

PROGRESS OF EXPERIMENT TO STUDY THE LIMITATIONS TO BEAM TRANSPORT IN A PERIODIC SOLENOID FOCUSING CHANNEL*

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Summary

An account is given of work carried out in preparation for an experiment on the transport of a high current electron beam through an array of solenoid lenses. The performance of grids for emittance enhancement, and measurements of beam profile and emittance are described. The effects of aberrations and nonlinearities are discussed.

1. Introduction

The proposed experiment to determine the limits to the current that can be carried by a periodic focusing channel has been reported previously.^{1,2} The immediate motivation is to supply information relevant to the heavy ion beam fusion concept, but the general problem is of wider interest. The theoretical analysis which gives guidance as to the region in parameter space where instability and emittance growth might be found has assumed the idealized Kapchinskij-Vladimirskij (K-V) distribution function for the beam particles, and a true paraxial focusing system.³ It remains to be seen how a more realistic distribution behaves. Indications from computational studies are that a distribution which in four dimensional transverse phase space is monotonically decreasing rather than hollow will be more stable.

The overall experimental arrangement at Maryland has been described earlier.^{1,2} A similar set-up, though different in detail, has been operating at the Rutherford Appleton Laboratory, where the emphasis has been on the study of grids for the enhancement of the beam emittance.

2. Parameter Range to be Studied

In the smooth approximation, the system is characterized by two parameters σ_0 and σ/σ_0 , where σ and σ_0 represent the tune shift per period in the absence and presence of space charge respectively. For particles of given energy, σ_0 is determined by the magnetic field in the solenoid lenses. The value of σ/σ_0 , on the other hand, depends on the mean beam radius a , the emittance ϵ , and the perveance, K . (K may conveniently be written as $2I/\beta^2\gamma^2I_A$, where $I_A = 4\pi mcA\beta\gamma/\mu_0 Ze$ is the Alfvén current.) The relation between these parameters is

$$\sigma/\sigma_0 = (1 + Ka^2/\epsilon^2)^{-1/2} \quad (1)$$

From the form of Eq. (1), it is evident that we have freedom in choosing a , ϵ , and K . The constraints on the values of these parameters arise from experimental requirements, discussed in Section 4. To cover the parameter range of interest, we need to vary σ_0 from about 0.3π to 0.7π , and at each value of σ_0 , to vary σ/σ_0 from about 0.1 to 0.5. The relation between the parameters may be expressed in several ways; we find it convenient to use a dimensionless parameter u containing σ_0 explicitly⁴

$$u = KS/2\sigma_0\epsilon \quad (2)$$

where S is the period length. In terms of this parameter

$$\sigma/\sigma_0 = [(1 + u^2)^{1/2} - u], \quad a/a_0 = [(1 + u^2)^{1/2} + u]^{1/2} \quad (3)$$

where a_0 is the radius of the beam in the absence of space charge, ($K = 0$, $u = 0$, $\sigma = \sigma_0$). These functions are plotted in Fig. 1. Comparison with results of accurate integration of the K-V equations shows that

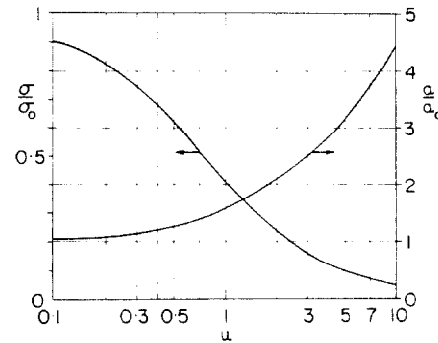


FIG. 1. The quantities a/a_0 and σ/σ_0 as a function of u .

the smooth approximation relations in Eq. (3) are very accurate even at the highest values of σ_0 of interest.

