

OPTIMISATION OF AVERAGE LUMINOSITY IN pp AND $\bar{p}p$ COLLIDERS

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Abstract

Algorithms are presented for the optimisation of the luminosity in pp and $\bar{p}p$ colliders. Their average luminosity is lower than the peak luminosity because the transverse dimensions of the beams increase due to intra-beam scattering, and the beam currents decay due to the collisions with the rest gas and the opposite beam. Assuming that the $\bar{p}$ are produced during the collision time links the $\bar{p}$ production rate to the maximum $\bar{p}$ current. Either the average luminosity or the average number of collisions with exactly one beam-beam event are optimised. The free parameters are the collision time, the initial luminosity, and the density of the beams. Numerical solutions are based on the parameters of the $\bar{p}p$ option of the Large Hadron Collider (LHC) and on the $\bar{p}$ production rate of the $\bar{p}$ source (ACOL) at CERN. Approximate analytical solutions are also presented.

1. Introduction

We study the optimisation of the luminosity L in pp and/or $\bar{p}p$ colliders such as the SPS [1], the LHC [2], or the SSC [3]. During the collision time T , the luminosity drops with time for reasons which will be discussed in detail below, and $\bar{p}$'s are collected in the $\bar{p}$ -source. The luminosity vanishes during the filling time F of the collider which includes all the times necessary to cycle the magnets, to inject, to accelerate, and to set up the beam-beam collisions and the detectors. The filling time of the LHC is about one hour.

An important collider parameter from the point of view of high-energy physics is the average number n of events in a single beam-beam collision. The view has been expressed [4] that all collisions with more than one event should be rejected in the data analysis. The parameter n is related to the standard machine parameters by the following relation:

$$n = L\Sigma/fk \quad (1)$$

Here Σ is the total cross section for pp or $\bar{p}p$ collisions, f the revolution frequency, and k the number of bunches in one beam. The probability of having exactly one event in a beam-beam collision is given by a Poisson distribution. Hence, the luminosity L_1 associated with these collisions is given by:

$$L_1 = L \exp(-n) = nfk \exp(-n)/\Sigma \quad (2)$$

It is easy to see that L_1 reaches a maximum at $n=1$, all other parameters being fixed. Therefore it is advantageous to start the run with n larger than unity because L_1 increases at the beginning, reaches a maximum when $n = 1$, and then decreases.

Our optimisation maximises either the average luminosity $\langle L \rangle$ defined by:

$$\langle L \rangle = (T + F)^{-1} \int_0^T L(t) dt \quad (3)$$

or a similar equation for $\langle L_1 \rangle$.

2. Mathematical formulation.

The two bunched beams collide head-on with zero crossing angle. The density distributions are Gaussian. For bunches with unequal bunch populations N_1 and N_2 ,

and unequal rms beam radii σ_{x1} , σ_{x2} , σ_{z1} , σ_{z2} , respectively, the instantaneous luminosity is:

$$L(t) = N_1 N_2 f k / 2\pi \sqrt{[(\sigma_{x1}^2 + \sigma_{x2}^2)(\sigma_{z1}^2 + \sigma_{z2}^2)]} \quad (4)$$

The subscripts x and z label the horizontal and vertical plane, respectively, and y may be either x or z . The normalised transverse emittances are $E_y = 4\pi\beta\gamma\sigma_y^2/\beta_y$. Here, β and γ are the usual relativistic factors, and β_y is the betatron amplitude function at the crossing points. The longitudinal emittance is $E_s = 4\pi E_0 \gamma \sigma_s \sigma_s / (\beta c)$, where E_0 is the rest energy, σ_s the relative rms energy spread, σ_s the rms bunch length, and c the speed of light.

