 IEEE Transactions on Nuclear Science, Vol. NS-30, No. 4, August 1983

DESIGN OF RFQ VANE TIP TO MINIMIZE SPARKING PROBLEMS

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Summary

Transverse and longitudinal focusing in radio-frequency quadrupole (RFQ) accelerators depends on V/R_0 (the vane voltage divided by the mean radial aperture). For maximum current carrying capacity, one generally designs for maximum V/R_0, which is limited by the peak surface electric field E_s $(= \kappa(V/R_0))$ that can be sustained without sparking. The value of the enhancement factor κ depends on the pole geometry and can be minimized by choosing an appropriate pole shape. A computer program POTRFQ will be described which derives the field potential $\phi(r,\theta,z)$ and the vane tip contour, for a range of input parameters. The effects on the beam dynamics of the higher multipole components resulting from the modified pole shapes will also be presented.

Introduction

The enhancement factor relates the peak surface electric field gradient on an RFQ vane to the V/r_0. For a conventional geometry as used in the Los Alamos POP RFQ[1] the enhancement factor is 1.355 for an unmodulated vane, but for a modulated vane can be greater than 1.6. Accelerator performance can be optimized by the proper choice of field gradient but in practical designs sparking between vanes prevents operation at this optimum. Improved performance is expected if the vane surfaces are shaped to reduce the enhancement factor and allow operation at higher fields. Intuitively what is wanted is to shave the vane tips to increase the minimum spacing between vanes without making the vanes sharp enough to increase the surface field.

RFQ Vane Surfaces and the Potential

From the viewpoint of beam particle dynamics any accelerator geometry is considered acceptable if it is able to establish a suitable radiofrequency quadrupole electric field potential $\Phi(r,\theta,z,t)$ in the vicinity of the beam axis. If the beam diameter is much less than the wavelength we can ignore rf magnetic fields near the beam axis and use a quasi-static approximation in which the time dependence can be separated.

$$\Phi(r,\theta,z,t) = \phi(r,\theta,z)\sin(\omega t)$$

Idealized vane surfaces have an asymmetric quadrupole geometry with the asymmetry being periodic in the z-direction with period 2L. The potential has the following boundary and symmetry conditions:

$$\phi(r,\theta,z+2L) = \phi(r,\theta,z)$$

$$\phi(r,\theta+\pi/2,z+L) = -\phi(r,\theta,z)$$

$$\phi(-x,y,z) = \phi(x,y,z)$$

$$\phi(x,-y,z) = \phi(x,y,z)$$

$$\left.\frac{\partial\phi}{\partial z}\right|_{z=0} = 0$$

A potential that satisfies these conditions can be expressed in the following series expansion

$$\phi(r,\theta,z) = \sum_{s=0}^{\infty} C_{0s}(kr)^{\lambda}\cos(\lambda\theta) + \sum_{m=1}^{\infty}\cos(kmz)\sum_{s=0}^{\infty} C_{ms} I_{\lambda}(kmr)\cos(\lambda\theta)$$

where $k = \pi/L$
and $\lambda = 4s$ if m is odd
$= 4s + 2$ if m is even
I_λ = a modified Bessel function of order λ.

If the C_{ms} are known we can evaluate the potential and electric field anywhere. In particular, the equipotential surfaces which could define the vane tips and the electric field at any point on these surfaces can be found.

POTRFQ

Mathematically the simplest form of RFQ is one in which only the terms C_{00} and C_{10} are non-zero (the two-term potential). This is not a practicable device, however, because the vane surfaces required to give this potential would have a hyperbolic cross section and an unbounded maximum surface field. A reasonable approximation can be obtained with cylindrical vane tips analogous to the poles of a magnetic quadrupole and any convenient shape beyond the tip. This was the design used in the Los Alamos POP. It has always been assumed that the higher order terms inherent in this design have a negligible effect on the beam.