3. Non-Paraxial Effects

Non-paraxial effects arising from lens aberrations and non-uniformity of the charge distribution in the beam must be carefully considered, since they are not included in the simple theory presented so far. The effect of spherical aberration in solenoid lenses in the absence of space-charge is well understood; third order terms in the trajectory equations focus rays away from the axis more strongly than those near the axis. This means that particles with large amplitudes of oscillation in the channel effectively have larger σ_0 .

In Eq. (1), it is seen that in scaled systems with the same value of σ/σ_0 , the ratio $K/(\epsilon^2/a^2)$ must be conserved. Spherical aberration can only be scaled if K and ϵ/a are separately conserved. This condition implies the angles that particles make with the axis remain unchanged.⁵ The second source of aberration arises from the non-uniformity of current density across the beam. Compared with a uniform distribution, a radially decreasing density with the same r.m.s. width reduces the effective σ of particles near the axis, and increases it for particles with large oscillation amplitude. Unlike spherical aberration, this effect scales without requiring K and ϵ/a separately to be conserved. The effect of these factors on our experiment is discussed in Section 6.

4. Choice of Experimental Parameters

The scale of the experiment was set by choosing a 5 cm diameter beam tube, with a beam diameter of about 1 cm. The solenoid magnet lenses are 7 cm long, and the first beam line is being assembled with a period length of $S = 14$ cm. The magnets have an inner diameter of 5 cm, and produce a field on the axis with

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half-width half-height about 2.2 cm. The maximum value of B_z on the axis is of order 0.01 T.

The beam radius will be about 0.5 cm. For reasons explained below it is difficult to make the emittance and hence the beam radius large, consequently a small beam radius is desirable. On the other hand, with a small beam, magnet and lining up errors are more serious, and diagnostics are more difficult.

Two Pierce guns have been used for generating the beam. The first is convergent with cathode radius 1.27 cm and waist radius 0.5 cm, and the second is planar with a gridded anode, and a cathode radius of 0.5 cm. The first gun has $K = 0.011$, in the second K can be varied from about 0.003 to 0.01 by varying the anode-to-cathode spacing. The operating voltage can be varied up to about 7 kV, and a pulse length of 5-10 μ sec ensures that there is negligible neutralization by positive ions. It has been found convenient to make most measurements at about 2 kV.

The emittance of these guns is determined by the temperature kT and radius R_c of the cathode. If eV is the beam voltage,

$$\epsilon = R_c (2kT/eV)^{1/2} \quad (4)$$

For $kT = 0.1$ eV, $eV = 2$ keV, $R_c = 0.5$ cm this is 5×10^{-5} m rad. With the above values of K and ϵ , and $a = 0.5$ cm, the rather large value of Ka^2/ϵ^2 implies [from Eq. (1)] that we are restricted to small values of σ/σ_0 . To overcome this limitation, a 'beam spoiler,' making use of transverse grids to increase the beam emittance has been designed and tested.

The best choice of values of a , ϵ , and K for the final experiment will not be known until some operating experience with a long channel has been obtained.

5. Enhancement of Emittance by Grids

The basic idea of grid action is illustrated in Fig. 2. First, we consider the idealized situation

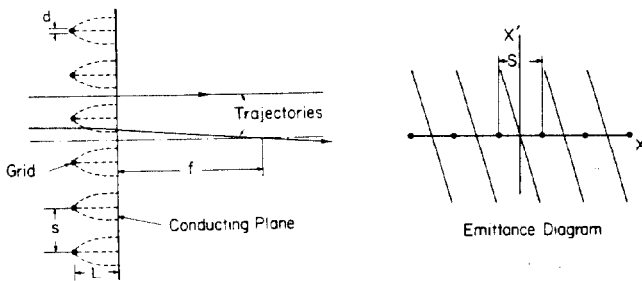


FIG. 2. Schematic diagram showing idealized grid configuration and emittance diagram after beam has passed through grid.

