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THE DESIGN PARAMETERS FOR A LITHIUM LENS AS ANTIPROTON COLLECTOR

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Summary

Maxwell's equations with appropriate boundary conditions are solved for a pulsed cylindrical conductor. The results are applied to the lithium lens used as an antiproton collector for the Fermilab $\bar{p}p$ collider. The magnetic field is expressed as a function of radial distance and time. The time corresponding to maximum linearity is calculated. A method for measuring the current density at the surface is discussed and the Joule heat produced per pulse is calculated.

Introduction

Recently there has been some interest in using the magnetic field inside a current-carrying cylindrical conductor to focus particle beams.[1] Applications include focusing targets and lithium lenses. The calculations described in this report were done in connection with the design of a lithium lens to focus antiprotons just downstream of the production target for the $\bar{p}p$ collider at Fermilab. However, many of the results are generally applicable for any pulsed cylindrical conductor.

For the simple case of a cylindrical conductor of radius r_0 carrying total current I with uniform current density J the magnetic intensity H is given by

$$H(r) = H(r_0)\frac{r}{r_0}$$

However, for many applications, including the lithium lens to be used as the antiproton collector, the Joule heating from direct current is prohibitively large. To minimize heating, these devices are often pulsed with a sine-like unipolar pulse whose width $\tau/2$ is small compared to the time between pulses. For the pulsed device an expression describing the magnetic intensity H as a function of radial position in the conductor and time can be derived by solving Maxwell's equations with appropriate boundary conditions. A solution applicable to a pulsed lithium lens with $I = I_0 \sin\omega t$ for $0 < t < \pi/\omega$ is given in Ref. 8. This paper assumes the cylindrical conductor is a component in an RLC circuit and has a pulse shape modified by a damping factor $e^{-\alpha t}$ where $\alpha = R/2L$. The mathematical description of the pulse form is $I = I_0 e^{-\alpha t}\sin\omega t$ for $0 < t < \pi/\omega$ where $\omega = 2\pi/\tau$ and $I = 0$ between pulses.

The paper is presented in three parts. In Part A an expression for $H(r,t)$ is presented and the time during the pulse corresponding to maximum linearity is calculated. In Part B an expression for the current density J_z is derived and a method for measuring J_z at the surface of the conductor is discussed. Part C describes Joule heating, including the radial dependence of temperature and the total heat deposited per pulse.

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A. Magnetic Field in a Pulsed Cylindrical Conductor

Assuming that the cylinder is coaxial with the z axis, that $\vec{H}$ has only an azimuthal (ϕ) component and that the magnitude of $\vec{H}$ depends only on r and t, manipulation of Maxwell's equations leads to

$$\frac{\partial^2 H}{\partial r^2} + \frac{1}{r}\frac{\partial H}{\partial r} - \frac{H}{r^2} = \sigma\mu\frac{\partial H}{\partial t} \qquad (1)$$

where H is the azimuthal component of $\vec{H}$, σ is the conductivity and μ is the permeability. The boundary conditions are

$$H(r,0) = 0;\ H(0,t) = 0$$

and

$$H(r_0,t) = H_0 \operatorname{Re}\{ie^{-\gamma t}\}$$

where $\gamma = \alpha + i\omega$ and H_0 is the maximum value of H for an undamped pulse.[9] A general solution to (1) is

$$H(r,t) = H_0 \operatorname{Re}\left\{\frac{iJ_1(\beta r)}{J_1(\beta r_0)}e^{-\gamma t}\right\} + \sum_j a_j J_1(\lambda_j r)\, e^{-\lambda_j^2 t/\sigma\mu} \qquad (2)$$

where

$$a_j = -4H_0\frac{r_0^2}{\delta^2}\frac{\lambda_j r_0}{[(\lambda_j^2-\sigma\mu\alpha)r_0^2]^2+4(r_0/\delta)^4}\frac{1}{J_0(\lambda_j r_0)}$$

and δ is the skin depth. $J_1(\beta r)$ is a complex first order Bessel function with β defined by $\beta^2 = \sigma\mu\alpha + 2i/\delta^2$ and $J_1(\lambda_j r)$ is a real first order Bessel function. The products $\lambda_j r_0$ are roots of $J_1(x) = 0$.

For the antiproton collection lens, the operating parameters[10] will be $\delta/r_0 \sim 0.45$ and $\alpha \sim 1300\ \mathrm{sec}^{-1}$. The time dependence of the penetration of the field into the lithium is shown in Fig. 1.