Most of the quantities appearing in (4) are functions of time. The bunch population of the i -th beam decreases because of beam-beam collisions in n_x collision points, and beam-gas collisions with a beam-gas lifetime τ_g :

$$dN_i/dt = -L\Sigma n_x/k - N_i/\tau_g \quad (5)$$

The rms beam radii change because of intra-beam scattering (IBS), i.e. the scattering of the p 's and/or $\bar{p}$'s in one beam on each other. Trivial modifications of the equations in [5] allow us to isolate $d\sigma_x/dt$, $d\sigma_z/dt$, and $d\sigma_s/dt$. All equations become independent of the number k of bunches, if instead of L and L_1 the new variables L/k and L_1/k are used. This observation expresses the intuitively obvious physical fact that the dynamics of one bunch does not depend on the presence of the other bunches, provided that n_x is fixed.

We thus have a set of eight first-order differential equations in the variables N_i , σ_{xi} , σ_{zi} , σ_{si} , with time as the independent variable. All other relevant quantities can be calculated from the eight variables. We integrate this set numerically up to a running time t , and thus find the instantaneous luminosities $L(t)$ and $L_1(t)$ from (1), (2) and (4). Simple numerical quadratures suffice to obtain the average luminosities $\langle L \rangle$ and $\langle L_1 \rangle$. They, in turn, are imbedded in the general-purpose minimisation program MINUIT [6]. The parameters to be minimised are $-L$ and $-L_1$.

The reason for this complicated hierarchy of numerical procedures is the absence of closed analytical formulae for the most general case. However, there are formulae for simpler special cases.

When IBS is negligible, i.e. when the beam sizes are constant, when the beams are equal and the beam-gas lifetime $\tau_g \rightarrow \infty$, the ratio of average to initial luminosity is:

$$\langle L \rangle / L_0 = (1 + f/t)^{-1} (1 + t)^{-1} \quad (6)$$

Here, $t = T/\tau_{bb}$, $f = F/\tau_{bb}$, and τ_{bb} is the initial beam-beam half life $\tau_{bb} = Nk/(L\Sigma n_x)$. The ratio reaches a maximum when the collision time T is equal to the geometric mean between the filling time F and the initial beam-beam half life τ_{bb} , with value $(1+f)^{-2}$. Under the same assumptions $\langle L_1 \rangle$ becomes:

$$\langle L_1 \rangle = (fk/\Sigma)(t+f)^{-1} \sqrt{\pi n_0/4} \times \{ \text{erf} \sqrt{n_0} - \text{erf} [\sqrt{n_0}/(1+t)] \} \quad (7)$$

The optimum values of t and n_0 , the value of n at $t=0$, were found numerically. The result is shown in Fig.3. A closed formula for $\langle L \rangle$ also exists when the beam sizes are constant and the beam-gas lifetime τ_g is finite [7], where $r = \tau_g/\tau_{bb}$:

$$\langle L \rangle / L_0 = (t+f)^{-1} \{ \ln[(1+r \exp(-rt))/r] + r(1+r)/(1+r \exp(-rt)) - (1+r) \} \quad (8)$$

Fig.1 shows result of a numerical optimisation of (8) with the scaled collision time t as free variable.

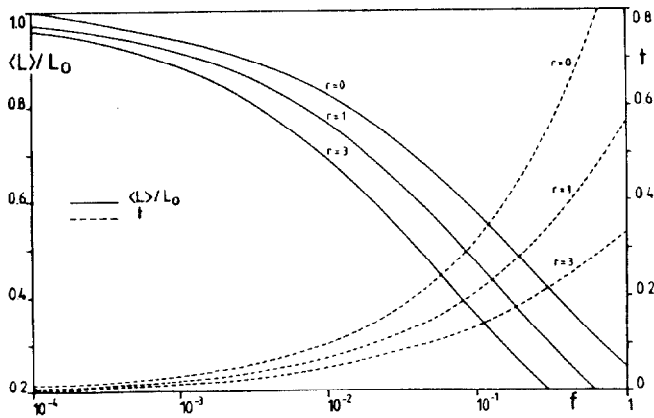


Fig.1 - Optimum scaled L and t versus f ($r = \tau_g / \tau_{bb}$).

