THE SPACE CHARGE COMPUTER PROGRAM SCHAR

R. J. Hayden and M. J. Jakobson
Department of Physics & Astronomy, University of Montana, Missoula, Montana 59812

The incentive to develop a program to simulate space charge effects came from the inability of the TRACE program to adequately predict beam transport for the LAMPF injector lines. Attempts had been made to determine if neutralization was the dominant factor in the failure of TRACE. The measurements made indicated neutralization was present, and vacuum dependent, but could not account for the major part of the dependence of the beam emittance on beam current.[1] The space charge computer program, SCHAR, uses macrofilaments or macroparticles to model the effects of space charge in ion transport. We have attempted to verify predictions of SCHAR by reproducing measurements made on the LAMPF H^+ injector line. For beams with appreciable space charge, when SCHAR is run with the KV distribution[2] utilized by TRACE, SCHAR agrees with TRACE. When the H^+ line measured beam profile is taken as the input to SCHAR, it reproduces the waist locations and beam emittance for the remainder of the transport line, something TRACE has failed to do. Various types of input beams, including KV and observed, may be utilized with the program. Beam transport elements incorporated are drifts, quadrupoles, solenoids, bend magnets (with pole face rotations) and rf bunchers. A hard edged model is used for quadrupole fields. Adequate agreement with measurement has been obtained without utilizing measured quadrupole fields. POISSON calculated fields are used for bend magnets and for solenoids, SUPERFISH fields are used for the bunchers.

Calculational Modes

SCHAR is not an optimization program. The beam line "tune" must be obtained from TRANSPORT or TRACE. The program calculates the transport properties of beams of non-relativistic charged particles using two alternative macroparticle formulations. The line mode formulation considers the beam to be composed of a manageable number of infinitely long charged filaments. The paths of these filaments are fixed in time but their positions as a function of distance along the line are determined by a fourth order Runge-Kutta time step integration. Durations of the time steps are adjustable for different parts of the beam line.

The point mode formulation considers the beam to be composed of a manageable number of charged macroparticles. These macroparticles have the same charge to mass ratio as the actual particles. In cases in which rf bunching has been imposed on the beam, the axial dimension of the swarm is taken as one wavelength of the particle waves (mean particle speed divided by rf frequency). In point mode, space charge effects are modeled solely by the Coulomb repulsion of the macroparticles. When two macroparticles approach closer than a specified distance the interaction force is limited to the value appropriate to that distance. The number of close collisions is printed as output. Changing the cut-off distance input parameter by two orders of magnitude had negligible effect on the results. The effects on particles of the swarm by charges which are not included in the swarm is approximated by considering, for space charge calculations only, two more identical swarms which are translated respectively one wave length ahead and one wave length behind the original swarm. If the beam current is small enough that space charge effects may be neglected, the calculational steps from SCHAR are identical for the two modes. In space charge cases the line mode will run a factor of two or three faster than the point mode but if beam bunching occurs its use is not appropriate.

Initial Conditions

The particles or filaments tested by SCHAR are described in a left-handed Cartesian coordinate system. The Z-axis is the propagation axis and bending magnets have their fields in the Y-direction. Initial values for X, Y, Z and for V_x, V_y, V_z are obtained by a pseudo random number generator. The ranges over which these values may vary are part of the macroscopic input data which must be supplied before running the program. Once these ranges have been specified there are several optional methods for spreading out the filaments or particles in phase space. These are:

1) Four-Volume Distribution: this is designed for use in line mode operation. All filaments are given the same (zero) initial Z-positions and the same initial speeds. The initial X, Y, V_x, V_y values are spread in a statistically uniform manner over the interior of a macroscopically deformable four dimensional ellipsoid. For essentially linear transport with sharply terminated edges (quads and drifts) in the absence of space charge, the volume of this distribution in four-space as calculated from a second order moment matrix remains constant. Space charge, in this case, increases this macrovolume irreversibly. Bends lead to difficulties because the volume problem becomes six dimensional.

2) KV Distribution: this is another and very special four dimensional input. It differs from the four volume distribution in that the input particles are spread over the surface of the initial ellipsoid in such a way that their projections on the X-V_x, Y-V_y, X-Y, V_x-V_y planes are all of uniform density. This is the distribution assumed by TRACE. For particles so distributed the effects of space charge may be replaced by an external potential function and consequently (Liouville theorem) space charge leads to no increase in four dimensional volume in quad-drift transport. SCHAR, like TRACE, finds essentially no four-volume increase in this instance. We have seen no evidence for experimental distributions of this type.

