LIMITATIONS OF BUNCH-CURRENT IN LEP BY TRANSVERSE MODE-COUPLING

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The most severe limitation of beam-current in LEP will be a fast transverse instability of single bunches, which can be explained as coupling of transverse modes. These predictions are verified with experimental data from several operating machines. The dependence of the threshold on beam and machine parameters is investigated in order to optimize the performance of LEP.

1. Introduction

We investigate the effect of transverse impedances on single bunches in high energy storage rings by extending the theory of coupled bunch-instabilities developed by F. Sacherer[1]. We thus restrict ourselves from the outset to a simplified model in which only the lowest ("most coherent") radial modes are included. On the other hand, we do include negative azimuthal mode numbers which appear to be most important for mode-coupling. We also assume that the mode-shape is known (Hermitian for Gaussian bunches) and that the actual impedance can be replaced by a broad-band resonator. Then the coherent frequencies can be found as the eigenvalues of a matrix which may be truncated to quite small dimensions without large error. If one takes only the 2x2 matrix containing the two modes which have the lowest thresholds, usually m=0 and m=-1, then the stability condition can be given explicitly and we find simple approximations for either very short or very long bunches. The lowest thresholds are found for bunches whose RMS lengths are of the order of the inverse resonant frequency. This formulation also suggests a number of possible means of increasing the threshold current which was found below design value in LEP. The theory has been verified by applying it to various existing large storage rings (PEP, PETRA) where the mode coupling instability has been observed.

2. Coherent Frequency Shift

In the absence of mode coupling, i.e. for very low beam currents, the frequency of oscillation of the m^{th} azimuthal mode is given by[1]:

$$\omega_m = \omega_\beta + m\omega_s + \Delta\omega_m \qquad (1)$$

where $\omega_\beta = \nu\omega_0$ is the betatron-frequency
$\omega_s = \nu_s\omega_0$ is the synchrotron-frequency

$$\text{and} \quad \Delta\omega_m = jDF_m Z_{mm}^{eff} \qquad (2)$$

is the coherent frequency shift. Here

$$D = \frac{\omega_0 \langle\beta\rangle \, I_0}{4\pi E/e} \qquad (3)$$

is a factor proportional to the average beam current I_c. The beta function $\langle\beta\rangle$ in the plane of oscillation is averaged over the circumference of the machine (weighted by the strength of the local impedances).

The form factor

$$F_m = 1/(|m|+1) \qquad (4)$$

was originally only derived for parabolic bunches, but we shall assume it to be a reasonable approximation also for a Gaussian. We define the effective transverse impedance of two coupled modes m,n analogously to the longitudinal case[2]:

$$Z_{mn}^{eff} = \frac{\sum Z_\perp(\omega_p) h_{mn}(\omega_p - \omega_\xi)}{B \sum h_{mn}(\omega_p - \omega_\xi)} \qquad (5)$$

where the summations over p extend from $-\infty$ to $+\infty$. and $Z_\perp$ is the transverse impedance per unitdisplacement. The spectral (cross) power-density:

$$h_{mn}(\omega) = \tilde{\lambda}_m^*(\omega)\tilde{\lambda}_n(\omega) \qquad (6)$$

is defined in terms of the Fourier transforms of the line-density modes $\tilde{\lambda}_m(\omega)$. The bunching factor B (average over peak current) has been included to facilitate normalisation which we take as:

$$B \sum h_{mn}(\omega_p) = 1 \qquad (7)$$

It turns out that the sum in the denominator of equation (5) is independent of the "chromatic frequency" $\omega_\xi = \xi\omega_\beta/\alpha$, and we get simply

$$Z_{mn}^{eff} = \sum Z_\perp(\omega_p) h_{mn}(\omega_p - \omega_s) \qquad (5')$$

The spectral frequencies of a single bunch are given by:

$$\omega_p = p\omega_0 + \omega_\beta + m\omega_s + \Delta\omega \qquad (6')$$

Since the mode coupling instability occurs also for vanishing chromaticity - contrary to the regular head-tail effect - we shall limit our discussion to the case $\xi=0$.