of a grid of wires close to a hypothetical transparent conducting plane, held at a different potential. A beam of particles incident from the left will be scattered, and the emittance diagram transformed as shown. If, now, the transparent plane is replaced by a grid consisting of a set of wires oriented at right angles, scattering will occur in both planes. This system has two disadvantages; first, the beam energy changes as it passes through the system, and second, the scattering does not have axial symmetry. The first difficulty can be overcome by having two pairs of grids, the outer grid of each pair being earthed, while the inner pair is connected together and held at a potential V_g , as shown in Fig. 3. An alternative possibility, also shown, is to combine the two central

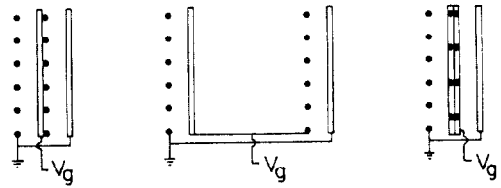


FIG. 3. Idealized sketches, not to scale, of grid configurations that have been studied. In the third sketch, wires of the central grid are oriented at $\pm 45^\circ$ to those of the outer grids.

grids into a single grid containing two sets of wires. Further variation can be obtained by rotating this central grid about the axis so that the wires are at angle to those in the outer grids.

The structure of beams passing through grids of this kind has been examined both experimentally and by means of a 'particle in cell' code in which the grids are represented as arrays of lenses. The focal length of the strip lenses for the system shown in Fig. 2 may be shown to be (in terms of parameters shown in the figure)

$$f = 2L(V_b/V_g) [1 + (S/2\pi L) \ln(S/\pi d)] \quad (5)$$

For lenses formed by two superposed grids, giving an array of squares, the focal length is twice the value given by Eq. (5). For the three grid system, these lenses combine to produce a 'spotty' structure in the beam, which varies in a subtle way with grid voltage. This structure can be varied by altering the orientation of the center grid, and by making the wire pitch different from that in the outer grids. A comparison of measured and computed structure is given in Fig. 4.

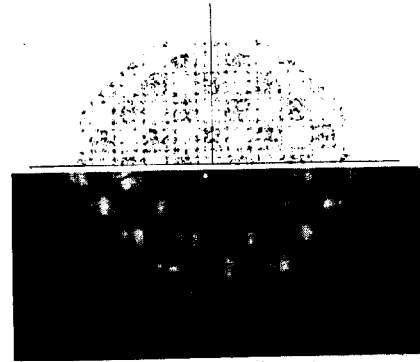


FIG. 4. Comparison of measured and computed structure in a drifting beam introduced by grids in the third configuration of Fig. 3; $V_g/V_b = 0.25$.

The use of two pairs of separated grids, rather than a central grid with two sets of wires, seems to give a beam with less marked structure. The examination of the phase-space distribution (from the computer print-out, or by use of a 'pepper pot' and fluorescent screen), reveals a quite distinct structure for large enhancements of the initial thermal emittance. This is hardly surprising; Liouville's theorem leads us to expect that if we 'chop up' and separate the initial low volume in phase-space into a number of four dimensional 'spots,' they cannot be merged again into a smooth distribution without severe non-linear filamentation. Over what length this structure remains in a long beam remains to be seen. An important question is, does this structure matter? If the overall emittance growth we are looking for occurs from initial fluctuations having scale length greater than the spot size, it is

perhaps not important. There is the practical difficulty that the structure makes emittance measurement difficult.

6. Lens and Space Charge Aberrations

The spherical aberration coefficient of the lens has been measured experimentally, by putting a mask with pinholes over the gun, and also computationally using both measured magnet fields and analytical approximations. Good agreement between all these methods has been found, and with tables given by El-Kareh and El-Kareh.⁶ The value of C_s/f for a parallel incident beam varies from 1.6 to 11 as the focal length increases from 4 to 12 cm, the range of interest in the experiments. It may readily be verified that except when σ/σ_0 is near unity, the nonlinear force arising from beam non-uniformity is likely to exceed that from lens aberrations.