3. Results

Our calculations are based on the parameters of the LHC [2], summarised in Table 1. Only parameters different from this standard set are explicitly mentioned.

Table 1 : Standard LHC Parameters

Momentum compaction factor	3.39E-4
Horizontal beta value at crossing	1 m
Vertical beta value at crossing	1 m
Machine circumference	26659 m
Number of collision points	4
Synchrotron tune	0.002
Betatron tune	56.25
Normalised horizontal emittance	5 π μ m
Normalised vertical emittance	5 π μ m
Normalised longitudinal emittance	2.5 eVs
Beam energy	8.136 TeV
Maximum beam-beam tune shift	0.0025
Filling time	1 h
Beam-gas lifetime	48 h
Total pp cross section	100 mb

The following variables are available for the optimisation : (i) the bunch populations N_i , (ii) the emittances E_{xi} , E_{zi} , E_{sj} , (iii) the number of bunches k , (iv) the collision time T . There is also a number of constraints :

- Experience with the SPS [8] has shown that the beam-beam tune shift has a sharp upper limit. Therefore, the bunch populations and emittances must be chosen such that the beam-beam tune shifts remain at or below that limit.
- The bunch populations and emittances must fall inside a range of phase-space densities which can be supplied by the p and $\bar{p}$ sources.
- The total number of $\bar{p}$'s, i.e. the product of bunch population and number of bunches, must be available from the $\bar{p}$ source, i.e. the intensity of the $\bar{p}$ source must be higher than the desired total number of $\bar{p}$'s.

- The total number of $\bar{p}$'s and the collision time must be compatible with the $\bar{p}$ production rate in the $\bar{p}$ source.

The first two constraints apply both to protons and antiprotons. They are taken into account by optimising with upper limits on the bunch populations. The last two only apply to the antiprotons because protons are usually abundantly available. It only takes four SPS cycles to fill the LHC [2].

Fig.2 shows how $\langle L_1 \rangle$ depends on the collision time for combinations of filling time and beam-gas lifetime. At small collision times the filling time is important, while at large collision times the dominant parameter is the beam-gas lifetime. The maximum of $\langle L_1 \rangle$ is reached for collision times of the order of a day.

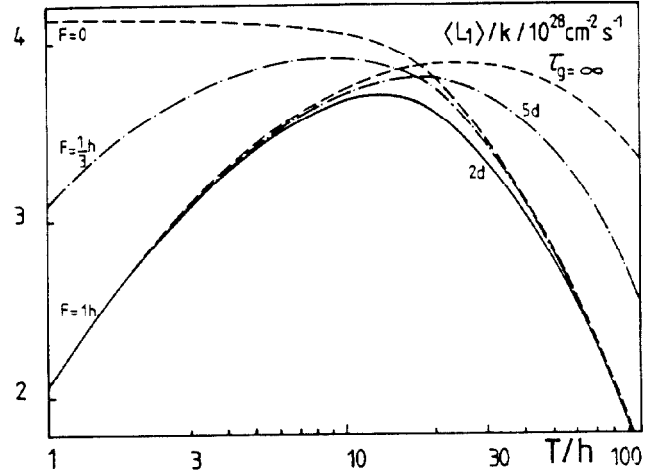


Fig.2 - $\langle L_1 \rangle / k$ versus T for combinations of F and τ_g .

