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BEAM SHAPE DISTORTION CAUSED BY TRANSVERSE WAKE FIELDS*

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Introduction

As a particle bunch in a storage ring passes through a region with a transverse impedance, it generates a transverse wake electromagnetic field that is proportional to the transverse displacement of the bunch in the region. The field acts back on the bunch, causing various effects (such as instabilities) in the motion of the bunch.[1]

In this paper, we study one of such effects in which a transverse impedance causes the beam to be distorted in its shape. Observed at a fixed location in the storage ring, this distortion does not change from turn to turn; rather, the distortion is static in time. To describe the distortion, the bunch is considered to be divided longitudinally into many slices and the centers of charge of the slices are connected into a curve. In the absence of transverse impedance, this curve is a straight line parallel to the direction of motion of the bunch. Perturbed by the transverse wake field, the curve becomes distorted. What we will find in this paper is the shape of such a curve.

The results obtained are applied to the PEP storage ring. The impedance is assumed to come solely from the RF cavities. We find that the beam shape is sufficiently distorted and hence that loss of luminosity due to this effect becomes a possibility.

Derivation

Assume the effect caused by the transverse impedance $Z_\perp$ is small so that we only have to evaluate effects up to first order in $Z_\perp$, or equivalently, first order in the beam current. Let y_z be the transverse displacement of the bunch as it passes through $Z_\perp$ in the limit of zero beam current. The dipole moment that is sampled by $Z_\perp$ every time the bunch passes by is

$$y_z\,\rho(\theta) \tag{1}$$

where $\rho(\theta)$ is the longitudinal charge distribution of the bunch with $\theta = z/R$ (z is the longitudinal coordinate relative to the center of the bunch, R is the average radius of the ring). The EM force excited by having this dipole moment passing through $Z_\perp$ turn after turn is[1,2]

$$F_\perp = e\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)_\perp = ie\omega_0\, y_z\, \delta(s) \sum_{p=-\infty}^{\infty} Z_\perp(p)\, \tilde{\rho}(p)\, e^{ip\theta} \tag{2}$$

where $Z_\perp(p)$ is evaluated at the frequency $p\omega_0$ with ω_0 the revolution frequency. The delta-function $\delta(s)$ indicates that the impedance is located at position $s=0$. The quantity $\tilde{\rho}(p)$ is the Fourier transform of $\rho(\theta)$:

$$\tilde{\rho}(p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\theta\, e^{-ip\theta}\, \rho(\theta) \quad . \tag{3}$$

In the static situation, a particle at θ experiences a transverse kick due to the impedance:

$$\Delta y'(\theta) = \frac{e}{\mathscr{E}} \int \left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)_\perp ds \tag{4}$$

where $\mathscr{E}$ is the particle energy. Substituting Eq. (2) into Eq. (4), we find

$$\Delta y'(\theta) = \frac{ie\omega_0}{\mathscr{E}}\, y_z \sum_p Z_\perp(p)\, \tilde{\rho}(p)\, e^{ip\theta} \quad . \tag{5}$$

The transverse kick (5) causes particle orbits to deform. Particles at different θ's see different kicks and will follow different closed orbits, leading to a distortion of transverse bunch distribution.

We emphasize the fact that the effect we are considering is a static effect. The beam is assumed to be stable in this statically distorted state. The distortion is a result of having the bunch passing by the impedance off-centered; the produced wake then acts on the particles, causing them to follow distorted closed orbits which depend on the longitudinal positions of the particles.

An analogy of this effect is well-known in the longitudinal case. A longitudinal impedance will distort the longitudinal distribution of the bunch; the distortion in this case is again static in time. Such an effect has been studied and is referred to as the potential-well distortion.[3]

Note that the charge distribution in the previously mentioned slices are not individually distorted; only their centers of charge are affected by the impedance. This is because the impedance we are considering samples and acts only on the dipole moments of the slices. The knowledge of impedance of a higher order that samples and acts on, say, the quadrupole moments of the slices is needed in order to describe the change in transverse charge distribution of individual slices.

A particle executing synchrotron oscillation in its θ-coordinate experiences kicks modulated by the synchrotron oscillation. The time averaged trajectory of the particle, however, is still described by the static picture as if particles were frozen in θ and kicked by $\Delta y'$ of Eq. (5). This is true even if the transverse radiation damping rate is smaller than the synchrotron oscillation frequency. On the other hand, it breaks down near synchro-betatron resonances.

Single Pass Wake

If we had precise information on the impedance $Z_\perp$, it should be possible to perform the summation over p in Eq. (5). Unfortunately, this is not the case for PEP for the time being. This is especially the case for the RF cavity impedance which consists of a large number of sharp peaks at many different frequencies ω. For each peak it is relatively easy to estimate the area under the $Z_\perp(\omega)$ versus ω curve but the locations of the peaks are not known precisely enough. For such a case we will contend ourselves with a statistical estimate,[4] i.e., we assume the locations of the impedance peaks are randomly distributed in ω. Effectively this means smearing the RF impedance peaks into a broad-band average, keeping the area under the $Z_\perp(\omega)$ versus ω curve the same.[5] Under this condition, the wake field is a single pass wake; multi-turn effects are ignored. The summation over p in Eq. (5) can be replaced by an integration over p. The head of the bunch is not perturbed since it sees no wake field.

For instance, we can represent the impedance by that of a broad-band resonator; i.e.,

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$$Z_\perp(p) = \frac{rh/p}{1 - iQ\left(\frac{p}{h} - \frac{h}{p}\right)} \qquad (6)$$

where r, h and Q are the parameters specifying the impedance. A broad-band resonator has a quality factor Q of the order of unity.

