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Signal Suppression Analysis for the Momentum Stochastic Cooling with a Multiple System

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Introduction

The Tevatron I project[1] at Fermilab depends very crucially on the momentum stochastic cooling to collect and store antiprotons produced by injecting protons on a target. This project differs from the similar and original one in CERN[2] by the fact that it requires larger $\bar{p}$-flux, that is a larger number of antiprotons collected per unit of time, and a larger final momentum density. Both of these quantities are about an order of magnitude larger in the Fermilab project. Moreover, the stochastic cooling design is made of several systems each with its own pick-up, kickers and chain of amplification. These systems could overlap in the frequency bandwidth as well in the beam reponse dynamics. It has been argued therefore that the performance of the momentum cooling could have been limited by the signal suppression which one derives when examining the cooling system in closed loop.

In this paper we make an analysis of the closed loop system for momentum stochastic cooling. In the closed loop configuration the electronic feedback depends not only on the electronic component characteristics but also on the beam intensity and energy distribution. An analysis for a single system has already been done[3,4,5] requiring basically the solution of one dispersion relation. Our results give the interrelation between the signals from the different parts of the system and their mutual enhancement or suppression. All this is described by a matrix notation. There is need now to solve a larger number of dispersion relations.

Derivation of the Vlasov Equation

Define the total beam density distribution $\psi_t(\theta,\varepsilon,t)$ where θ is the angular position around the main closed orbit and ε is an energy difference variable. The conservation law requires

$$\frac{\partial\psi_o}{\partial t} + \mathrm{div}\,(\vec{u}\psi_t) = 0 \qquad (1)$$

Where $\vec{u} \equiv (\dot\theta,\dot\varepsilon)$ and $\dot\theta = \Omega(\varepsilon)$ is the rotation frequency and $\dot\varepsilon = \dot\varepsilon(\varepsilon,t)$ the energy gain per unit of time. Split the total distribution $\psi_t = \psi_o + \psi$ as the sum of an unperturbed distribution $\psi_o = \psi_o(\varepsilon,t)$ and a perturbation $\psi = \psi(\theta,\varepsilon,t)$. Also $\dot\varepsilon = \dot\varepsilon_o + \dot\varepsilon_p$ where $\dot\varepsilon_o$ are the external cooling forces and $\dot\varepsilon_p$ are the forces created by the perturbation.

The cooling equation is derived from

$$\frac{\partial\psi_o}{\partial t} + \frac{\partial}{\partial\varepsilon}\,(\dot\varepsilon_o\psi_o) = 0$$

There is a term $\partial/\partial\varepsilon(\dot\varepsilon_p\psi)$ which of second order magnitude and it will be neglected. Another term that is also neglected is $\partial/\partial\varepsilon(\dot\varepsilon_o\psi)$ which represents the effect of cooling on the perturbation.

From (1) one then derives the following (Vlasov) equation for the perturbation ψ

$$\frac{\partial\psi}{\partial t} + \Omega(\varepsilon)\,\frac{\partial\psi}{\partial\theta} + \frac{\partial}{\partial\varepsilon}\,(\dot\varepsilon_p\psi_o) = 0 \qquad (2)$$

The next step is the calculation of $\dot\varepsilon_p$.

Calculation of the perturbation Force

We assume the cooling device is made of N_s systems. In Fig. 1 we show a case with $N_s = 5$. A dark line shows one particular system. This system joins pick-up station #1 with kicker station #2. The station numbers should not be confused with the system number. In principle it should be possible to break up the analysis to the smallest detail which is made of a system connecting one single pick-up to one single kicker. Indeed pick-ups (and kickers) are distinguishable from each other because at least they occupy different locations around the ring. Nevertheless here for simplicity we shall consider a system as connecting one group of pick-ups to another group of kickers. But in our derivaton we will not neglect the fact that there are two or more groups of pick-ups and kickers.

The j-th system includes N_{pj} pick-ups at the location θ_{pj} connected to N_{kj} kickers at the location θ_{kj}. The extension of the pick-ups and kickers is here obviously neglected. The perturbation current in exit of the pick-up station is

$$I_j(t) = e\int\psi(\theta_{pj},\varepsilon,t)\,S_{pj}(\varepsilon)\Omega(\varepsilon)d\varepsilon \qquad (3)$$

where S_{pj} is the pick-up energy response function. The voltage applied at the kicker location θ_{kj} by the perturbation itself is

$$V_j(t) = \frac{1}{2\pi}\int G_j(t-t')\,I_j(t')dt' \qquad (4)$$

where $G_j(t)$ is the transfer Green function between pick-ups and kickers of the same system. We prefer to express quantities in the frequency domain, so that introducing the overall transfer impedance function $Z_j(\omega)$

$$G_j(t) = \int Z_j(\omega)e^{i\omega t}\,d\omega \qquad (5)$$

and

$$Z_j(\omega) = A_j\,\sqrt{N_{pj}N_{kj}R_{kj}R_{kj}}\;g_j(\omega)e^{-i\omega t_j} \qquad (6)$$

where A_j is an overall linear gain, R_{pj} the effective

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pickup impedance, R_{kj} the effective kicker impedance, t_j a delay time between pick-ups and kickers and g_j includes special filter, amplifier, pick-up, kicker and any other items in the system with special frequency dependence.

