STUDY OF LAMPF H^- BEAM POSITION MOTION AND A PREDICTIVE ALGORITHM FOR NOISE REDUCTION*

V. Yuan, AT-2 (MS H816) and J. D. Bowman, MP-4 (MS H846)
Los Alamos National Laboratory, Los Alamos, NM 87545 USA

Summary

The position noise in the LAMPF H^- beam has been studied through examination of the autocorrelation and spectral density functions[1] for position motion. The LAMPF H^- beam is a pulsed 800-MeV beam, and the position of each beam pulse was treated as a discrete value. Also investigated was the reduction in position noise that can be attained with an algorithm that predicts beam position from the measured positions of previous beam pulses. The basis for this predictive algorithm is the calculation of the optimal linear estimator (see the appendix). We conclude that the majority of low-frequency noise occurs within the 10- to 15-Hz frequency range, and that, with a simple, 2-pulse predictive correction system, one could improve the noise in this frequency region by as much as a factor of 15.

Introduction to Noise Study

The noise study was done using wire-chamber beam profile data taken, at 120 Hz, in the External Proton Beam line by Experiment 792, Search for Parity Violation in Polarized p-p scattering at 800 MeV. Beam periodicity was 1/120th of a second. Beam motion is believed to result primarily from low-frequency mechanical oscillations in the linac drift tubes. Wire-chamber profiles were analyzed to determine the position of discrete beam macropulses, and data samples taken over a one week period were examined to determine how the noise changes with time.

In addition to the 10- to 15-Hz noise, there exists a significant amount of 60-Hz position noise. We also found a δ-function component of the autocorrelation function which arises from an undetermined combination of measurement error and high-frequency position noise. An optimal reduction of the overall noise will require minimizing the 60-Hz motion and δ-function component by independent means.

Autocorrelation Plots

An example of an autocorrelation spectrum

$$R(\tau) = \int x(t)x(t+\tau)dt$$

for one of the data samples (Run 4442) is shown in Fig. 1. The spectrum covers the range of τ = 1/120 s to τ = 50/120 s. The autocorrelation integral has been approximated by numerical averaging over a 5000 pulse (~1 min) time span. In calculating the Fig. 1 spectrum, 60-Hz position motion was eliminated by averaging the position of consecutive beam pulses. This averaging provides a smoothed set of data in which the slower frequency components of beam motion can be more easily resolved.

All $R(\tau)$ values in these and subsequent plots have been normalized to $R(0)$. As can be seen in Fig. 1, the $R(\tau)$ curve does not extrapolate to a value of 1 at τ = 0, indicating the presence of a δ-function-type high-frequency noise that has its source in either real position motion or detector measurement error. Whatever the source, such a high-frequency component cannot be reduced by means of predictive correction.

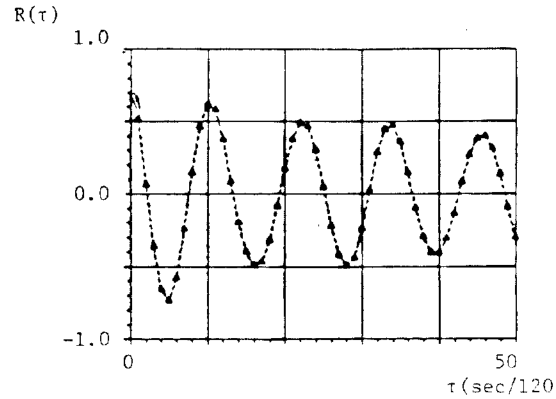

Fig. 1. Autocorrelation plot for Run 4442. Plot is calculated from position data that has been smoothed to eliminate the contributions of 60-Hz noise.

Autocorrelation plots for beam motion at two different times of accelerator operation (indicated by different run numbers) are shown in Fig. 2. The range of the plots has been expanded to τ = 500/120 s so that the envelope of the autocorrelation function can be easily discerned. The plots show the presence of a damped component as indicated by the shrinking of envelope amplitude with increasing τ. They also show the presence of multiple frequencies as indicated by the presence of beats. The change in envelope shape from sample to sample is evidence of changes in both the damping and in the frequencies of the noise contributants.

Spectral-Density Plots

Details showing typical low-frequency noise components in position motion are shown in the spectral-density function (SDF) plot of Run 4442, shown in Fig. 3. The SDF plot shows a large peak at ~10.4 Hz, with several smaller peaks at higher frequencies in the 10- to 15 Hz frequency range. Other data samples were analyzed also, and the frequencies for the smaller peaks varied from one run to another.

Based on the general form of the spectral density functions, fits to the autocorrelation functions of each run examined were made using a fit function of the form:

$$R_{fit} = A_1 e^{-\gamma t}\cos(2\pi f_1 t) + A_2\cos(2\pi f_2 t) + A_3\cos(2\pi f_3 t) + A_4\cos(2\pi f_4 t) \ .$$

The fits were made over a range of pulse spacings from 1 to 150, and starting values for amplitudes and frequencies used in the fits were estimated from the spectral density plots. Damping for the fits varies from a value of 0.1/s to a value of 0.3/s. The fit values for Run 4442 are presented in Table I.