The calculation of the coefficients C_{ms} for an arbitrary vane shape is tedious but has been done for a few simple shapes using the computer program CHARG3D written by K. Crandall at LANL. An alternative technique is used by the program POTRFQ which evaluates the potential and the vane shape from specified source terms A_{mn}. This program is approximately 100 times faster than CHARG3D but the design process still has an element of trial and error, the problem being to find source terms that result in small enhancement factors.

POTRFQ evaluates the potential $\phi(r,\theta,z)$ in terms of 4 sets of source terms A_{mn} along lines with $(x,y) = (a,0), (0,a), (-a,0), (0,-a)$ using

$$\phi(r,\theta,z) = \sum_{j=1}^{4}\sum_{m=0}^{M} \chi_{jm}\cos(kmz)\sum_{n=0}^{N} A_{mn} f_{nm}(r_j)\cos n\theta_j$$

where $\chi_{jm} = (-1)^{j-1}$ if m is even
$= 1$ if m is odd
$k = \pi/L$
$f_{nm}(r_j) = \log(kr_j)$ for $m=n=0$
$= (1/kr_j)^n$ for $m=0,\ n>0$
$= \dfrac{K_n(kmr_j)}{K_n(kma)}$ for $m>0$

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The C_{ms} are related to the A_{mn} by the following:

$$C_{0s} = \frac{4}{(ka)^{\lambda}} - \frac{A_{00} - \log(ka)}{\lambda} \sum_{n=1}^{N} \binom{-n}{\lambda} A_{0n}(ka)^{n}$$

where $\lambda = 2 + 4s$ if m is even
$= 4s$ if m is odd

$$\binom{-n}{\lambda} = (-1)^{\lambda} \frac{(n+\lambda-1)!}{\lambda!(n-1)!}$$

and for $m > 0$

$$C_{ms} = \frac{4}{1+\delta_{\lambda 0}} \sum_{n=0}^{N} \frac{K_{n+\lambda}(kma) + K_{|n-\lambda|}(kma)}{K_n(kma)} A_{mn}$$

where $\delta_{\lambda 0} = 1$ if $\lambda = 0$
$= 0$ otherwise

Vane surfaces are defined by solving

$$\phi(r,\theta,z) = \pm V/2$$

where V is the voltage between vanes. The surface field is given by

$$\sqrt{(E_R^2 + E_\theta^2 + E_z^2)}$$

where $E_R = -\partial\phi/\partial r$
$E_\theta = -\frac{1}{r}(\partial\phi/\partial\theta)$
$E_z = -\partial\phi/\partial z$

Choice of A_{mn} to Fit a Specified Equipotential Surface

The technique of choosing suitable A_{mn} is not trivial so a separate program SPOTRFQ was written to do a least squares fit for the A_{mn} necessary to provide a specified equipotential surface. This program allows the points to be weighted and permits linear constraints on the A_{mn} to be specified.

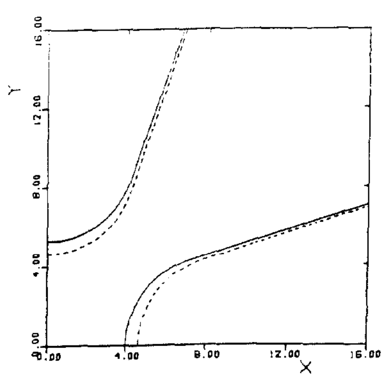

Fig. 1 Two sections through equipotential surfaces to be fit using SPOTRFQ.
solid line at z = 0
dashed line at z = L/2.

As an example consider the pole tip shown in Fig. 1 which is described at the 3 positions: quadrupole symmetry, maximum, and minimum aperture. Points are chosen on these curves, weighted more heavily near the tip, and constrained to fit at the minimum aperture. Sets of A_{mn} for two values of n_{max} are shown in Table 1 and the resulting equipotentials shown in Fig. 2. Either set gives an acceptable fit.

