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Emittance Growth of High Current Beams in Transport Lines

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Abstract

The rms emittance increase of high current beams in transport lines, caused by nonlinear space charge forces, can be understood as the development of the initial particle distribution to an equilibrium one. For a small beam radius, we get less rms emittance increase and the equilibrium distribution is reached earlier than for a large beam radius. The beam can now be transported over a long distance without further rms emittance growth in all 3 directions. These results are incorporated in the design of a funneling section for the high current linear accelerator of the HIBALL concept.

Demonstration of the effects in a drift space

During the last few years relations were found[1] which allow the design of high current linacs or periodic transport lines[2] without substantial transverse and longitudinal emittance growth. Most of these relations are based on simple models (e.g. equipartitioning[3], smooth approximation) or derived from special phase-space distributions (e.g. KV-distribution) of the beam. Not very much is known about the equilibrium distribution[4], which lies behind the smaller observed emittance increase.

For a non-periodic transport system, there do not exist such simple design relations, almost nothing is known about the equilibrium distribution.

In order to simplify here the situation emittance growth of high current beams are studied in a drift space. We used a Bi^{2+} (N = 209)-beam with E_{kin} = 1.7MeV/N, I_p=100 mA, f = 54 MHz. All calculations are done with the fully 3 dimensional multiparticle simulation code MOTION[5].

In Fig. 1 - 4, the rms emittances and the total beam envelopes are shown for two different cases. The phase space filling was ellipsoidal, uniform in the four transverse, and independent uniform in the two longitudinal coordinates. In every case, we calculated the emittances also for an uniform distribution, having at 8 m the <u>same</u> rms values as the transported distribution from the beginning. These results are marked with *.

The initial beam ellipses were chosen in such a way that the beam is first focused to a waist and then spread out due to the large space charge forces. By varying the waist position, we get almost no emittance increase (Fig. 4) or about 100 % increase (E_y-emittance in Fig. 2).

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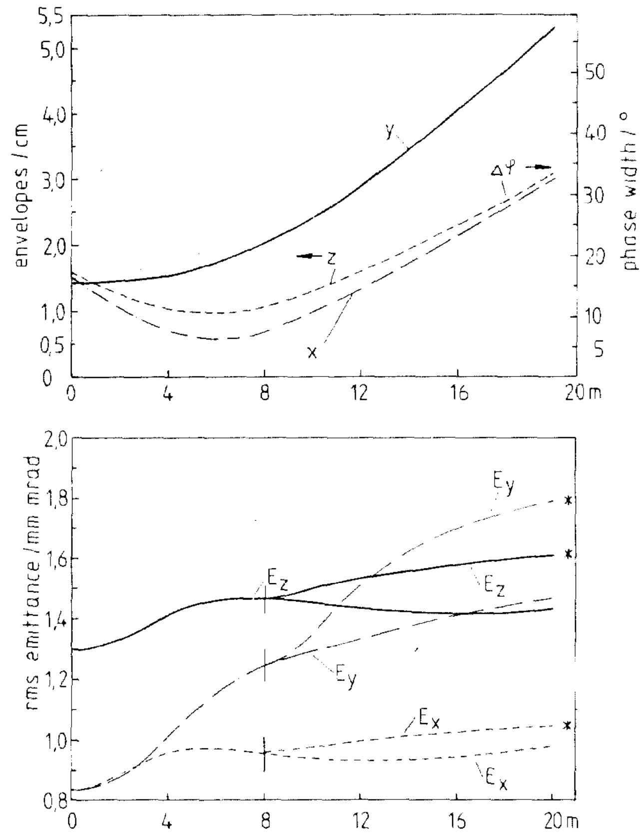

Fig. 1,2: envelopes and rms emittances for a drift; *: restart at 8 m with an uniform distribution.

These results can be interpreted as follows: the uniform phase space filling used for initial condition corresponds to a particle density[6]

$$\rho(x,y,z) = \rho_0 \left(1 - \left(\frac{x}{x_{max}}\right)^2 - \left(\frac{y}{y_{max}}\right)^2\right) \cdot \sqrt{1 - \left(\frac{z}{z_{max}}\right)^2} \qquad (1)$$

which leads to nonlinear and coupled space charge forces. Under the influence of these nonlinearities the beam adapts now itself to an equilibrium distribution in all 3 directions. This equilibrium distribution is reached ealier and with less emittance increase for small beam radii.

