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THE INVESTIGATION OF ACCELERATING TUBES WITH PERIODIC FIELD

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Abstract

Two types of periodically modulated field for high voltage accelerating tubes are presented. The trajectories of secondary electrons emitted from electrodes have been computed. It is shown that the maximum energy obtained by the electrons decreases with the increasing field modulation. Therefore, it is possible to suppress the electron loading effect of high voltage accelerating tubes by these means.

Introduction

A method of suppression of electron loading is to set up a transverse field in an accelerating tube. For instance, the secondary electrons are deflect away from axis by the transverse electrical field in an inclined field accelerating tube and by the transverse magnetic field in a magnetically suppressed tube. Both the kinds of the accelerating tubes have been successful in effective suppression of the electron loading[1,2]. In this paper, two types of periodic field for accelerating tubes are proposed(Fig.1 and 2), in which the secondary electrons emerging from electrodes get transverse momentum from its transverse field component and then hit the electrodes after travelling a rather short axial distance. In principle, this sort of tubes with periodic electrical field is similar to one with special diaphragm electrodes[3], however, its field period is much shorter and all the aperture diameters of the tube electrodes are the same. By the way, the periodic field also could produce some focusing effect, which may enhance the acceptance of the accelerating tube for the ion beam.

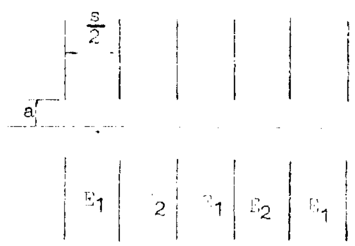
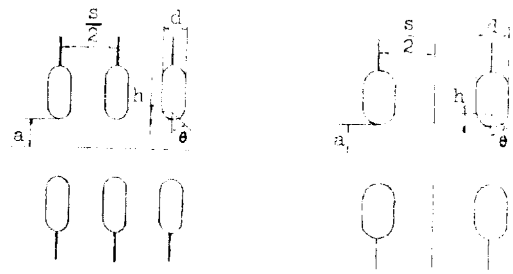


Fig.1



(a) (b)

Fig.2

Electron trajectories in accelerating tube with uniform field

The equation of electron trajectories (relativistic) is

$$y''+\frac{eT}{T^2-T_r^2}(1+y'^2)\left(\frac{\partial V}{\partial x}y'-\frac{\partial V}{\partial y}\right)=0 \tag{1}$$

where $y'=\frac{dy}{dx}$, y and x are transverse and axial coordinates respectively, V is the electric potential, T_r and e are the rest energy of the electron and the absolute value of the electronic charge respectively. T is the total energy of the electron as the following

$$T=T_0+e(V-V_0) \tag{2}$$

where T_0 is the initial energy of the electron, V_0 is electric potential at the position where the electron is emitted. For uniform accelerating field, $\frac{\partial V}{\partial y}=0$, $E=\frac{\partial V}{\partial x}$ is the field strength. Thus the solution of equation (1) is

$$y-y_0=\frac{y'_0}{eE}\frac{\sqrt{T_0^2-T_r^2}}{\sqrt{1+y_0'^2}}\log\frac{T_0+eEx+\sqrt{(T_0+eEx)^2-T_0^2+(T_0^2-T_r^2)/(1+y_0'^2)}}{T_0+\sqrt{(T_0^2-T_r^2)/(1+y_0'^2)}} \tag{3}$$

If $W_0=(T_0-T_r)/T_r\ll 1$

$$y-y_0\simeq\frac{y'_0}{eE}\frac{T_r\sqrt{2W_0}}{\sqrt{1+y_0'^2}}\log\left[1+W+\sqrt{(1+W)^2-1}\right] \tag{3'}$$

where $W=eEx/T_r$. By equation (3) we can see that if the initial direction of the secondary electron emerging from an electrode is away from the axis of the tube ($y'_0>0$), the electron immediatly hits at the next electrode and then stop travelling further. Otherwise, namely $y'_0<0$, the electron is travelling a long distance along the axis in the meantime it can get more energy through the accelerating field. Fig.3 shows the energy increasing of the electrons versus transverse displacements for $T_o-T_r=100ev$, $E=1.5Mv/m$. It can be seen clearly that it is difficult to stop the secondary electrons toward the axis of the tube with uniform field, even thought the electrode aperture is small.

