THRESHOLD BEHAVIOR FOR LONGITUDINAL STABILITY OF INDUCTION LINAC BUNCHES*

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Summary

The induction linac bunches of heavy ion fusion scenarios are strongly influenced by the longitudinal space charge impedance. This is in distinct contrast to relativistic bunches in storage rings where most of the data on stability have been obtained. Simulation results reveal that when space charge effects are large, the stability requirement of small growth rate relative to the synchrotron frequency for relativistic bunches is replaced by the relaxed condition of small growth rate relative to the frequency spacing of the space charge wave modes on the bunch. Dispersive effects from finite pipe size tend to make the lower frequencies less susceptible to instability than higher frequencies. Since induction modules have a high resistive component only for the lowest bunch modes, stability is better than would occur for a broadband impedance of comparable magnitude. These results indicate that long term longitudinal bunch stability is realizable for induction linac drivers for heavy ion fusion.

Introduction

The final lens system for focusing an intense beam of heavy ions for inertial confinement fusion requires a very small longitudinal velocity spread. For a uniform beam the currents envisioned would be above threshold for longitudinal instability driven by the induction module and space charge impedance, and would contribute to growth of the velocity spread. For bunched beams, however, there is clear evidence[1,2] that the coupling of growing waves and damping waves through bunch end reflections can have a stabilizing influence. However, for sufficiently fast growth rates it has been observed in relativistic storage rings that this reflection mechanism breaks down. These issues have been addressed through a particle simulation code[3] which can model an arbitrary machine impedance including space charge forces (with finite pipe size effects) in an induction linear accelerator. Results indicate that the end reflection process does indeed improve the long term stability characteristics of an induction linac bunched beam. Ultimately, an instability threshold is reached, but at values of the module impedance which are higher than required for typical induction linac drivers. Dispersive effects introduced by the finite pipe size reduce the group velocity of high frequency perturbations, making their threshold requirements more severe. The thresholds observed are consistent with a mode coupling model of the bunch instability analogous to Sacherer's[4] analysis of storage ring bunch instabilities.

Coherent Longitudinal Bunch Dynamics

The coherent propagation of a longitudinal perturbation on an induction linac bunch is driven by the induction module impedance and space charge. The impedance of the induction module is well represented over the most important frequency range by a parallel RLC circuit. The dispersive space charge force law of a beam in a circular pipe can be approximated by an electric field whose k dependence is given by

$$E \propto \frac{ik}{1 + \alpha k^2} \tag{1}$$

where $\alpha = (B/2.4)^2$ for pipe radius B. This expression is obtained from a Bessel function expansion of the longitudinal field in a finite pipe, with the value 2.4 the first zero of J_0. For a purely rectangular, cold bunch of length L, space charge wave modes can be obtained from a fluid model. The eigenfrequencies of these modes are given by

$$\omega_n^2 = v_0^2 (k_n)^2/(1 + \alpha k_n^2) \tag{2}$$

$$k_n = \frac{\pi n}{L} \tag{3}$$

where n is any integer and L is the bunch length. The plasma wave velocity, V_0, is given by

$$v_0^2 = q^2 g\lambda/m \quad , \tag{4}$$

for charge q, g = 1 + 2Ln (B/A), line density (λ) beam radius A, and mass m. Note that at high frequencies the modes coalesce, and the group velocity tends to zero.

Microwave Instability

For a non-relativistic uniform beam the threshold for longitudinal instability for machine impedance per unit length Z' and wave vector k is given by the Landau damping[5] condition

$$v_0^2(Z'v/k) > 2\pi F \, v_{th}^2 \tag{5}$$

where v_{th} is the full width, half height velocity spread, v is the beam velocity, and F is a form factor of the order of unity.

For relativistic bunched beams in a storage ring it has been found that if the analogous condition is satisfied for modes of wavelengths shorter than the bunch, there is instability only if the associated growth rate exceeds the synchrotron angular frequency. Sacherer[4] has analyzed this phenomenon in terms of a perturbation expansion, and found that the above condition corresponds to strong coupling of the unperturbed modes of the bunch by the machine impedance Z'. The coupling is strong if the associated growth rate exceeds the local mode spacing. For hot bunches, where the thermal velocity greatly exceeds the plasma wave velocity, the mode spacing is the synchrotron angular frequency.

Simulations of the hot bunch microwave instability were performed, with the machine impedance modeled with the usual Q = 1 resonator which has been found to well represent most storage rings. A threshold for instability was found with the above form factor

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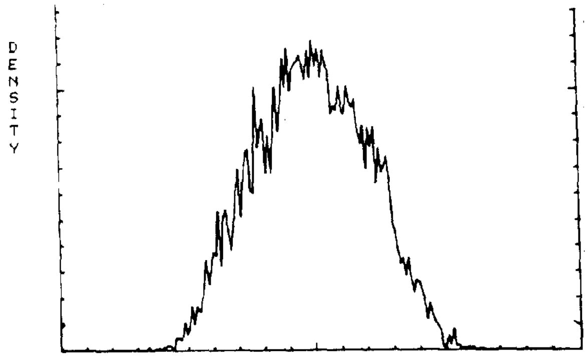

Figure 1: Equilibrium longitudinal density of an initially parabolic distribution below microwave instability threshold.