The total energy gan per unit of time is

$$\dot{\varepsilon}_p = e\ \Omega\ (\varepsilon) \sum_j \delta(\theta - \theta_{kj})\ V_j(t)\ S_{kj}\ (\varepsilon) \qquad (7)$$

where the summation is done over all the systems. $\delta(\theta - \theta_{kj})$ is a delta function which expresses the fact that the gain is applied at the kicker location and $S_{kj}(\varepsilon)$ is the kicker energy response.

For obvious reasons of convenience, in the following we shall make use of the expansions

$$\delta(\theta - \theta_{kj}) = \frac{1}{2\pi} \sum_m e^{im(\theta - \theta_{kj})} \qquad (8)$$

and

$$\psi = \sum_m e^{im\theta} \int \tilde{\psi}_m(\varepsilon, \omega)\ e^{i\omega t}\ d\omega \qquad (9)$$

where $\tilde{\psi}_m(\varepsilon, \omega)$ could either be the Fourier transform of $\psi(t)$ if one desires to investigate the stability of the beam or the Laplace transform. In the first case the perturbation is present from $t = -\infty$ on, in the latter it appears only at $t = 0$ and $\psi = 0$ for $t < 0$.

Derivation of Dispersion Relations

The transform functions $\psi_m(\varepsilon, \omega)$ are normal to each other with respect to both the mode number m and the frequency ω. Therefore it is possible to substitute (7) with (8) and (9) in the Vlasov equation (2) and obtain an equation for $\tilde{\psi}_m$

$$\tilde{\psi}_m = \frac{\psi_m(0)}{i(\omega + m\Omega)} + \qquad (10)$$

$$- \frac{e^2}{2\pi} \sum_j e^{-im\theta_j} \frac{Z_j(\omega) H_j(\omega)}{i(\omega + m\Omega)} \frac{d}{d\varepsilon}\left[\psi_0(\varepsilon) S_{kj}(\varepsilon)\ \Omega(\varepsilon)\right]$$

where $\psi_m(o) = 0$ if the Fourier transform is taken and

$$\psi_m(o) = \frac{1}{2\pi}\ \psi_m(\varepsilon,\ t{=}0^+)$$

applies with the Laplace transform. Moreover

$$H_j(\omega) = \sum_n e^{in\theta_{pj}} \int \tilde{\psi}_n(\varepsilon', \omega)\ S_{pj}(\varepsilon')\ \Omega(\varepsilon')\ d\varepsilon' \qquad (11)$$

is the Fourier or Laplace transform of the perturbation current

$$I_j(t) = e \int H_j(\omega) e^{i\omega t}\ d\omega. \qquad (12)$$

We now (i) multiply both sides of (10) by $\exp(im\theta_{ps})$, (ii) sum both sides over all m's, (iii) multiply both sides by $S_{ps}(\varepsilon)\ \Omega(\varepsilon)$, and (iv) integrate both sides over ε. We obtain the following N_s dis persion relations ($s = 1,2,\ldots N_s$)

$$H_s(\omega) = B_s(\omega) + \sum_j J_{sj}(\omega)\ H_j(\omega) \qquad (13)$$

where

$$B_s(\omega) = -i \int d\varepsilon\ S_{ps}(\varepsilon) \Omega(\varepsilon) \sum_m \frac{\psi_m(o)\ e^{im\theta_{ps}}}{\omega + m\Omega(\varepsilon)} \qquad (14)$$

and the dispersion integrals

$$J_{sj}(\omega) = \frac{e^2 i}{2\pi} Z_j(\omega) \int \frac{d}{d\varepsilon'}\left[\psi_0(\varepsilon') S_{kj}(\varepsilon') \Omega(\varepsilon')\right] \Omega(\varepsilon') \times$$

$$\cdot\, S_{ps}(\varepsilon') \sum_m \frac{\exp[im(\theta_{ps} - \theta_{kj})]}{\omega + m\Omega(\varepsilon')}\ d\varepsilon' \qquad (15)$$

It is convenient to make use of vector and matrix rotation. Define the vectors

$$\vec{B} \cdot (B_1, B_2, \ldots B_{NS})$$
$$\vec{H} \cdot (H_1, H_2, \ldots H_{NS})$$

and the maxtrix D with elements

$$D_{sj} = \delta_{sj} - J_{sj}$$

where $\delta_{sj} = 0$ for $s \neq j$ and $\delta_{ss} = 1$. Eq. (13) can also be written simply

$$\vec{B} = D\ \vec{H} \qquad (16)$$

which can be inverted to give

$$\vec{H} = D^{-1}\vec{B} \quad \text{or} \quad H_j = \sum_s D_{js}^{-1}\ B_s \qquad (17)$$