The time at which the field is most linear depends on δ/r and α. This time can be calculated as follows.[11] Define the quantity

$$(\Delta H)^2 \equiv [H(r,t) - G(t)\, r]^2$$

as a measure of the deviation of the field from linearity, with $G(t)$ equal to the slope of the best fit to a straight line. The expectation value of $(\Delta H)^2$ is

$$\langle(\Delta H)^2\rangle = \frac{1}{\pi r_0^2}\int_0^{2\pi}\int_0^{r_0}(\Delta H)^2\, r\,dr\,d\phi \qquad (3)$$

The value of G corresponding to a minimum expectation value is found by solving the equation

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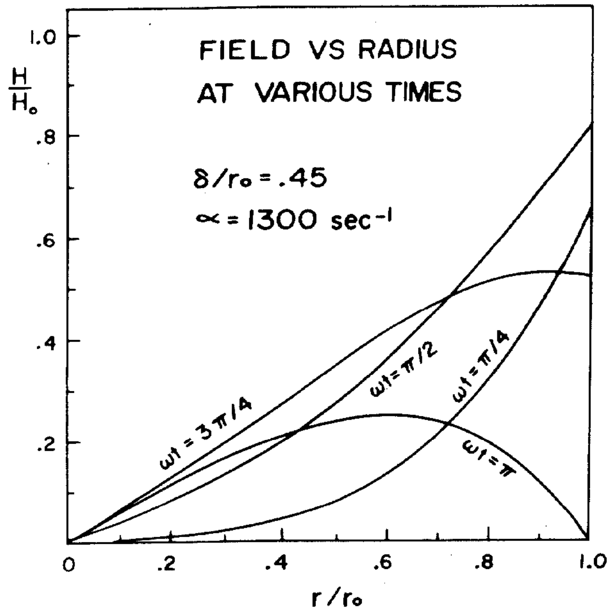

Fig. 1. Magnetic intensity vs radius at several times during the pulse. H_0 has units amp/meter and is the value of H corresponding to I_0 in the expression $I=I_0e^{-\alpha t}\sin\omega t$. For an undamped pulse it is the maximum value of H at the surface of the cylinder.

$$\frac{\partial}{\partial G} \langle(\Delta H)^2\rangle = 0$$

The result is

$$G = \frac{4}{r_0^{\,2}} H_0 \mathrm{Re}\left\{\frac{ie^{-\gamma t}}{\beta}\,\frac{J_2(\beta r_0)}{J_1(\beta r_0)}\right\} - \sum_j \frac{a_j}{\lambda_j} J_0(\lambda_j r_0) e^{-\lambda_j^{\,2} t/\sigma\mu}$$

A measure of the goodness of fit to a straight line is found by substituting this expression for G into (3) and performing the integration. The time at which $\langle(\Delta H)^2\rangle$ is minimum is the time at which the field is most linear. Fig. 2 shows the time corresponding to maximum linearity vs δ/r_0.

B. Current Density in a Pulsed Cylindrical Conductor

An expression for the current density may be derived by taking the curl of the expression for $\vec{H}$ given in (2). The result is

$$J_z(r,t) = H_0 \mathrm{Re}\left[ie^{-\gamma t}\,\frac{\beta J_0(\beta r)}{J_1(\beta r_0)}\right] + \sum_j a_j\lambda_j J_0(\lambda_j r) e^{-\lambda_j^2 t/\sigma\mu} \tag{4}$$

Figure 3 shows J_z vs ωt for various values of r/r_0. The curve describing J_z at the surface of the conductor vs ωt is of particular interest because it is related to the potential difference between two points on the surface of the conductor via the equation $V = \int\vec{E}\cdot d\vec{\ell} = \rho\int\vec{J}\cdot d\vec{\ell}$ where ρ is the electrical resistivity of the conductor. Consider a line segment of length L, parallel to the axis of the cylinder and having as its endpoints two points on the surface of the cylinder. The potential difference between these