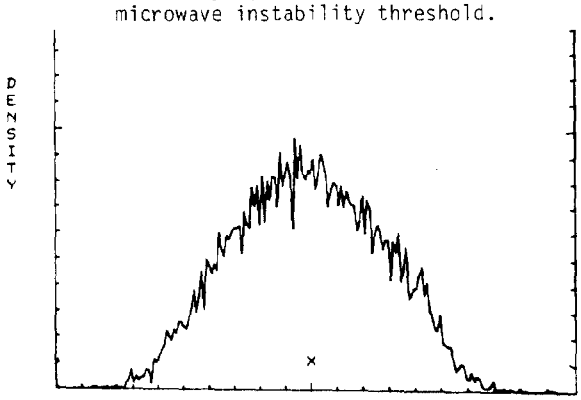

Figure 2: Equilibrium longitudinal density of an initially parabolic distribution above microwave instability threshold.

F = 0.6. Figures 1 and 2 show the density distribution after saturation of a parabolic bunch below and above the stability threshold, respectively. The initial velocity spread and bunch length were the same in both cases, with the impedance doubled in the latter to exceed the instability threshold. Below threshold, there has been some softening of the sharp edged parabolic distribution, but no appreciable bunch lengthening. Above threshold, the bunch lengthens until an equilibrium is reached.

Instability Thresholds for Induction Linac Bunches

In the simulations shown, induction linac bunches are modeled by a uniform density with 10% parabolic ends. The thermal spread was constant over the uniform region with a value of approximately 0.1 of the wave velocity v_0. In the parabolic ends the phase space distribution is elliptic.[6] The beam pipe radius is .05 of the bunch length. Figure 3 illustrates the resulting phase space distribution. Runs were performed using a resistance with a high frequency rolloff. The magnitude of the resistance is best expressed in terms of e-folding lengths L_e of the nondispersive uniform beam longitudinal instability. The e-folding length is inversely proportional to the resistance in the parameter regime studied. Figures 4, 5, 6 are phase space plots of runs with resistive e-folding lengths of 0.6, 0.3, 0.15 bunch lengths and at times corresponding to 12, 6, and 3 plasma wave traversals along the bunch. Thus, each figure represents the same total

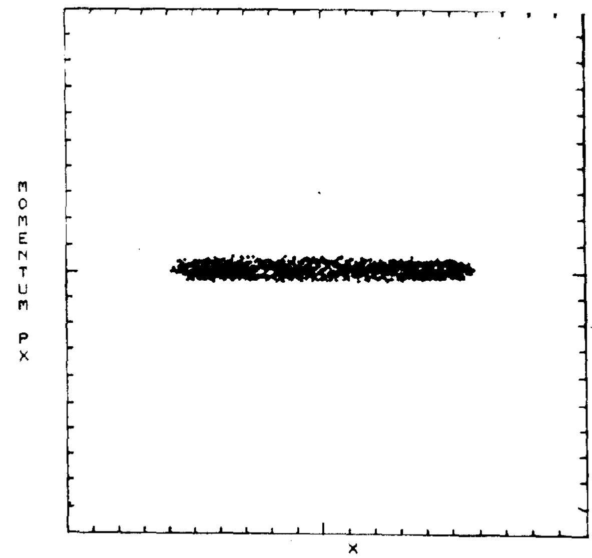

Figure 3: Initial phase space distribution for a model induction linac bunch.

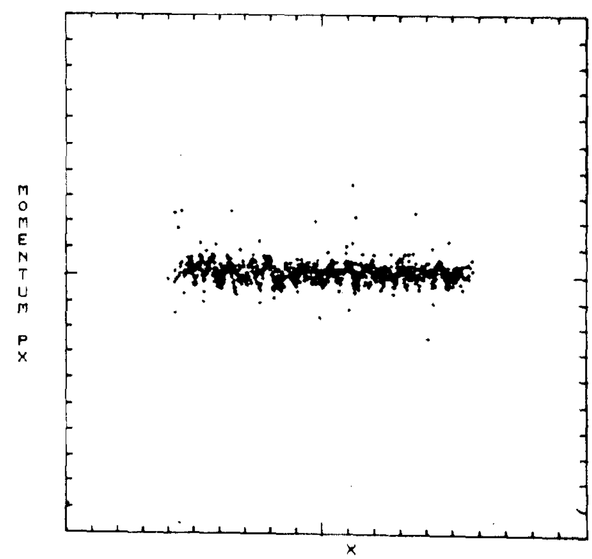

Figure 4: Phase space distribution for L_e = .6 after 12 plasma wave traversals.

exponential growth if there is no reflection stabilization. (All cases are well above the Landau damping threshold as determined primarily by the space charge force, which is unchanged from case to case.) It is clear that a threshold for instability has been crossed. A calculation of the ratio of the uniform beam growth rate γ to the spacing $\Delta\omega_n$ of the modes of Eq. (2) is shown in Table 1, and suggests a criterion similar to that of Sacherer's. The higher frequency modes are more closely spaced because of the space charge dispersion, and low frequencies should be relatively less susceptible to instability. This appears to be borne out in that low frequency growth is more dominant for the shortest e-folding length.