3) Six-Volume Distribution: this is designed for point mode operation but may also be used in line mode. Initial X, Y, Z, V_x, V_y, V_z values are spread in a statistically uniform manner over the interior of a six dimensional ellipsoid which has specified maximum ranges. In the absence of space charge, six dimensional volume is preserved thru interactions with static fields.

4) "Observed" Distribution: this also is designed for point mode operation. The X-V_x and Y-V_y initial coordinates are distributed in such a way as to approximate the X-V_x and Y-V_y values measured at a particular (initial) location along the beam. The charges are spread uniformly over an X-Y ellipse and no other coupling exists.

Elements of the Beam Transport System

The filaments or macroparticles follow paths slightly different from each other through three different modes. These are:

1) A focusing mode which has a sequence of regions which can be sequentially magnetic quadrupole, solenoid, or field free. In the quadrupole region the fields vary in the X and Y directions only. The Z-axis follows the X-Y saddle points and edge effects are neglected. POISSON calculated data fields are used for the solenoids. The sensitivity of a solenoid to misalignment can be calculated using an alignment parameter (ALN).

2) A magnetic bending mode which significantly changes the mean direction of the beam. The magnetic fields in the centers of these magnets are assumed to be in the Y direction so that most of the beam deflec-

0018-9499/83/0800-2540$01.00© 1983 IEEE

tion lies in the X-Z plane. The mean particle beam may enter or leave these magnetic fields at finite angles in the X-Z plane. POISSON fringing fields for these magnets are utilized. These fringing fields are abruptly terminated after a specified number of gap widths, usually 3. The direction, radius, angle of bend, gap and pole face rotations are input parameters for a bend. SCHAR calculates and prints out the correct magnetic field. After passing through a bending magnet the coordinates of the beam line are rotated so that the mean beam again follows the Z-axis.

3) A "buncher" mode in which bunchers are driven by sinusoidally varying rf electric fields. This demands the point mode calculation option. The wave length generated by this buncher is long compared to the region over which the field effectively acts. SCHAR approximates the effects of the buncher by spreading the particles linearly in Z over one wavelength as they enter the first buncher. SUPERFISH fields are used for the bunchers. Aperture constraints are placed on the beam where calculated data fields are used. If a particle reaches the boundary of the data field, its demise is recorded, the identity of the particle lost is printed out and the calculation continued with the remaining particles.

Analysis of Output Data

The properties of the beam at any specified point along its path are assumed to be derivable from a knowledge of the position and velocity coordinates of each of the filaments or macroparticles when near the specified point. The macroscopic significance of a tabulation of the six coordinates for each macroparticle is, however, usually not obvious. Such significance is made more clear by the consideration of averages and moments of the individual particle data or by two dimensional plots which show correlations between any two of the six coordinates. SCHAR prints out averages, moments, and r.m.s. deviations for the six coordinates after each physically distinguishable region of the beam has been traversed. After this, output of two dimensional particle plots using a plotter is optional.

In addition, the volume in phase space occupied by the particles is estimated by two distinct methods. Firstly, the products and moments of inertia about the mean in six-space are used to obtain the volume of the corresponding ellipsoid. This method also is used to calculate four dimensional volume in line mode and area in the X-V_x, Y-V_y, Z_y-V_z planes. Clearly this method cannot give the correct phase space volume unless the particle distribution is uniform and ellipsoidal in phase space.

The second method of volume estimation is applied only in the X-V_x, Y-V_y, Z-V_z cases. A convex polygon is constructed about the outer particles and the area of this polygon is calculated. These results are incorrect when, as in filamented distributions, a significant part of the periphery is concave.

An override (OR) option is available so that any or all of the input particles can be given specified input parameters rather than values from a random number generator. A single particle option (SOP) will provide a printout of any particle's coordinates and velocity components at each step in the calculation so that individual particles may be traced as rays. The total distance along the Z axis travelled by the centroid of the distribution is printed out at the end of each region. Input and output are in MKSI units.

Space Charge Defocussing for Special Beams

Emittance measurements on the LAMPF H^+ injector line provided the observed values to check SCHAR calculations.[1] An abnormal beam line tune for which space charge effects were extreme was made. A 25 ma beam emerging from the LAMPF TBM01 45° bend magnet was focussed sharply to a 1.5 mm radius at the prebuncher. The beam diverged to about 1.5 cm radius as it was transported through the TCOL01 triplet and the TCOL02 quadruplet which refocussed the beam to ∿.5 cm radius. The transport distance involved was about 6.5 meters. X and Y emittances were measured at the two emittance monitors, EM2 and EM3, in that portion of line. The emittances were measured by scanning a slit across the beam and measuring the current to wires parallel to the slit. An orthogonal slit and wires were used to measure the emittance in the orthogonal dimension. At the EM2 emittance station the spatial distribution could be approximated by a paraboloid distribution. At EM3 the beam emittance in either plane had characteristic S or "hysteresis" shape tails. When the low intensity portion of the beam was excluded, the remaining intense 85% of the beam, had a characteristic tilted "bow tie" appearance indicative of two peaks in the emittance plot.

