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CHOICE OF FOCUSING STRENGTH AND APERTURE FOR HIGH ENERGY SYNCHROTRONS AND COLLIDERS

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Introduction

At low energies and low intensities the considerations in the choice of the focusing strength (betatron tune ν) for a synchrotron are the beam size and the orbit distortion due to field errors which, together, generate a geometrical requirement on the size of the beam pipe and the good field aperture. Indeed, the strong focusing principle was invented to reduce the necessary magnet aperture, thereby the cost of the magnets. The closed orbit distortion increases with ν and was cited[1] as the factor counteracting the desire to reduce the beam size indefinitely by going to arbitrarily strong focusing. At high energies and intensities and with modern technology this is no longer true. The beam size is generally negligibly small and the orbit distortion can be corrected to arbitrary desired accuracy. Studies of field errors and orbit distortions are now used for sizing the correction magnet system rather than the aperture. Other types of geometrical demands on the aperture arise from beam manipulations such as stacking and resonant extraction. These requirements tend to be local and can usually be satisfied by local lattice insertions (high- or low-β, high- or zero-dispersion etc.).

In high energy synchrotrons or colliders for which there is no geometrical demand on aperture why, then, can one not apply arbitrarily strong focusing (high ν) and use millimeter apertures for millimeter beams? The reason is the electromagnetic requirements which must be satisfied in order to stably transport a high current beam of whatever size through a conducting pipe.* The beam current induces a voltage through an "impedance" of the beam pipe. This voltage can act back on the beam as positive feedback and make it unstable. Low frequency components of these coherent instabilities can be damped by negative electronic feedback systems, but high frequency components can only be checked by Landau damping derived from a spread in the natural frequencies of individual particles in the beam which causes the instability to lose coherence. The larger is the frequency spread and the smaller is the "impedance" the more stable is the beam.

The "impedance" depends and, therefore, imposes demands on the material, the structure, the shape and the size of the beam pipe. The tune spread $\Delta\nu$ is limited by non-linear resonances. The excitations of high order resonances by magnetic field errors are small and negligible beyond the octupole. But in colliders the excitation by beam-beam forces is large and resonances up to the 7^{th} order must be avoided. This imposes a severe limitation on the allowable tune spread. This excitation is, however, independent of the orbit functions and hence makes no demand on the focusing strength. The dynamics of coherent instabilities of beams is a complex, multidimensional problem. To make our discussion understandable we will resort to using simplified semi-quantitative descriptions.

*We ignore here the economic considerations. Extremely strong focusing, millimeter-aperture synchrotrons may be extremely costly. A tiny lady's watch costs more and is much less reliable than a man's pocket railroad watch.

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Conditions for Stabilities and Implications on Focusing Strength

The condition for longitudinal stability is[2,3], at high energies

$$\left\langle \frac{|Z_\ell|}{n} \right\rangle < F_\ell \, \frac{E/e}{I} \, \frac{1}{\nu^2} \left(\frac{\Delta p}{p}\right)^2 \tag{1}$$

where we have used the approximation

$$\frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \cong \frac{1}{\gamma_t^2} \cong \frac{1}{\nu^2}$$

and where

γ_t = transition energy in units of rest energy
F_ℓ = beam distribution form-factor of order unity
Z_ℓ = longitudinal impedance
n = mode number = number of instability waves around the ring
$\langle\ \rangle$ denotes value weighed by the mode spectrum
E = energy of beam
I = peak current of beam
$\frac{\Delta p}{p}$ = FWHM of momentum spread.

The condition for transverse stability[4] is

$$\left\langle |Z_t| \right\rangle < \pi F_t \, \frac{E/e}{I} \, \frac{\nu}{R} \, \Delta\nu \tag{2}$$

where

F_t = beam distribution form-factor of order unity
Z_t = transverse impedance
R = radius of ring
$\Delta\nu$ = tune spread in beam.

There are two main types of contribution to the impedance. The beam contribution depends on the energy and the dimensions of the beam and is non-zero even when the beam pipe is removed. This is generally small for the range of parameters in consideration. The wall contribution is that due to the charge and current induced by the beam on the pipe wall and depends, therefore, on the material and the geometry of the beam pipe. The wall contributions of the longitudinal and the transverse impedances are related through the geometry of the pipe. For a circular beam pipe of radius b it is[4]

$$Z_t = \frac{2R}{b^2} \frac{Z_\ell}{n} \tag{3}$$

There are two types of terms in the wall contribution to Z_ℓ/n. The "smooth" term, usually known as the resistive wall term, is that of a perfectly uniform and smooth pipe and depends on the size and the skin depth of the pipe. It is rich in low frequencies and is generally small. The most important is the "interruptions" term arising from discontinuities in the pipe and from various beam sensing and manipulating devices inserted in the pipe. This term in Z_ℓ/n is not sensibly dependent on the pipe size. For further discussion we shall consider Eq. (3) as the approximate relation between the total contributions to the impedances.