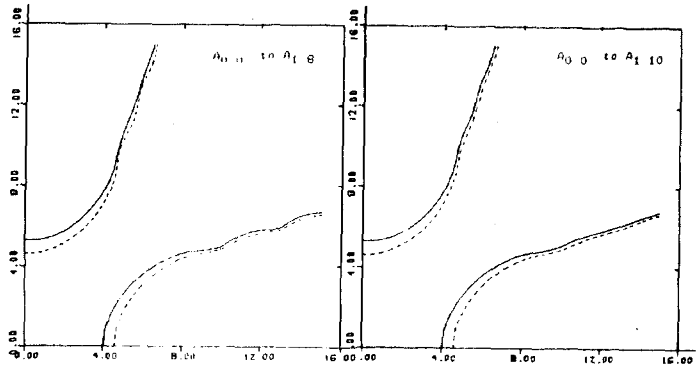

Fig. 2 Equipotential surfaces defined by source terms A_{mn}.

Table 1

Source Terms Calculated by SPOTRFQ

	m = 1, n = 8	m = 1, n = 10
A_{00}	-0.08469	-0.08479
A_{10}	-0.00086	0.00068
A_{01}	0.00374	0.00334
A_{11}	-0.00036	-0.00005
A_{02}	0.01215	0.01239
A_{12}	0.00107	0.00083
A_{03}	-0.00122	-0.00142
A_{13}	-0.00027	-0.00013
A_{04}	0.00177	0.00191
A_{14}	0.00038	0.00031
A_{05}	-0.00030	-0.00038
A_{15}	-0.00029	-0.00007
A_{06}	0.00022	0.00026
A_{16}	0.00007	0.00007
A_{07}	-0.00003	-0.00005
A_{17}	-0.00001	-0.00001
A_{08}	0.00001	0.00002
A_{18}	0.00001	0.00001
A_{09}		0.00000
A_{19}		0.00000
A_{010}		0.00000
A_{110}		0.00000

	m = 1, n = 8	m = 1, n = 10
C_{00}	0.03272	0.03272
C_{10}	0.00657	0.00655
C_{01}	0.08850	0.09006
C_{11}	0.37167	0.38505

Optimum Radius of Curvature for an Unmodulated Pole

POTRFQ has been used to find the radius of curvature of a vane tip to minimize the enhancement factor. For the simplest case where only A_{00} is non-zero, the pole tips have a radius of curvature depending on the value of (a/R_0). Figure 3 shows the enhancement factor κ plotted against (a/R_0) where a minimum of $\kappa = 1.265$ occurs at $a/R_0 = 1.7$ corresponding to a radius of curvature at the tip of 0.7 R_0. Figure 4 shows how the harmonic terms vary with a/R_0. κ vs a/R_0 (Fig. 3) is fairly flat near its minimum so a smaller harmonic contribution can be obtained at a slightly larger a/R_0 for a small penalty in κ. A value of $\rho = 3/4\ R_0$ is a good compromise for the optimum radius of curvature.

The variation of the enhancement factor with vane tip radius of curvature as calculated using Crandall's CHARG3D for RFQ1[2] cell number 120 (the region of maximum enhancement) shows essentially the same conclusion that the optimum value of $\rho/R_0 = 3/4$.

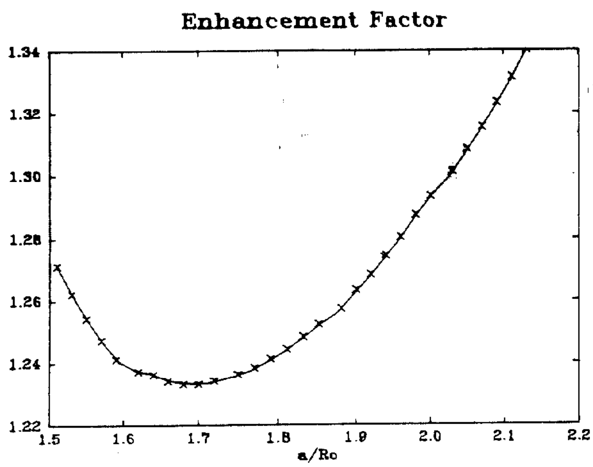

Fig. 3 Enhancement factor vs a/R_0.