When the behavior of the beam was calculated using SCHAR, the input spatial distribution was taken as parabolic at the prebuncher and the input emittance chosen to approximate the observed emittance at the first emittance station. SCHAR indicated the 25 ma beam became hollow near its maximum size at the center of the quadruplet. The hollow beam gave rise to the double peaked emittance plots at EM3.

Beam behavior for H^+ injector line tunes used for production runs was also duplicated with SCHAR. The beam waist at the prebuncher was ∿3 to 4 mm radius and the maximum beam excursion was less than for the abnormal tune. The calculated beam emittance agreed quite well with the observed emittances. The range of currents for which calculations agreed with measurements indicated less than twenty percent neutralization in regions where the beam was small.

A characteristic "bow tie" emittance can also be generated in a buncher if the beam at the buncher is too large or off center.

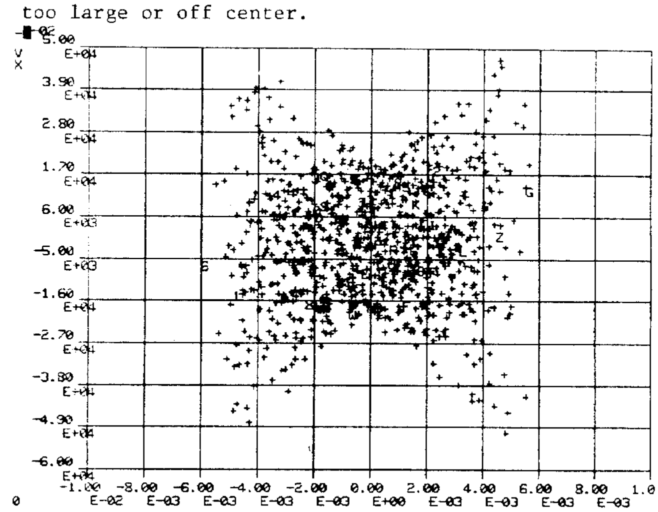

Fig. 1. SCHAR calculated emittance plot for the beam emerging from the LAMPF buncher with SUPERFISH fields and no space charge. Illustrates emittance growth due to too large a beam at the buncher.

Solenoid Fields

The analytical evaluation of ion transport thru a solenoid's field is difficult and has been done only for a uniform field and for paraxial paths with a few special field distributions. SCHAR was used to obtain numerical solutions with and without space charge for solenoid fields of different shapes for which the fringe field regions were comparable in length to the "constant" field region. The solenoids had an aperture of 7.5 cm (dia) and a length between half value points for $B_z(r=0)$ of 9.1 cm. The maximum value for B_z was 0.43 Tesla.

For the first solenoid calculations a linear "ramp" approximation was used to fit the measured field values. In the fringe region,

$$B_x = \frac{B_o}{2F}\, x\, \left[\frac{z}{\mathrm{abs}(z)}\right] \qquad B_y = \frac{B_o}{2F}\, y\, \left[\frac{z}{\mathrm{abs}(z)}\right]$$

$$B_z = \frac{B_o}{F}\,\left[(L_{eff}+F)/2-\mathrm{abs}(z)\right]$$

whereas in the central region, $B_x=B_y=0, B_z=B_o$. When an 80 KV H^- ion beam of 7π mr-cm emittance and negligible current was sent through the solenoid, filling roughly half its diameter, negligible growth in effective emittance occurred and no filamentation.

A second set of calculations used a Bessel function solution to Laplace's equation in cylindrical coordinates which approximated the observed axial magnetic field.

$$B_r = \frac{B_o}{2}\,\sin(kz)\, I_1(kr) \qquad B_z = \frac{B_o}{2}\,[1+\cos(kz)\, I_o(kr)]$$

where k was determined by the measured half value points for $B_z(r=0)$. When a run previously calculated with the ramp field was repeated with this new solenoid field, effective emittance growth occurred due to the non-linear fields. Approximately one quarter of the particles form the non-linear "tails".

