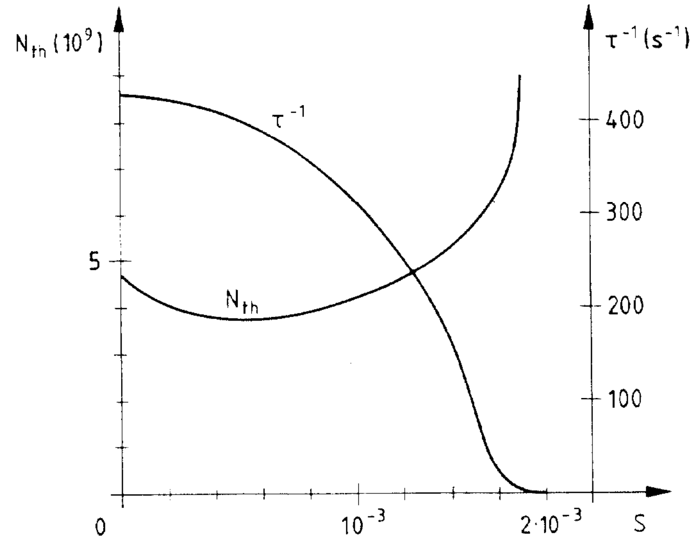

Fig.8 - Landau damping by betatron frequency spread.

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