*Work supported by U.S. Dept. of Energy.

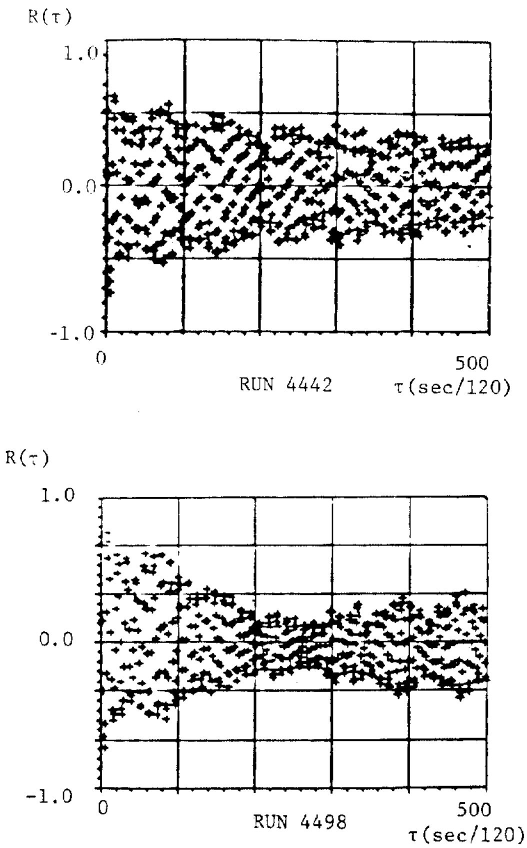

Fig. 2. Autocorrelation-spectrum envelopes for data samples taken at different times, Run 4442 and Run 4498.

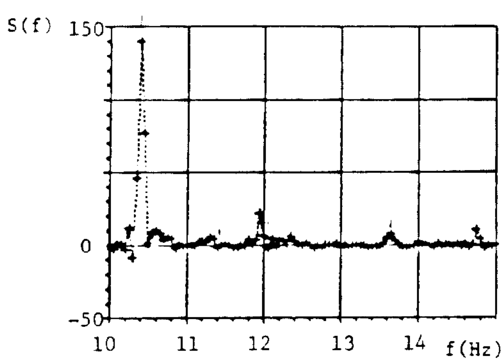

Fig. 3. Spectral-density function in the 10- to 15-Hz range for Run 4442.

Predictive Algorithm

A predictive algorithm was derived based on a formalism calculating the optimal linear estimator. Details of the formalism are presented in the appendix. The predictive algorithm can use any number N of previous beam positions to calculate its prediction. It requires only that the autocorrelation function up to R(N) be known and that an N x N matrix be inverted. From the formalism, a theoretical noise reduction (TNR) also can be calculated.

We tested the predictive algorithm on three representative samples of beam motion using predictions based on the 2, 5, and 20 previous positions. As was the case in calculating the noise spectrum, 60-Hz noise was not treated and its effects were eliminated through smoothing of consecutive pulse positions. The success of the predictive algorithm for Run 4442 is shown in Fig. 4. Actual noise reductions (ANR)

$$\mathrm{ANR} = \frac{\sigma\ (\text{raw position motion})}{\sigma\ (\text{corrected position motion})}$$

were calculated for all of the trial cases, and they are presented in Table II. Also presented in Table II are the theoretical noise-reduction values for each case as calculated from the formalism. The theoretical noise reduction can be regarded as an upper limit to possible noise improvement, and the actual noise reduction is smaller, as expected, because of the δ-function-type noise described earlier. Should this δ-function-type noise be dominated by measurement error rather than actual position motion, one can expect to improve the ANR through the development of improved position detectors.

The results presented in Table II show that a prediction based the positions of only the 2 most recent beam pulses does nearly as well as predictions based on 20 previous positions: the respective noise reductions in the 10- to 15-Hz range are 14 and 18 for the two algorithms. Also, the 2-pulse predictive coefficients for all the sample cases are the same; this suggests that a universal 2-pulse algorithm using these coefficients can be implemented. Such an universal algorithm makes possible the reduction of position noise without the need for a real-time analysis of past positions. The fact that a 2-pulse universal algorithm can effect nearly-maximal noise reduction is consistent with the exact solution for predictive coefficients in the case of a pure sinusoidal autocorrelation function without damping (see the appendix).

TABLE I

FIT VALUES TO AUTOCORRELATION FUNCTION OF SAMPLED RUN 4442.