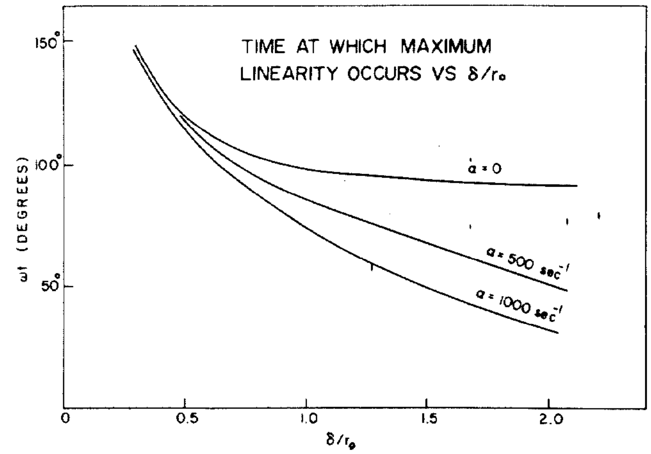

Fig. 2. The time during the pulse at which maximum linearity occurs. The pulse lasts for a time interval corresponding to $0 < \omega t < 180°$.

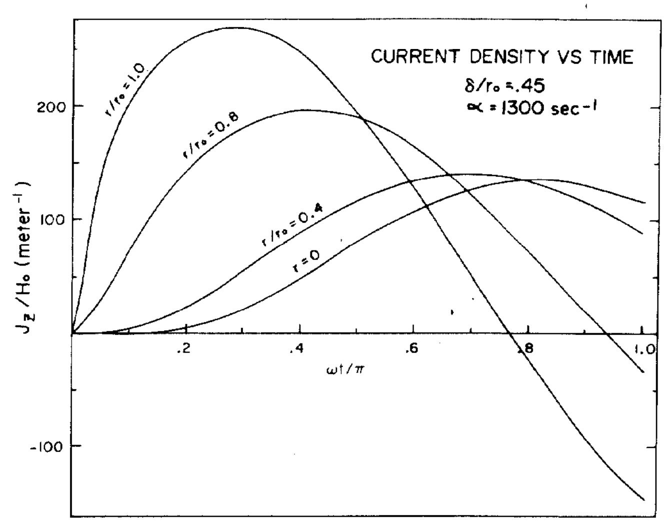

Fig. 3. Current density vs time at various distances from the axis.

points at time t is $L\rho J_z(r_0,t)$. Measurement of this potential difference provides a test of whether or not the device is producing the expected field.[12]

C. Joule Heating in a Pulsed Cylindrical Conductor

An expression for heating due to ohmic losses can be found by evaluating the integral $\int\vec{J}\cdot\vec{E}$ dtdV. This will be done assuming constant resistivity during the pulse and then a method for taking into account a changing resistivity will be given. The radial distribution of heat is given by

$$q_0(r) = \rho\int_0^{\pi/\omega} J_z^2 dt \tag{5}$$

Using 4 for J_z one obtains

$$q_0(r) = \frac{H_0^{\,2}}{4\sigma}\left[\mathrm{Re}\left(\frac{\beta^2 J_0^{\,2}(\beta r)}{\gamma J_1^{\,2}(\beta r_0)}\right) - \frac{1}{\alpha}\left|\frac{\beta J_0(\beta r)}{J_1(\beta r_0)}\right|^2 (e^{-2\alpha\pi/\omega}-1)\right]$$

$$+2\,\frac{H_0}{\sigma}\,\mathrm{Re}\left[i\beta\frac{J_0(\beta r)}{J_1(\beta r_0)}\sum_j \frac{a_j\lambda_j J_0(\lambda_j r)}{\gamma+\lambda_j^{\,2}/\sigma\mu}\right](1+e^{-\pi(\alpha+\lambda_j^{\,2}/\sigma\mu)/\omega})$$

$$+ \mu \sum_j \sum_n \frac{a_j a_n \lambda_j \lambda_n}{\lambda_j^2+\lambda_n^2} J_0(\lambda_j r)\, J_0(\lambda_n r)(1-e^{-\pi\delta^2(\lambda_j^2+\lambda_n^2)/2})$$