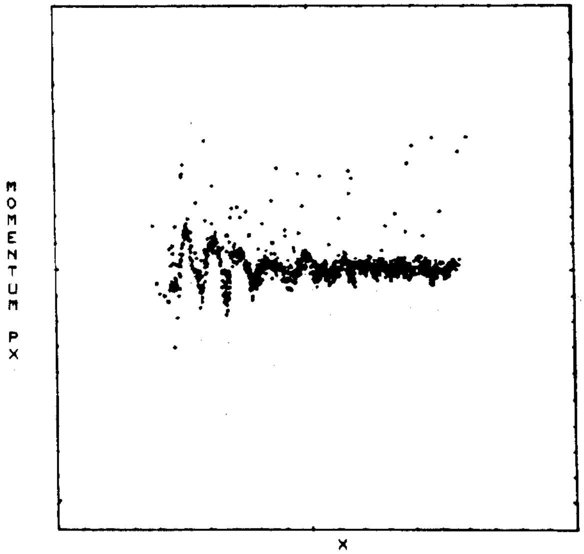

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Figure 5: Phase space distribution for L_e = .3 after 6 plasma wave traversals.

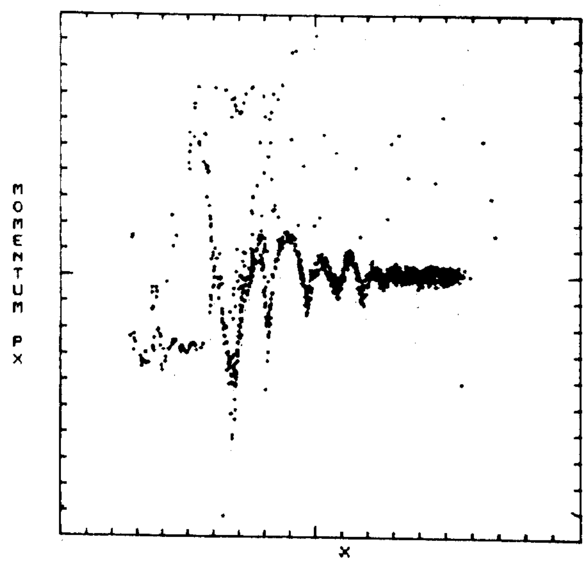

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Figure 6: Phase space distribution for L_e = .15 after 3 plasma wave traversals

Table 1
Uniform Growth Rate/Mode $\Delta\omega_n$ Spacing

Mode Number	e-folding length = .6	.3	.15
2	.65	1.3	2.6
6	.65	1.3	2.6
10	.70	1.4	2.8
14	.75	1.5	3.0
18	.85	1.7	3.4

It should be noted that the above criterion corresponds to a calculation of the e-folding length using the group velocity and comparing the values obtained to the bunch length. The mode spacing provides the proportionality constant.

Several runs were made to explore this threshold phenomenon and the conjecture that instability occurs only if the ratio $S = \gamma/\Delta\omega_n$ is of the order unity or greater. Increasing the bunch length reduces the spacing of modes and increases S. It was found that doubling the bunch length for L_e = 0.6 does indeed induce instability, with a time-scaled growth comparable to the short bunch, L_e = 0.3 case. As another test of the criterion, a narrowband impedance was investigated, with peak values corresponding to the fifth and fifteenth mode. Because of the space charge dispersion, it was possible to choose a single value for the peak impedance such that for mode 5, $S < 1.3$, and for mode 15, $S > 2$. The lower S valued case was stable, whereas the higher was found to be unstable. When the dispersion was enhanced by increasing α, the fifth harmonic could be driven unstable, with again $S > 2$.

Stability Requirements for HIF Induction Linacs

The impedance of HIF induction modules is large at low frequencies, with a frequency halfwidth corresponding to only several harmonics on the bunch. It is at these frequencies that dispersive effects are at a minimum and, therefore, stability is at a maximum. Runs modeling the induction module impedance exhibit stability for growth lengths of 0.3 bunch lengths (calculated from the peak module resistance).

These results indicate that module impedances of several hundred ohms/meter, which provide good linac efficiency, can be tolerated for a variety of induction linac driver parameters. Since the threshold value of Z' scales inversely with bunch length, short bunches are to be preferred from the point of view of longitudinal stability. The numerical studies presented are strictly applicable to single beam transport, and include neither acceleration nor multiple beam interactions. Matching of the end longitudinal focusing has been idealized. These issues may be of importance in determining stability thresholds and beam quality, and are currently being explored.

In conclusion, numerical results suggest that long term longitudinal stability is realizable for induction linac HIF drivers. The magnitude of the longitudinal machine impedance cannot be disregarded in obtaining an optimal design, but there is sufficient flexibility in the choice of machine parameters to permit both high linac efficiency and long term longitudinal beam stability.

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