RUN	I	A(I)	γ	f(I)
4442	1	0.571	0.313	10.43
	2	0.003		11.43
	3	0.080		12.06
	4	0.004		13.65

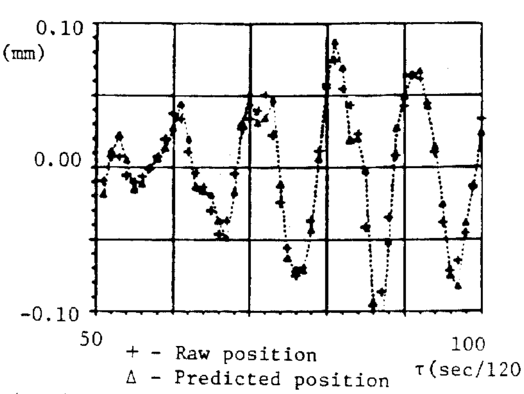

Fig. 4. Comparison of raw position data (60 Hz removed) with algorithm predictions for Run 4442. Predicted beam positions calculated from 2-pulse algorithm.

TABLE II

NOISE REDUCTION LIMITS FOR PREDICTIVE ALGORITHMS.

RUN	PULSES[1]	TNR[2]	ANR[3]
4132	2	14.9	5.9
	5	14.9	4.6
	20	18.5	5.9
4442	2	13.9	3.9
	5	14.1	4.0
	20	15.4	4.4
4582	2	14.1	3.3
	5	14.7	3.0
	20	19.2	3.3

[1]Number of previous positions used in algorithm to make prediction.

[2]Theoretical noise reduction. This is upper limit to noise reduction that can be attained in the 10- to 15-Hz region.

[3]Actual noise reduction; attained over full frequency range.

Appendix

Optimal Linear Estimator

Consider a discrete random process y_k defined on the positive and negative integers. We assume that the process is stationary:

$$E(y_k y_\ell) = R(k-\ell)$$

where E denotes the expectation value and R is called the autocorrelation function. We assume that y_k is ergodic: that ensemble averages are equal to time averages. We assume $E(y_k) = 0$.

Given the value of y up to k-1 we wish to predict y_k as a linear function of m previous values: y_{k-1}, y_{k-2}, ... y_{k-m}.

$$p_k = \sum_{\ell=1}^{m} G_\ell y_{k-\ell} \ .$$

For fixed m, we define the minimum-variance set of prediction coefficients G_ℓ such that

$$E\left((y_k-p_k)^2\right) = \text{minimum}.$$

Then as $m \to \infty$, p_k will be the minimum variance estimator of y_k given the past history of the process.

The equations for the G_ns are

$$R(s) = \sum_{n=1}^{m} G_n R(s-n)$$

$$G_n = \sum_{s=1}^{m} R(s) M_{s,n}$$

where $M_{s,n}$ is the symmetric matrix that is the inverse of $R_{s,n} = R(s-n)$:

$$\sum_{n=1}^{m} R(s-n) M_{n,q} = \delta_{s,q} \qquad 1 \le s \le m, \quad 1 \le q \le m \quad .$$

The theoretical noise reduction ν is then

$$\nu^{-2} = \frac{E\left((p_k-y_k)^2\right)}{E(y_k^2)} = \frac{R(0) - \sum_{s=1}^{m} R(s)G_s}{R(0)}$$

Application to Examples

Examples of cases that can be easily worked out are

(1) $R(s) = a^{-|s|}$; exponential decay.

The matrix $M_{s,n}$ is nonzero along the main diagonal and the two adjacent diagonals. It is zero everywhere else. It can be shown

$G_1 = a$, and $G_\ell = 0$, otherwise.

Hence, $\nu^{-2} = 1-a^2$.

In the special case when the damping is large

$a = 0$, and, hence,

$R(s) = \delta_{s,0}$, and

$\nu^{-2} = 1$.

So noise that has a δ-function autocorrelation cannot be reduced by a predictive correction.

(2) $R(s) = \cos(2\pi fts)$; undamped sine wave, f is the frequency and t is the pulse spacing.

For m = 2,

$$M = \frac{1}{\sin^2(2\pi ft)} \begin{pmatrix} 1 & -\cos(2\pi ft) \\ -\cos(2\pi ft) & 1 \end{pmatrix} ,$$

$G_1 = 2\cos(2\pi ft)$,

$G_2 = -1$, and

$\nu^{-2} = 0$.

For m > 2, the matrix R(s-n) is singular. Because the autocorrelation function for the processes being considered (LAMPF beam motion) is to a good approximation undamped, it is to be expected that the m = 2 solution given above will be a good approximation to the optimal estimator. The damping per pulse interval is ~1 - (0.2)/(120) = 0.998.

It is interesting to calculate the response of $E(|y_k-p_k|)/E(|y_k|)$ as a function of frequency f when y_k is a sine wave at frequency f,

$G_1 = 2\cos\left(2\pi(10.4)(1/120)\right) = 1.71$, and

$G_2 = -1$.

Although the response at 10.4 Hz is zero, the 60-Hz component of the signal will be enhanced by a factor of 3.70. This is the price of suppressing the 10- to 15-Hz noise.

References

1. W. B. Davenport, Jr. and W. L. Root, An Introduction to the Theory of Random Signals and Noise, (McGraw-Hill Book Company, Inc., New York, 1958).