Substituting Eq. (3) in Eq. (2) we can rewrite the condition for transverse stability as

$$\left\langle \frac{|Z_\ell|}{n} \right\rangle < \frac{\pi}{2} F_t \frac{E/e}{I} \left(\frac{b}{R}\right)^2 \nu\Delta\nu. \tag{4}$$

Solving Eqs. (1) and (4) for ν we see that the choice of ν is hemmed in by longitudinal and transverse stability requirements as

$$V\left\langle \frac{|Z_\ell|}{n} \right\rangle < \nu < U\left\langle \frac{|Z_\ell|}{n} \right\rangle^{-\frac{1}{2}} \tag{5}$$

with

$$\begin{cases} U \equiv \left(\frac{E/e}{I}\right)^{\frac{1}{2}} \frac{\Delta p}{p} \propto \left(\frac{E}{I}\right)^{\frac{1}{2}} \frac{\Delta p}{p} \\ V \equiv \frac{2}{\pi} \frac{I}{E/e} \left(\frac{R}{b}\right)^2 \frac{1}{\Delta\nu} \propto \frac{I}{E} \left(\frac{R}{b}\right)^2 \end{cases} \tag{6}$$

where, consistent with the approximation, we have put $F_\ell = F_t = 1$. The available range for ν shrinks to zero when

$$\left\langle \frac{|Z_\ell|}{n} \right\rangle = \left(\frac{U}{V}\right)^{\frac{2}{3}} = \left(\frac{\pi}{2}\right)^{\frac{2}{3}} \frac{E/e}{I} \left(\Delta\nu \frac{\Delta p}{p}\right)^{\frac{2}{3}} \left(\frac{b}{R}\right)^{\frac{4}{3}}. \tag{7}$$

At this point the single allowable value of ν is

$$\nu = (U^2V)^{\frac{1}{3}} = \left[\frac{2}{\pi} \frac{1}{\Delta\nu} \left(\frac{\Delta p}{p}\right)^2 \left(\frac{R}{b}\right)^2\right]^{\frac{1}{3}}. \tag{8}$$

This is a good value to choose for ν in any case because it allows the largest value of the impedance $\langle |Z_\ell|/n \rangle$.

Numerical Example and Scaling Laws

For a 20 TeV proton collider, assuming

E = 1 TeV (injection energy)*

I = 5 A (10^{11} protons in a 1 m long bunch)

$\frac{\Delta p}{p} = 2\times10^{-4}$ (ε_ℓ = 1 eVsec, $\Delta\ell = \frac{3}{2}$ m)

R = 8 km (using 10 T dipoles)

b = 0.0254 m (1 inch radius aperture)

$\Delta\nu$ = 0.01 (limited by resonances)

*We consider here only the injection energy. However, condition (5) must be satisfied over the entire operating energy range.

we get

$$(31.6\ \Omega^{-1})\left\langle \frac{|Z_\ell|}{n} \right\rangle < \nu < (89.4\ \Omega^{\frac{1}{2}})\left\langle \frac{|Z_\ell|}{n} \right\rangle^{-\frac{1}{2}}$$

and the maximum allowable value of

$$\left\langle \frac{|Z_\ell|}{n} \right\rangle = 2\ \Omega$$

with a choice of

$$\nu = 63.2.$$

The "impedance" of 2 Ω is achievable but not without difficulty. If the aperture were reduced by a factor 2 the maximum allowable "impedance" would be down to the nearly impossible value of ~0.8 Ω. It is interesting to take a look at how the conditions given by Eq. (5) scale with respect to different parameters.

A wider range of acceptable ν value would allow larger values of $\langle |Z_\ell|/n \rangle$. Hence we would like U to be large and V to be small. To increase U and decrease V we should

1. Increase E/I. This extends the acceptable ν-range at both ends. This also shows that the tightest constraint occurs at injection when E is lowest. Reducing I helps, but the luminosity suffers.

2. Increase $\Delta p/p$. This raises the upper limit of the ν-range, but requires either blowing up the longitudinal emittance or a huge increase in rf voltage (as the 4th power of $\Delta p/p$). Neither alternative is very attractive.

3. Increase b/R. Because of the squared dependence this is very effective in lowering the lower limit of the ν-range. Since the stored energy in the magnet ring is proportional to $B^2 x (b^2R) \propto b^2/R$ (B = magnetic field), to minimize the increase in stored energy it is more desirable to reduce R than to increase b.

References

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2. E. Keil and W. Schnell, CERN Report, ISR-TH-RF/69-48 (1969)
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