By comparing runs with different emittances, we found out that the quantity to compare is

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not the beam radius but the phase shift $\Delta\Psi(L)$

$$\Delta\Psi(L) = \int_0^L \frac{ds}{\beta(s)} \qquad (2)$$

where $\beta(s)$ is either the longitudinal or transverse β-function. For an elliptical distributed beam the β-function is given by[7]

$$\beta = \frac{\langle x^2 \rangle}{E^{rms}} \qquad (3)$$

where E^{rms} is the unnormalized rms emittance[7].

$$E^{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

The phase shift $\Delta\Psi$ determines the rotation of the beam ellipses; for a periodic system the tune σ is the phase shift $\Delta\Psi$ at the end of the period.

For getting the equilibrium distribution after 8 m (see Fig. 4) the average $\bar{\beta}$ value should be smaller than

$$\bar{\beta} < 20 \text{ m},$$

which corresponds to a phase shift value

$$\Delta\Psi = \frac{8 \text{ m}}{20 \text{ m}} = 23°.$$

After reaching an equilibrium distribution the beam can now be transported over 12 m (see Fig. 4) without further emittance increase.

The obtained equilibrium distribution is not an uniform distribution, which can be seen from the curves that result when starting again at 8 m with uniform distributions. The beam behaviour is the same in all 3 directions.

The drift space results can partly be interpreted in terms of temperature exchange (equipartitioning[3]) . For a periodic system one expects no emittance growth due to temperature exchange if

$$E_t^{rms}\,\sigma_t = E_l^{rms}\,\sigma_l \qquad (4)$$

where $\sigma_{t,l}$ are the tune's and $E_{t,l}^{rms}$ are the unnormalized rms emittances in transverse and longitudinal directions. $E^{rms}\cdot\sigma$ is proportional to the over one period averaged temperature $\bar{T}$.

For a non-periodic system, it is more convenient to calculate the temperature T directly from the definition

$$T_i \sim \langle v_i^2 \rangle = (\beta c)^2 \cdot \langle x'^2 \rangle_i \qquad (5)$$

where βc is the beam velocity. For an elliptical distributed beam the average value $\langle x'^2 \rangle$ is given by[7]

$$\langle x'^2 \rangle_i = E_i^{rms} \cdot \gamma_i = E_i^{rms} \left(\frac{1+\alpha^2}{\beta}\right)_i \qquad (6)$$

Therefore one expects an emittance increase of E_2^{rms} if

$$E_1^{rms} \left(\frac{1+\alpha^2}{\beta}\right)_1 > E_2^{rms} \left(\frac{1+\alpha^2}{\beta}\right)_2 \qquad (7)$$

all along your line.

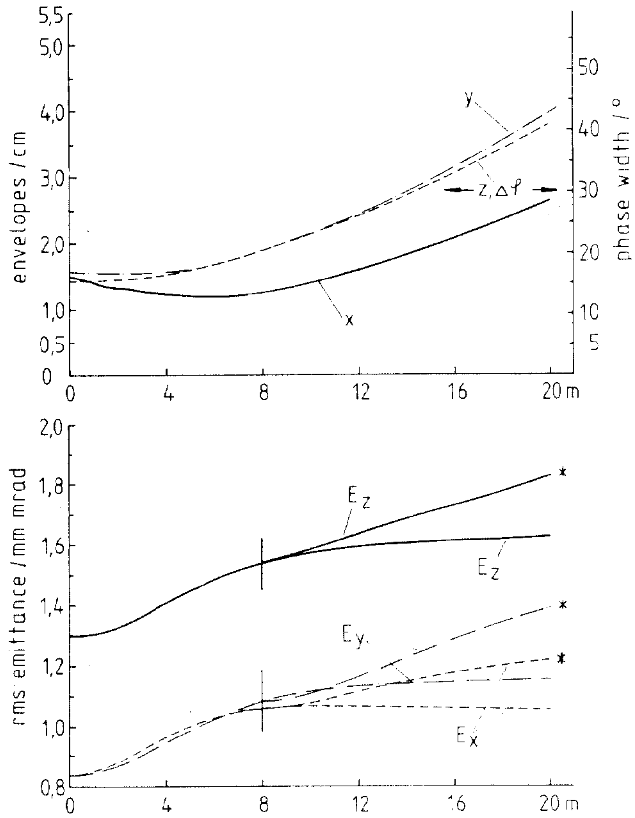


Fig. 3,4: envelopes and rms emittances for a drift; *: restart at 8 m with an uniform distribution

The quantity $E^{rms}\left(\frac{1+\alpha^2}{\beta}\right)$ is the local temperature at every point.