3. Mode - Coupling

The coherent frequencies can be found from the zeros of the infinite determinant:

$$|M_{mn} + (\omega_\beta + m\omega_s - \omega)| = 0 \qquad (7)$$

where:

$$M_{mn} = jDF_m Z_{mn}^{eff} \qquad (8)$$

They are the eigenvalues of the matrix $M+(\omega_\beta+m\omega_s)I$ Without large error, the infinite matrix may be truncated to small dimensions including only the modes which are coupled and their neighbours. For low enough beam-currents, the off-diagonal elements - which are all proportional to I_0 - are negligible, and we obtain equation (2) since $\Delta\omega_m = M_{mm}$. The frequency shifts are negative for long bunches which sample the inductive low-frequency impedance. Because of this negative azimuthal mode-numbers are coupled first. For long bunches, the shifts are shown in fig.1a: the m= 0 and m= -1 mode cross over when $\Delta\omega=2\omega_0$.

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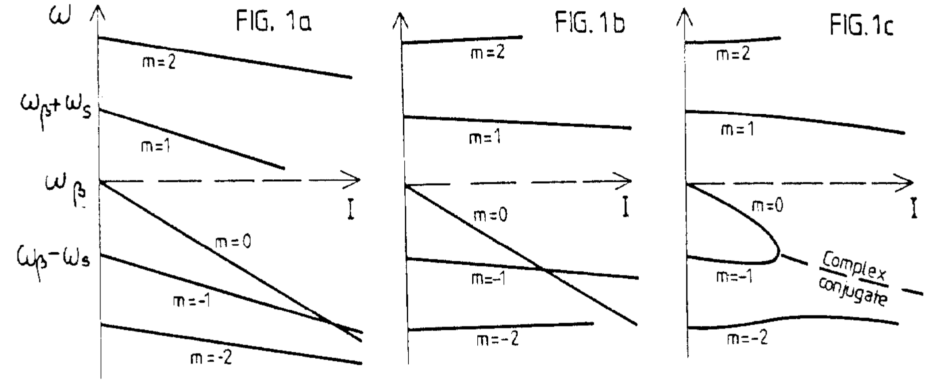

Fig: 1 Coherent frequency versus current

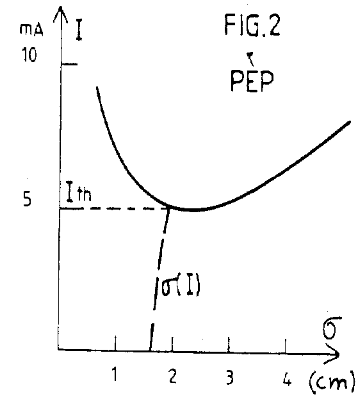

Fig: 2 Threshold vs bunch length in PEP

For shorter bunches, only the m=0 mode will have a negative slope since it samples frequencies around zero. The other modes sample increasingly higher frequencies, and tend to become capacitive when the peak of the power-density lies above the peak impedance. This will make the slope positive for larger m as shown in fig 1b: the cross-over occurs near $\Delta\omega=\omega_0$. A further reduction of the shift is obtained when the off-diagonal elements are included. Their presence makes the lines ω versus I become curved, and more realistic pictures are shown in Fig 1c. Limiting ourselves to the 2x2 matrix containing the modes concerned, the coherent frequencies are given by the solutions of:

$$\begin{vmatrix} \omega-\omega_m & M_{m,m+1} \\ M_{m+1,m} & \omega-\omega_{m+1} \end{vmatrix} = 0 \qquad (9)$$

From equation (6) we see that $h_{nm}=h^*_{mn}$ and thus $M_{mn} = -M^*_{mn}$ Then equation (9) yields a quadratic which has real solutions (assuming real matrix elements M) if

$$2|M_{m,m+1}| \leq |\omega_s+\Delta\omega_{m+1}-\Delta\omega_m| \qquad (10)$$

This is an approximate stability criterion for transverse mode-coupling. It agrees essentially with an expression derived by Kohaupt[3], but is should be understood to include negative mode numbers which are the most unstable.