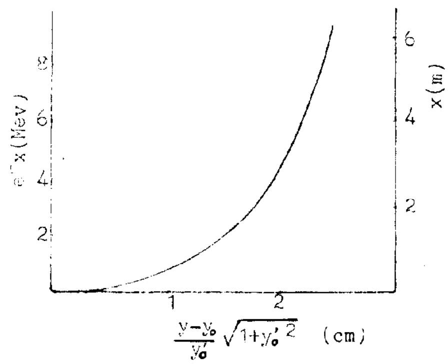


Fig.3

The production of the periodic field

Two types of periodic field have been proposed. For the first type two different field strength E_1 and E_2 are applied alternately on two adjecent gaps to form a series of focusing-defocusing aperture lenses as shown in Fig.1. Where s/2 is the length of the gap, 2a is the aperture diameter of the electrode. The origin of the coordinates is at the mid-plane of the stronger field, where electrical potential is set to zero.

Using the variable-seperation method, we can get the space potential distribution as follows

$$V(y,x)=\frac{E_1+E_2}{2}x+\sum_{m=0}^{\infty}A_{2m+1}I_o(k_{2m+1}y)\sin k_{2m+1}x, \quad y\leqslant a \qquad (4)$$

where

$$\begin{cases} A_{2m+1}I_o(k_{2m+1}a)=(-)^m\frac{(E_1-E_2)s}{\pi^2}\left[\frac{1}{(2m+1)^2}+\sum_{n=1}^{\infty}\frac{B_{2n}}{(2m+1)^2-(2n)^2}\right] \\ \sum_{n=1}^{\infty}\frac{B_{2n}}{(2m+1)^2-(2n)^2}\left[\frac{2n}{2m+1}\,\frac{I_o(k_{2m+1}a)K_o(k_{2n}a)}{I_o(k_{2m+1}a)K_o(k_{2n}a)}-1\right]=\frac{1}{(2m+1)^2} \\ m=0,1,2,3, \end{cases} \qquad (5)$$

$k_q=2q\pi/s$, I_o and K_o are the first and second kind of the zero order Bessel function respectively.

Thick electrodes are used in the second type of field to form a series of gap lenses as shown in Fig.2. In order to analyse the effect of the field on electron motion we take rotatory symmetry field as the two dimension field so that we can manipulate with approximate analytical method.

Making an analytical transfer from the complex plane $z=x+jy$ to the complex plane $w=u+jv$ we obtain

$$\frac{dz}{dw}=A\left(\sqrt{\frac{\sin^2 w+\sinh^2 v_1}{\sin^2 w+\sinh^2 v_2}}+B\right) \qquad (6)$$

where $Re(Esw/\pi)$ is the potential and E is the average field strength of the accelerating tube. The parameters A,B,v_1,v_2 introduced can be expressed as follows

$$\begin{cases} A(2I_1+B\ )=s \\ A(I_2+Bv_2)=a \\ A(2I_1-\ \ )=d \\ AB(v_1-v_2)=h \end{cases} \qquad I_1=\int_0^{\frac{\pi}{2}}\sqrt{\frac{\sin^2 u+\sinh^2 v_1}{\sin^2 u+\sinh^2 v_2}}\,du \qquad I_2=\int_0^{v_2}\sqrt{\frac{\sinh^2 v_1-\sinh^2 v}{\sinh^2 v_2-\sinh^2 v}}\,dv \qquad (7)$$

The parameters s,a,d and h are shown in the Fig.2. The maximum value of electrical field E_{max} is given in equation (8)

$$E_{max}/E=1+2I_1/\pi B \qquad (8)$$

The electron trajectories

The electron trajectorics in the first type of periodic field are shown in Fig. 4 for $a/s=0.25$, $E_1+E_2=3Mv/m$ and $K=2(E_1-E_2)/(E_1+E_2)$ (the modulation degree of the electrical field).

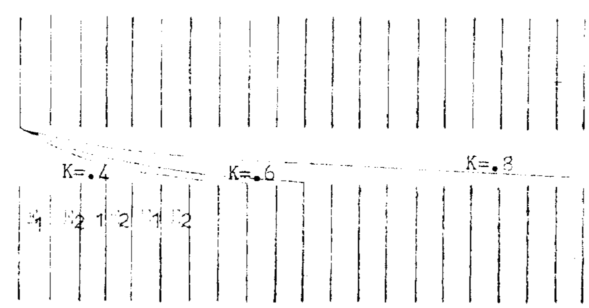


Fig.4

As $T_o-T_r<<e(E_1+E_2)s/2$ initial energy and divergence effect on the electron motion is very weak. Fig.4 shows the trajectories of electrons emitted from the electrode surface at the side with higher field, but the electrons at the side with weaker field will hit the next electrodes after moving against the direction of force lines. The energy of the electrons decreases with the increasing modulation degree of field.