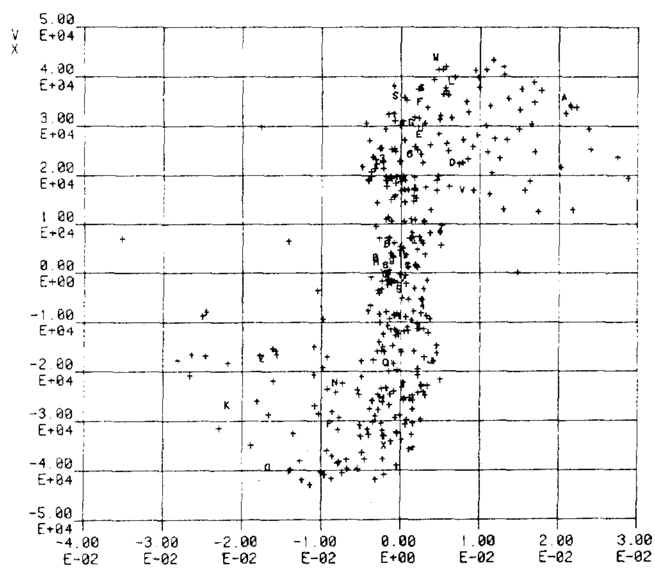

Fig. 2. SCHAR X-V_x emittance at the solenoid "focus" for 4VOL input and Bessel function fields.

Runs were then made using POISSON calculated data fields. SCHAR gives essentially the same ion paths for the POISSON solenoid data fields as for the Bessel function field approximations.

Solenoid Alignment

Tolerences for alignment and displacement errors for the solenoid of the previous section were obtained for the same maximum field and input beam. If the solenoid is rotated an angle θ about the Y axis thru its center, the solenoid misalignment will cause the beam to be deflected an angle $\sim\theta/2$ with the rotation occurring about the x axis. The effect is essentially linear since the transverse field component causing the deflection is proportional to the alignment angle. If the solenoid is displaced transversly to the beam, the non-linearity of the focussing fields becomes more pronounced and the beam's effective emittance is increased. This effect is approximately quadratic with a 20% increase in effective emittance occurring for a 1 cm lateral displacement for the beam conditions selected.

Solenoid Aberrations

The override and single particle options provide a convenient method of measuring solenoid aberrations. For spherical aberration, a number of particles of the input swarm are sent in parallel to the central axis at given X displacements from it. Their positions and velocities are printed out at each step and the axial distance at which they cross the X axis is obtained by interpolation. This can be done as a function of the beam current or of spatial distribution of input particles (KV, 4VOL or observed). Since the SCHAR numerical calculation includes all orders and does not distinguish between them the standard aberration coefficient has not been calculated. Instead, a plot is presented.

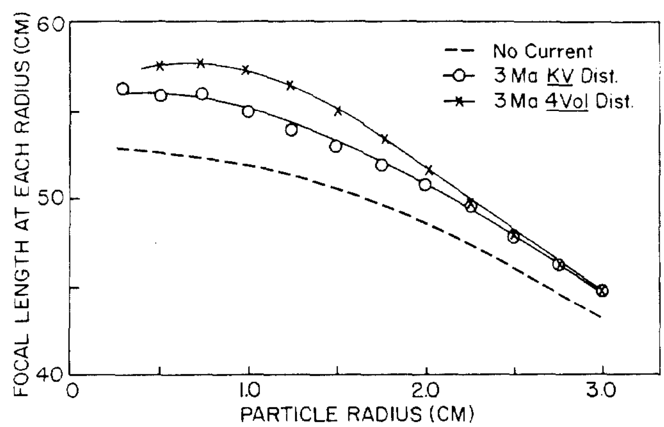

Figure 3 illustrates the dependence of the solenoid focal length on beam current and on the spatial distribution of the input beam. The 4VOL distribution has more current concentrated near the center of the beam than the KV distribution and provides a greater change in focal length per unit current.

Remarks

The number of macroparticles required to determine a waist is less than the number required to predict an accurate effective emittance increase and subsequent small beam intensity loss. The dependence of the "moment of inertia" phase space on the number of macroparticles was checked for a typical H^+ beam line tune with 15 ma of protons. The ratio of the output/input phase space was constant, within statistics, for 200 macroparticles or more. The polygon area phase space is more dependent on the number of macroparticles. With 400 particles the output/input phase space was still increasing slightly. A beam waist location can be reasonably well determined by forty macroparticles and possibly fewer: The fact that so few particles are required to locate a waist suggests that it may be practical to modify the program to optimize parameters and tune beam transport lines.

Acknowledgements

We are indebted to LAMPF personnel for discussions, to Donald Hagerman for suggesting the problem, to Andrew Browman and James Hurd for making the beam line measurements, and to Harold Butler for assistance with those calculations which were made at LAMPF.

1. A. Browman and J. Hurd: private communication.
2. M. Kapchinskij and V. V. Vladimirskij, Conference on High Energy Accelerators and Instrumentation, CERN, Geneva (1959).