For simplicity, we assume a uniform bunch distribution

$$\rho(\theta) = \begin{cases} \frac{Ne}{2a} & \text{if } |\theta| \le a \\ 0 & \text{otherwise} \end{cases} \qquad (7)$$

where $a = \sqrt{3}\,\sigma_z/R$ with σ_z the rms bunch length and N is the total number of particles in the bunch. Substituting Eqs. (6) and (7) into (5) and replace the summation over p by an integral, we obtain

$$\Delta y'(\theta) = \frac{Ne^2\omega_0}{\mathscr{E}}\, y_z \frac{r}{2aQ} S\left(\frac{\theta}{a}, ha, Q\right) \qquad (8)$$

where the function S is found to be

$$S(x,y,Q) = 1 - \exp\left[-\frac{y}{2Q}(1-x)\right]\left\{\frac{\sin\left[y(1-x)\sqrt{1-(1/4Q^2)}\right]}{\sqrt{4Q^2-1}} + \cos\left[y(1-x)\sqrt{1-(1/4Q^2)}\right]\right\} \quad (|x| \le 1). \qquad (9)$$

Bunch shape is distorted as a result of the kicking angle. The bunch head has $\theta = a$ (i.e. $x = 1$) and $\Delta y' = 0$.

Experimentally, the displacement of the bunch as a whole is measured. The transverse impedance gives rise to a measured closed-orbit as a function of beam intensity. From Eq. (8), the bunch as a whole sees a kick

$$\overline{\Delta y'} = \frac{Ne^2\omega_0}{\varepsilon}\, y_z \frac{r}{2aQ} \bar{S}(ha, Q) \qquad (10)$$

with $\bar{S}$ obtained by averaging S over the bunch:

$$\begin{aligned} S(y,Q) &= \frac{1}{2}\int_{-1}^{1} dx\, S(x,y,Q) \\ &= 1 - \frac{1}{2yQ}\left[1 - e^{-y/Q}\cos\left(2y\sqrt{1-(1/4Q^2)}\right)\right] \\ &\quad - e^{-y/Q}\left(1 - \frac{1}{2Q^2}\right)\frac{\sin\left(2y\sqrt{1-(1/4Q^2)}\right)}{2y\sqrt{1-(1/4Q^2)}} . \end{aligned} \qquad (11)$$

Numerical Estimate for PEP

The equivalent broad-band impedance for PEP is taken to be the sum of two resonator impedances as described by Eq. (6). The parameters for the two contributions are obtained by fitting the calculated impedance for the PEP cavities:[6,7]

$$\begin{aligned} r_1 &= 2.55\ \text{k}\Omega/\text{m per cavity cell}, \\ r_2 &= 0.24\ \text{k}\Omega/\text{m per cavity cell}, \\ h_1\omega_0/2\pi &= 1.5\ \text{GHz}, \quad h_2\omega_0/2\pi = 5.5\ \text{GHz}, \\ Q_1 &= 1.5, \quad Q_2 = 1.0 . \end{aligned} \qquad (12)$$

In the calculation below, we have not included the possible impedance contributions from the vacuum chamber discontinuities other than that of the RF cavities. If we further take single bunch current = 10 mA

$$\begin{aligned} \mathscr{E} &= 15\ \text{GeV} \\ y_z &= 2\ \text{mm} \\ a &= \sqrt{3}\,\sigma_z/R = 10^{-4} \quad (\sigma_z = 2\ \text{cm}) \end{aligned} \qquad (13)$$

the kicking angle as a function of θ across the bunch is obtained from Eq. (8) and is shown in Fig. 1.

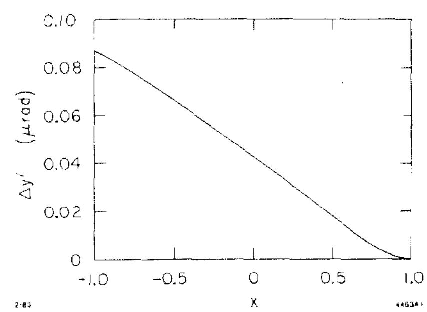

Fig. 1. The kicking angle, Δy', received by a particle as it traverses an RF cavity cell off-centered by 2 mm versus the location of the particle in the beam bunch. A particle at the head of the bunch (x = 1) receives no kick.

At the tail of the bunch, the kicking angle is 0.087 μrad for each cavity cell. Since there are three RF cavity locations in PEP and each location has 40 cavity cells, the net kicking angle is taken to be 0.087 μrad × $40\sqrt{3}$ = 6.0 μrad.

The closed-orbit of the bunch tail differs from that of the bunch head by an amount of the order of $6.0\sqrt{\beta_{obs}\beta_{cav}/2}$ μrad, where β_{obs} and β_{cav} are the beta-functions at the observation point and at the cavity, resprectively. At the interaction point, $\beta_{obs} = 0.11$ m. Taking $\beta_{cav} = 40$ m, we find that the bunch head and bunch tail displace transversely from each other by an rms of ~8.9 μm. Actual bunch tilts in different interaction regions can be different, causing different luminosity values. If we then collide two 10 mA bunches, luminosity decreases somewhat since the head-tail separation is comparable to the vertical beam size, which is about 20 μm for the same machine parameters and optimum coupling.

The average kick seen by the bunch as a whole is 3.0 μrad, giving rise to a closed-orbit of the order of 0.23 mm at a point of high beta-function $\beta_{obs} = 300$ m. It should be possible to observe this orbit distortion at the high-β positions. The discussion of the subject of this paper in more details can be found in work.[8]

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