Because it is very complicated to discribe the periodic field of second type analytically, we try to draw the electron trajectories in the w-plane, where the equation of electron trajectories is

$$d\left(\tan^{-1}v'+\tan^{-1}\frac{\frac{\partial V}{\partial u}}{\frac{\partial V}{\partial v}}\right)+v'\,d\log\sqrt{T^2-T_r^2}=0 \qquad (9)$$

Fig.5 shows some typical trajectories of electrons for the tube, where $a/s=0.25$, $d=2h=0.317s$ and $E=1.5$Mv/m. The angle θ corresponds to various initial positions of electrons emerging from arc-shaped edge of the electrode. Table 1 and 2 show the number N of the gaps to be traveled by the electrons before stopping, where $E=1.5$Mv/m, $a/s=0.5$ and 0.25 respectively. It can be seen that the number N decrease when enhancing the modulation degree of the field. If the electrodes get thin enough, the tube returns to uniform field accelerating tube, and then it is very difficult to stop the secondary electrons. In general, the higher the average field E, the more number of gaps that the electrons would go through, however, it is not sensitive to the average field strength.

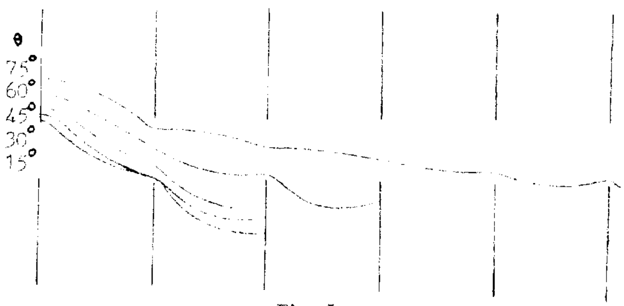


Fig.5

Table 1 ($a/s=0.5$)

$\frac{d}{s}$	$\frac{E_{max}}{E}$	θ 15	30	45	60	75
.071	1.70	16	14	16	23	/
.206	1.84	6	6	7	12	/
.329	2.03	5	5	6	11	/
.445	2.27	4	4	6	14	/
.550	2.60	3	4	6	/	1

Table 2 ($a/s=0.25$)

$\frac{d}{s}$	$\frac{E_{max}}{E}$	θ 15	30	45	60	75
.081	1.70	4	3	4	6	/
.148	1.80	2	2	3	/	/
.267	1.91	2	2	2	4	/
.317	1.98	2	2	2	3	/
.380	2.10	2	2	2	4	/
.433	2.22	2	2	2	5	1

At last we should show that there is some error to replace the rotatory symmetrical field with a plane field in above calculation. However, it is still true that periodic field can get electrons moving transversely, and thus it is useful to limit their energy.

Conclusions

Some conclusions have been drawn from the above discussions.

(1) It is possible to set up the periodic field with proper modulation degree when the aperture of the electrodes is small and the field period is short as can be seen from table 1 and 2.

(2) Periodic field could be used to limit the energy obtained by the secondary electrons emerging from accelerating electrodes. The suppression effect increases with the increasing modulation degree of field.

(3) The above two kinds of periodic field are compared as follows. (a) In type 1 the voltage sustained between the insulating rings should be increased, however, in type 2 only the electrical field inside the tube is enhanced. (b) Because of having stronger suppression the type 2 is more effective than the type 1. If electrode edge near the aperture is designed carefully, maybe the suppression effect could get stronger.

(4) Unfortunately, the periodic field has no suppression effect for the electrons produced in the residual gases, but it has some focusing action. Besides, the periodic field will increase the acceptance for the electrons coming from outside of the accelerating tube. Naturally it could be limited by means of appling an suppressing voltage on the last two electrodes of the tube.

(5) According to the electrostatic focusing principle, the periodic field can increase the acceptance for the accelerated beam, but such an action is obvious only for the first several electrodes.

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