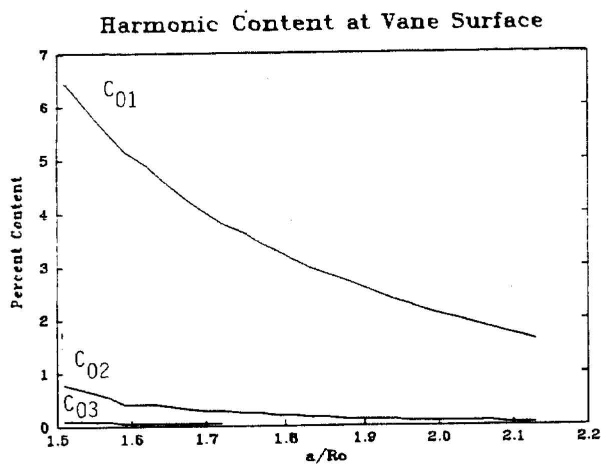

Fig. 4 Higher order harmonic terms of potential vs a/R_0.

Effects of Higher Order Terms

Vane shapes which reduce the enhancement factor will also increase the harmonic content. While the higher order terms are not necessarily harmful, it is important to check that the gain from a reduced enhancement factor is not offset by these effects.

To illustrate the influence of the higher order terms consider our RFQ1 design which has a large ratio of bore radius to cell length (R_0/L) = 0.8 at input) and should be very sensitive to high order effects.

Initially, all calculations were done with the two term potential, assuming POP type vanes. For a 90 mA input beam, the calculated output current varied from 30 mA to 80 mA when the peak electric field gradient was varied from 1.1 to 1.65 times the Kilpatrick limit. At the design field of 1.37 times Kilpatrick, predicted output current was 70 mA (transmission = 78%). However, when the multipole potential with the 8 terms C_{00}, C_{10}, C_{20}, C_{30}, C_{01}, C_{11}, C_{21}, C_{32} was used, transmission at 1.37 times Kilpatrick fell to about 67% (60 mA out for 90 in).

With POP style vanes, the maximum enhancement factor was 1.48. We then looked at the case of vanes with a tip radius of curvature = 0.75 R_0. For these, the maximum enhancement factor is 1.32, allowing operation at a 12% higher vane voltage than with POP style vanes. However, the advantages of higher vane voltage are almost negated by the harmful effects of the higher multipole components of the field and at 1.37 times Kilpatrick, transmission is still about 67%. (The C_{11} and C_{20} terms are approximately 3 times as large for a tip radius of 0.75 R_0 as for POP style vanes.) C_{11} was identified as the term responsible for the reduced transmission by omitting the higher order terms in the dynamics calculation one at a time.

We have looked in some detail at the particle trajectories with and without the higher order terms. In the majority of cases, the betatron oscillation frequency is slightly increased by these terms (which would normally improve performance) but in addition the coupling between longitudinal and transverse motion is increased. This seems to cause more particles to drop out of the phase stable accelerating region, followed eventually by an increase in transverse amplitude and loss to the vanes.

Conclusions

A computer program has been developed to calculate the potentials for a variety of vane shapes, making it easier to search for an optimum design for a high current RFQ. By appropriate choice of vane tip shape, the enhancement factor can be reduced significantly from that of the POP type design but the higher multipole terms of the potential will be larger. For a high current RFQ with a large ratio of bore to cell length, the higher multipole terms have a significant effect on beam dynamics even for the POP type vanes. For the designs studied to date the improvements from reduced enhancement factor have been largely offset by higher multipole effects.

Acknowledgements

The authors would like to thank W.L. Michel for his work in running the computer codes and tabulating the results.

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