To optimise $\langle L_1 \rangle / k$ for pp colliders as a function of the filling time F , we use the bunch populations N_i and the collision time T as variables. The results are shown in Fig.3, which compares three cases. When the filling time f increases, the optimum collision time t grows, and $\langle L_1 \rangle / k$ decreases. The initial value of n is always above unity, and also grows

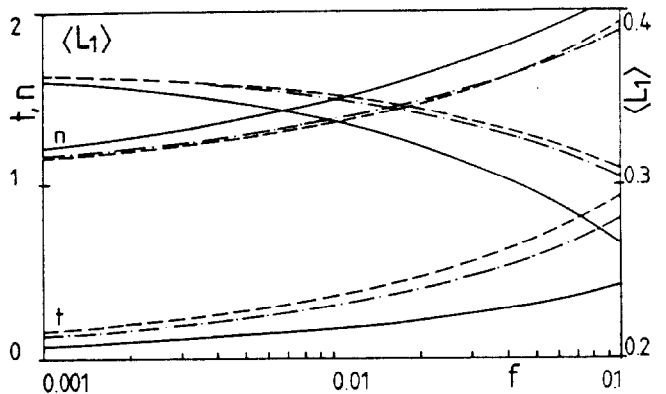


Fig.3 - Optimum scaled L_1 , t and n_0 versus f for : (- - -) no IBS and $\tau_g \rightarrow \infty$, (-.-.-) IBS and $\tau_g \rightarrow \infty$, (—) IBS and $\tau_g = 2d$.

with the filling time. The effect of IBS is rather small, but the effect of beam-gas scattering is noticeable. It reduces the optimum collision time by almost a factor of two, and $\langle L_1 \rangle$ by up to 20% at extremely large filling times. For typical LHC parameters, the reduction of $\langle L_1 \rangle / k$ is about 5%. Increasing the emittances beyond the nominal values has a very small effect on $\langle L_1 \rangle / k$, because it only increases the intra-

beam scattering and beam-beam lifetimes which themselves have rather small effects. The total $\langle L_1 \rangle$ is simply proportional to the number of bunches. In order to optimise the luminosity $\langle L \rangle$ for pp colliders, the intensity of the bunches must be as high, and their transverse emittances must be as low as is compatible with constraints a) and b).

In the optimisation of $\langle L \rangle$ and $\langle L_1 \rangle$ for $\bar{p}p$ colliders the $\bar{p}$ accumulation rate A and the $\bar{p}$ intensity N^- available from the $\bar{p}$ source must be taken into account. This is done by optimising $\langle L \rangle/k$ and $\langle L_1 \rangle/k$ with variable bunch populations N_i and collision time T as for pp colliders, and then choosing the maximum number of bunches k which is compatible with A and N^- , given by $k < AT/N_0$ and $k < N^-/N_0$, respectively, where N_0 is the initial $\bar{p}$ bunch population. We do not discuss upper limits on k which may arise from the bunch spacing required for the filling procedure or the separation of the p and $\bar{p}$ beams. The optimum $\langle L_1 \rangle$ obtained when A , k or N^- , respectively, are held constant is given in Fig.4 which shows that $\langle L_1 \rangle$ is proportional to any one

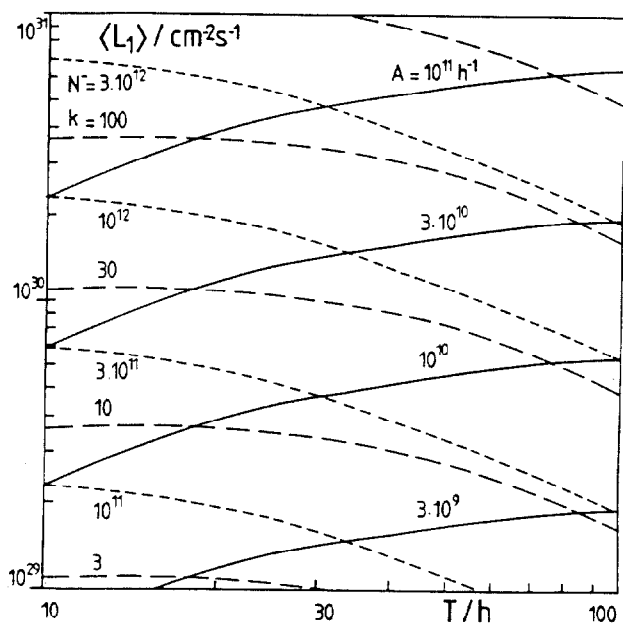


Fig.4 - $\langle L_1 \rangle$ versus T for constant A (—), constant k (---) and constant N^- (.....).