4. Broad-band impedances

For a single-bunch effect we are only concerned with short range fields. In the frequency domain, this permits us to replace the actual large number of sharp resonances by a single broad-band one with the same total R/Q value. Since the impedance then is a slowly varying function of frequency, we can justify neglecting the frequency shift in its argument. The transverse impedance of a resonater will be written as:

$$Z_\perp = \frac{\omega_r}{\omega} \frac{R_\perp}{1+jQ_\perp\left(\frac{\omega}{\omega_r}-\frac{\omega_r}{\omega}\right)} \qquad (11)$$

characterised by three parameters: the transverse shunt impedance $R_\perp$, the quality factor $Q_\perp$, and resonant frequency ω_r. For a Gaussian beam we assume that the line-density modes are Hermitian with the Fourier transforms.

$$\tilde{\lambda}_m(\omega) = j^{-m} C_m (\omega\tau)^m \exp(-\omega^2\tau^2/2) \qquad (12)$$

The normalisation condition equation (7) yields:

$$C_m^2 = \sqrt{2\pi}\,/\Gamma(m+1/2) \qquad (13)$$

To a very good approximation, the infinite sums in the expression for the effective impedance can then be evaluated analytically[4].

5. Threshold estimate

For short bunches ($\omega_r\tau \ll 1$) the absolute value of the argument of the complex error function[5] is small compared to unity and we may use the first terms of the power series. Keeping only the largest term, we then obtain an approximation for the diagonal terms of the impedance matrix:

$$Z^{eff}_{mm} = j\,\frac{\sqrt{2\pi}}{1/2-|m|}\;\frac{\omega_r R_\perp}{\omega_0 Q_\perp}\,\omega_r\tau \qquad (14)$$

The off-diagonal elements are proportional to $(\omega\tau)^2$ and hence negligible for short bunches, except for the elements:

$$Z^{eff}_{0,+1} = -2\pi j\,\frac{\omega_r R_\perp}{\omega_0 Q}\,\omega_r\tau \qquad (15)$$

which are of same order as the diagonal elements. The stability criterion then can be written:

$$I_0 > \frac{\sqrt{2}}{\omega_r\tau}\;\frac{\omega_s E/e}{\langle\beta\rangle\omega_r R_\perp/Q_\perp} \qquad (16)$$

For long bunches we can use the asymptotic series for the complex error function. A large number of terms cancel and the lowest remaining one yields:

$$Z^{eff}_{mm} = j\,\frac{2\pi}{\omega_0\tau}\,\frac{R_\perp}{Q_\perp} \qquad (17)$$

The off-diagonal elements are all negligible for long bunches and the stability criterion becomes:

$$I_0 > 2\sqrt{2\pi}\,\frac{\omega_s E/e}{\langle\beta\rangle\omega_r R_\perp/Q_\perp} \qquad (18)$$

Thus the threshold current as a function of bunch length first decreases as $1/\tau$ and later increases with τ. The minimum threshold occurs near $\tau \approx 1/\omega_r$ and is of the order

$$I_{th} = \frac{\sqrt{2\pi}\,\omega_s E/e}{\langle\beta\rangle\omega_r\ R/Q} \qquad (19)$$

More exact values for the threshold are obtained by numerical evaluation of the truncated equation (7) by computer. However, the parameter dependence is better seen from the approximation which also yields a simple first estimate.

6. Application to storage rings

Thresholds have been calculated numerically for a number of existing e+e- storage rings. The transverse impedance is usually not well known and its estimate is the largest source of error. The RF cavity impedances are obtained by scaling from the LEP-cavity impedance of 3 - 4KΩ per cell.

For PEP with 120 RF cells with an average hole radius of 5cm we find the contributions of the RF cavities about 0.5MΩ/m and a similar amount for the vacuum-chamber. With ν_s=0.041, α=0.002,$\langle\beta\rangle$=62m we obtain the threshold curve shown in fig.2, with a minimum of about 5mA for an RMS bunch length of 18mm.