Discussion - System Stability

To understand Eq. (17), which is here our major result, we propose to look at the cases of $N_s = 1$ and 2 systems which are represented with the diagrams of Fig. 2 and 3. With only one system we actually get from Eq. (13)

$$H_1 = \frac{B_1(\omega)}{1 - J_{11}(\omega)}$$

B_1 being the perturbation in the open loop case. If the loop is closed the signal is effectively reduced (or enhanced) as one can customary derive by inspecting the diagram of Fig. 2. There is only one signal here to worry about and the analysis is straight forward. But for two systems, both of them in closed loop, the actual signal from say pick-up station #1 will depend on the perturbation not only at that station but also at the other one #2. In general for $N_s > 1$ the actual signal from one particular system will be given as a linear combination of all the open loop signals from all the systems.

Another issue is the system stability against possible errors added to the transfer function. We want first of all to point out that for those frequencies such that

$$\text{Determinant } D(\omega) = 0 \qquad (18)$$

the whole cooling device is intrinsically unstable. We assume here that this is not the case. To estimate the stability of the j-th system we can still solve an equation similar to (18) which is obtained by multiplying all the elements J_{sj} of the j-th column by an error function $E(\omega)$. An ideal system with no error has E = 1. In general $E(\omega)$ is a complex function, and stability diagrams can be obtained by plotting the imaginary versus the real part of it.

Signal Suppression

This is best described by the Fourier transform $U(\omega)$ of the energy gain per turn which we can write as the following summation over all the systems

$$U(\omega) = e\sum_s S_{Ks}(\varepsilon) Z_s(\omega) P_s(\omega) B_s(\omega) e^{i\omega t_s} \tag{19}$$

where

$$P_s(\omega) = \sum_j \frac{S_{Kj}(\varepsilon) Z_j(\omega)}{S_{Ks}(\varepsilon) Z_s(\omega)} D_{js}^{-1}(\omega)\, e^{i\omega(t_j - t_s)} \tag{20}$$

is a complex factor which measures the suppression (or the enhancement if $|P_s| > 1$) of the contribution to the total voltage from the s-th system. In absence of any closed loop effects obviously $P_s(\omega) = 1$ and Eq. (19) reduces to the usual formula which is used to estimate the energy gain per turn.

A computer code (SSBS) is now available here at Fermilab that calculates not only the stability diagram of each system of a stochastic cooling device, but also the various suppression factors (20). A crucial part in these calculations is the inversion of the complex matrix D which also requires a careful analysis[5] of the dispersion integrals (15). This has been done now also for the case of multiple poles (the zeroes at the denominator of (15) in a specified energy range).

References

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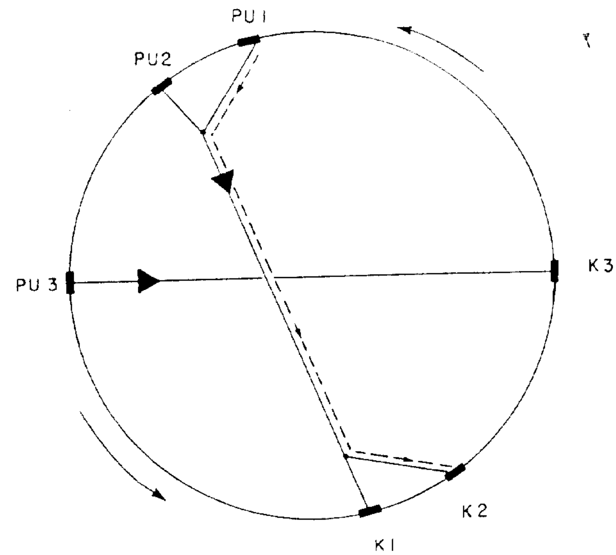

Fig. 1 Schematics of a Stochastic Cooling Device with $N_s = 5$ systems

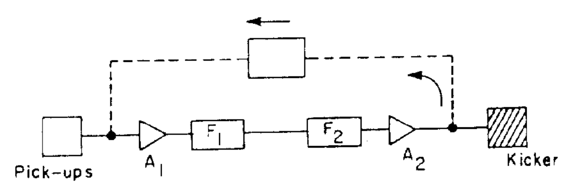

Fig. 2 Closed Loop Diagram with $N_s = 1$ system

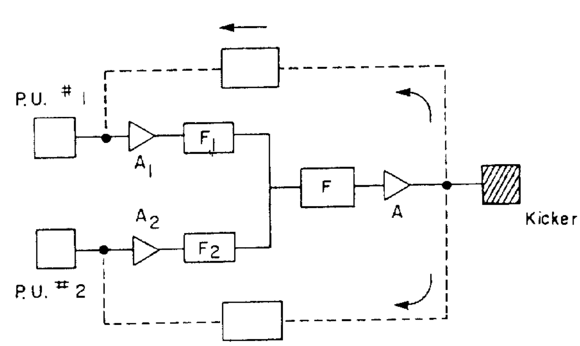

Fig. 3 Closed Loop Diagram with $N_s = 2$ systems