of these three parameters. However, the optimum value is only achieved when all three parameters have the appropriate values. The effect of the emittance on $\langle L_1 \rangle$ for fixed A and hence variable k is shown in Fig.5. It can be understood with the help of (1) and (2). When the $\bar{p}p$ collider is operated at the beam-beam limit, L depends on $N_0 k = N^- AT$, and hence is independent of k . Therefore a dependence of $\langle L_1 \rangle$ on the emittance arises only from the exponential in (2). When the parameters are such that $n \approx 1$, a reduction of the emittance increases $\langle L_1 \rangle$ because the concomitant increase in k reduces the argument of the exponential. However, $\langle L_1 \rangle$ becomes independent of the emittance when $n \ll 1$ because the argument of the exponential is already small. In order to optimise $\langle L \rangle$ for $\bar{p}p$ colliders, we again choose the intensity of the bunches as high, and their transverse emittance as low as is compatible with constraints a) and b), and we satisfy constraints c) and d) by adjusting k .

4. Conclusions

Our results for the LHC are summarised in Table 2. We either optimise the average luminosity $\langle L \rangle$

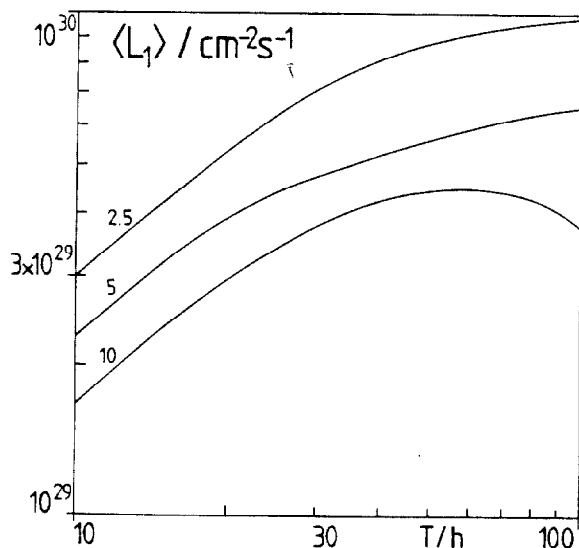


Fig.5 - $\langle L_1 \rangle$ versus T for various emittances in units of $\pi \mu m$ with $A = 5 \times 10^{10} h^{-1}$ and variable k .

or the average probability of having exactly one event in a collision $\langle L_1 \rangle$. The fixed parameters are underlined. For pp collisions, we adopt the maximum number of bunches compatible with a bunch spacing of 25 ns, and use the collision time as a free variable. For $\bar{p}p$ collisions, we fix the collision time $T = \frac{25}{h}$, and the $\bar{p}$ production rate $A = 5 \times 10^{10} h^{-1}$ [9], and calculate the number of bunches k . In order to achieve higher values of $\langle L_1 \rangle$ and $\langle L \rangle$, all three parameters A , k and the total number of particles N would have to be increased in proportion.

Table 2 : Optimum settings of the LHC

Collisions	pp	pp	$\bar{p}p$	$\bar{p}p$
Optimum	$\langle L_1 \rangle$	$\langle L \rangle$	$\langle L_1 \rangle$	$\langle L \rangle$
k	3564	3564	64	48
$\langle L_1 \rangle / 10^{30} cm^{-2} s^{-1}$	130	96	2.2	1.6
$\langle L \rangle / 10^{30} cm^{-2} s^{-1}$	370	960	6.6	7.9
T/h	13.1	4.3	25	25
$A / 10^{10} h^{-1}$	-	-	5.0	5.0
$N / 10^{12}$	60	91	1.3	1.3

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