For PETRA with 60 RF cavities of 5 cells each with a hole radius of 6cm we get a higher impedance (1.2MΩ/m), but also a higher resonant frequency (2.2GHz). An operation with ν_s=0.063, α=0.0027 and $\langle\beta\rangle$ = 15m yields the threshold curve shown in fig. 3. The minimum of 4mA occurs near a RMS bunch length of 14mm. This figure also shows that the coupling moves to modes m= -1 and m = -2 for longer bunches.

A similar estimate has been made for the threshold in the much smaller machine DCI (Orsay). Up to the maximum current of 300 mA no instability could be found, in agreement with observation.

The thresholds for LEP-phase I (640 RF cells) are summarized in fig.4 for operation with a 60° phase-shift per lattice period:

A minimum of 0.5mA/ bunch occurs for an RMS bunchlength of 2cm, slightly below the design current of 0.75 mA/bunch. Several means to increase the thresholds are under discussion (see below). For LEP phase II twice as many cavities are added. The situation would become catastrophic for copper cavities, but the foreseen "spherical" superconducting cavities can have much bigger beam holes (7.5 cm radius) and have lower impedances.Hence the total transverse impedance of the machine is only increased by about 30%, and the thresholds decrease accordingly.

7. Conclusions

A simplified theory of transverse mode-coupling has been developed. The thresholds of instability can be calculated by finding the eigenvalues of matrices which may be truncated to a rather small size. The approximate expression for the threshold current suggests a number of means to raise the threshold:

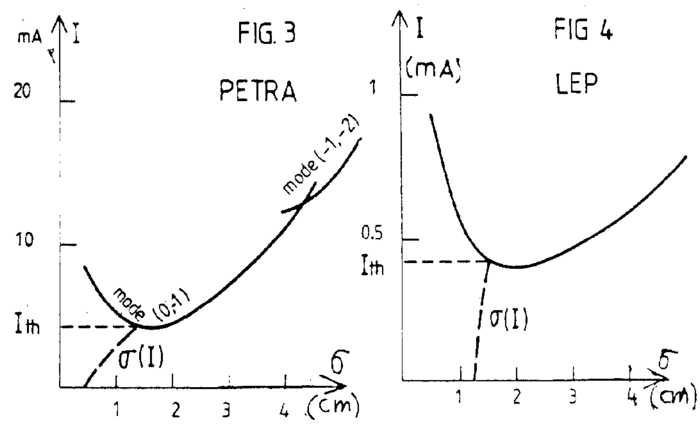

Fig 3/4: Thresholds in PETRA/LEP 60°

a. increased synchrotron frequency (higher RF voltage).
b. increased bunch-length (change of partition number, non-linear wigglers or higher harmonic RF system)
c. reduced betafunction at RF cavities.
d. reactive feedback system (avoid coupling by keeping m=0 mode-frequency constant).
e. decreased bunch length (needs experimental verification).
f. higher injection energy.

Most of these cures have side-effects, e.g. when the synchrotron frequency is increased at injection, it can no longer be kept constant during acceleration and it becomes necessary to jump synchro-betatron resonances. A higher harmonic RF system for LEP would be rather costly. A non-linear (dipole-octupole or quadru-sextupole) wiggler may be a more economic alternative. The brute-force method of reducing the betafunctions at the RF cavities is rather limited before problems occur with the machine optics.

A factor of 50% in one plane could be gained by simply placing the copper cavities only near the F-quads where the vertical betafunctions are small. A reactive feedback system has been developed at the ISR and should in any case be useful for keeping the coherent tune constant. Its effectiveness needs to be tested on existing machines. Finally, the somewhat surprising fact that the stability becomes better for shorter bunches - i.e for higher peak currents - may be limited by higher mode losses in sensitive components, even if the required short bunch lengths can be reached. In any case, the wide variety of possible cures need experimental verification under clean conditions, and support from computer simulations which are now in progress.

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