The in Fig. 2 observed strong increase of E_y can be interpreted as temperature exchange from the (x,z) to the y direction. The emittance increase in Fig. 4 can, however, not be explained in this way, because the temperatures here are almost equal along the line. The higher temperature $T_{x,z}$ in Fig. 2 means less emittance increase.

Emittance Behaviour in a Real Transport Line

In Fig. 5,6, the E_y transverse rms emittance and the envelopes are given in a realistic transport line (funneling section[3]) for the HIBALL[9] concept. After the transport line we have added an 8 m long drift space. The beam parameters here are the same as for the test drift space (Bi^{2+} - beam, E_{kin} = 1.7 MeV/N, I_p = 100 mA, f = 54 MHz) except for the longitudinal rms emittance (E_z=12 mm mrad).

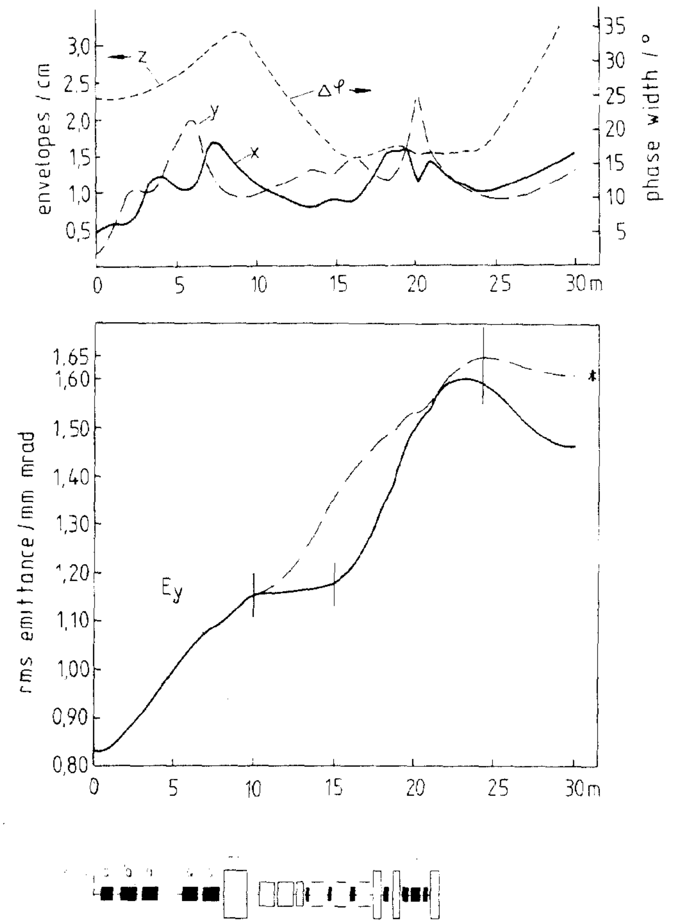
The longitudinal emittance is not shown but stays constant along the line, the E_x emittance is not shown but looks very similar to E_y.

The $\dot{E}_y$ emittance behaviour in Fig. 6 can be explained in a very simple way. Along the first 10 m the beam adapts itself to an equilibrium distribution, which again is not a simple uniform distribution as shown by the * marked curve. For the next 5 m where the beam radius is small enough the emittance stays constant. Then due to a too large beam radius, the equilibrium distribution is destroyed which leads to emittance increase. At the end the beam again adapts itself to a new equilibrium distribution due to the small beam radius. No filamentation effects are seen at the end[8]. The longitudinal emittance stays constant due to its large initial value.

Although the longitudinal temperature is roughly 100 times larger than the transverse ones the transverse emittances stay constant some of the time.

Conclusions

Transport lines for high current beams can be designed in such a way that the beam adapts itself under the influence of strong non-linear space charge forces to an equilibrium distribution in all 3 directions with no further emittance increase. Further studies will be done in order to analyze this observed distribution.



R1(2) 54(108) MHz rebunching cavity
Q quadrupole
S,D bending and deflecting elements which here act like drift spaces.

Fig. 5,6: envelopes, transverse rms emittance and position of the elements for a transport line; *: restart at 10 m with an uniform distribution.

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