The units for q_0 are Joules/m^3 and the temperature rise $\Delta T(r)$ can be calculated by dividing $q_0(r)$ by the heat capacity c. The total heat generated per pulse unit length is found by integrating $q_0(r)$ over the cross sectional area. The result of this integration is

$$Q_0 = \frac{H_0^2\pi}{4\sigma} \frac{(\beta r_0)^2}{\gamma} \left(\frac{J_0^{\,2}(\beta r_0)}{J_1^{\,2}(\beta r_0)} + 1\right) (e^{-2\pi\alpha/\omega}-1)$$

$$- \frac{H_0^{\,2}\pi r_0}{\sigma} \frac{\beta\beta^*}{\beta^2-\beta^{*2}} \frac{e^{-2\alpha\pi/\omega}-1}{\alpha} \left\{ i \ \mathrm{Im}\left(\frac{\beta J_0^*(\beta r_0)}{J_1^*(\beta r_0)}\right)\right\}$$

$$+ \frac{4\pi H_0 r_0}{\sigma} \mathrm{Re} \left\{ i\beta^2 \sum_j a_j\lambda_j \frac{J_0(\lambda_j r_0)}{\beta^2-\lambda_j^{\,2}} \frac{1+e^{-\pi(\alpha+\lambda_j^2/\sigma\mu)/\omega}}{\gamma+{}_j^2/\sigma\mu} \right\}$$

These results were derived assuming a constant resistivity. An approximation which can be used to take into account the temperature dependence of resistivity is given by Knoepfel.[13] The resistivity can be parameterized by $\rho = \rho_0\ (1 + bQ)$ where b is the heat factor and Q the increase of heat content relative to 0°C. In the solid phase $Q = c\Delta T$. In the case of lithium, one uses the slope of a ρ vs T curve and the value $c = 2.0 \times 10^6$ Jm^{-3}/°C to calculate $b = 2.4 \times 10^{-9}$ m^3/J. If Q_0 is the heat per unit volume calculated above, then the "corrected" value is $Q = (\exp(bQ_0)-1)/b$. Figure 4 shows the radial distribution of heat that deposited during a single pulse of a

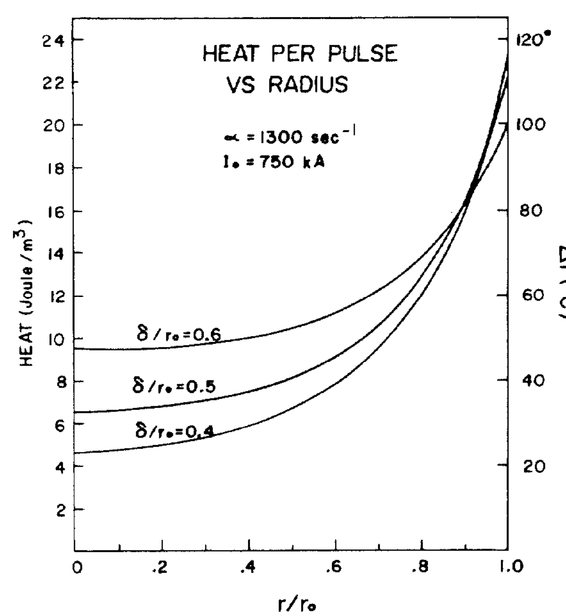

Fig. 4. Heat per unit volume per pulse vs radius for a lithium cylinder of radius 1 cm. The ΔT scale was calculated assuming $c = 2 \times 10^6$ Jm^{-3}/°C.

lithium cylinder with I_0 = 750 kA. It includes the correction for changing resistivity. For a 1 cm radius 15 cm long cylinder and δ/r_0 = .45 the total heat per pulse Q is 4570 Joules.

Some Conclusions

In order to understand the focusing properties as well as the power and cooling requirements for operating the antiproton collection lithium lens, it is necessary to understand the penetration of a magnetic field into a pulsed lithium cylinder. Maximum linearity occurs after the current has peaked. To achieve the required gradient at the time of maximum linearity, the peak current has to be scaled accordingly. (For the collection lens, 670 kA peak current is required if the current is to be 500 kA when the field is most linear.) Joule heating increases in a nonlinear manner from a minimum along the axis to a maximum at the surface of the cylinder. It can be decreased by decreasing the pulse width, and hence the skin depth δ but this is done at the expense of linearity.

References & Footnotes

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10. For details of these and other lens parameters, see paper K251, G. Dugan, et al., this conference.

11. This approach to the problem is taken from Ref. 8.

12. For a brief description of an actual measurement using a tantalum cylinder, see Fermilab $\bar{p}$ Note No. 262, January 1983 (unpublished), by A. J. Lennox.

13. H. Knoepfel, <u>Pulsed High Magnetic Fields</u>, North-Holland Publishing Company, London, 1970, p. 88.