7. Measurement Program

Extensive measurements have been made of the behavior of the beam in the presence of one, two or three lenses, and emittance enhancing grids. In the absence of grids, so that the beam emittance is small, beam profile and envelope curves have been compared with computations which include third order terms in the equation of motion. Space-charge effects have been included by finding, at each point along a trajectory, the fraction F of the current lying within a circle through the point. For a paraxial system, the flow is laminar and F remains constant; when, however, third order terms are included, orbit crossing occurs and F varies with z . The radial equation, including third order terms, may be written

$$r'' + \kappa r [1 + r'^2 + \kappa r^2 - (B'/B) r r' - (B''/2B) r^2] + \kappa F/r = 0 \quad (6)$$

where

$$\kappa = [eB(z)/2B\gamma m_0 c]^2. \quad (7)$$

Comparison of calculated and measured envelopes is shown in Fig. 5. Using the same code, transverse

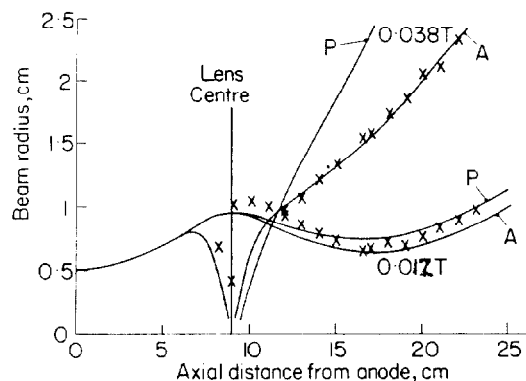


FIG. 5. Comparison of measured and computed envelopes for a beam with space charge passing through a single lens. The labels A and P refer to accurate calculation (with third order terms) and paraxial approximation. The peak value of B_z is shown on the curves; the beam voltage and current were 5 kV and 190 mA.

density distributions near a focus have also been obtained. Very marked features are often found; these include hollow profiles, and beams with an intense core surrounded by a halo. These have been confirmed by beam-scanning measurements using a pinhole and Faraday cup. These results apply to much sharper foci than are expected in the final experiment, where the

aberration effect is much less serious. They will be described elsewhere.

Measurements have also been made of emittance. The traditional 'slit and pinhole' method can be used, but this becomes impossible if there is grid induced structure in the beams. In this case, the rather simpler 'pepper pot' technique, in which the pattern of beamlets incident on a scintillator is analyzed, gives a more direct result. The spacing between spots gives information about beam divergence at the screen, and the ratio of diameter to spacing contains information about the emittance. Such images are illustrated in Fig. 6. Figure 7 shows a preliminary measurement of the variation of emittance with grid volts for the system shown in Fig. 2. It would appear that emittance enhancement by a factor of at least six should be readily obtainable.

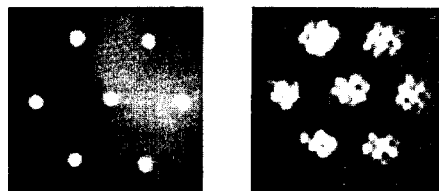


FIG. 6. 'Pepper pot' image on fluorescent screen of beam in which the ratio of hole size to spacing is 10, showing spreading arising from emittance. In the first picture, the emittance is of thermal origin; in the second, it is induced by grids.

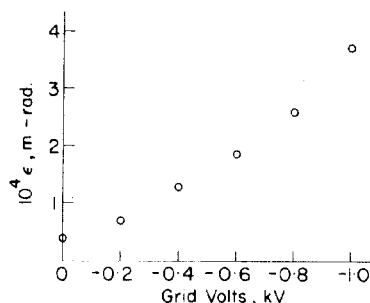


FIG. 7. Results of 'pepper pot' measurement of the emittance of a typical beam with 2.5 kV and 35 mA.

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9. References

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