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RADIO-FREQUENCY QUADRUPOLE VANE-TIP GEOMETRIES*

K. R. Crandall, R. S. Mills, and T. P. Wangler, AT-1, MS H817
Los Alamos National Laboratory, Los Alamos, NM 87545

Summary

Radio-frequency quadrupole (RFQ) linacs[1,2] are becoming widely accepted in the accelerator community. They have the remarkable capability of simultaneously bunching low-energy ion beams and accelerating them to energies at which conventional accelerators can be used, accomplishing this with high-transmission efficiencies and low-emittance growths. The electric fields, used for radial focusing, bunching, and accelerating, are determined by the geometry of the vane tips. The choice of the best vane-tip geometry depends on considerations such as the peak surface electric field, per cent of higher multipole components, and ease of machining.

We review the vane-tip geometry based on the "ideal" two-term potential function and briefly describe a method for calculating the electric field components in an RFQ cell with arbitrary vane-tip geometry. We describe five basic geometries and use the prototype RFQ design for the Fusion Materials Irradiation Test (FMIT) accelerator as an example to compare the characteristics of the various geometries.

Vane-Tip Geometry from Two-Term Potential Function

As a starting point for obtaining electric fields and vane-tip geometry in RFQ linacs, we take

$$U(r,\theta,z) = \frac{V}{2}\left[\left(\frac{r}{r_0}\right)^2 \cos 2\theta + A\, I_0(kr) \cos kz\right] \qquad (1)$$

as the time-independent portion of the two-term potential (TTP) function.[3] In this expression, V is the intervane potential difference, and $k = \pi/L$, where $L = \beta\lambda/2$ is the length of one "cell" of the RFQ. Although the cell length and other geometrical characteristics change gradually throughout the linac, the field analysis is done as if each cell were one element in a completely periodic structure.

Let $z = 0$ at the beginning of a cell in which the horizontal vanes (centered at $\theta = 0$) are at the minimum displacement, a, from the z axis; $z = L$ at the end of the cell where the horizontal vane-tip displacement is ma, and m, the modulation parameter, is ≥ 1. The boundary conditions

$$U(a,0,0) = U(ma,0,L) = V/2$$

are used for calculating A and r_0 from Eq. (1):

$$A = \frac{m^2 - 1}{m^2 I_0(ka) + I_0(mka)} \quad ; \qquad (2)$$

$$r_0 = a[1 - A\, I_0(ka)]^{-1/2} \quad . \qquad (3)$$

If a and m are specified, A and r_0 can be calculated directly. However, because focusing and acceleration depend upon r_0 and A, these quantities usually are determined by beam-dynamics requirements, and a and m are calculated by iterating Eqs. (2) and (3).

Ideally, vane-tip geometries should correspond to the ±V/2 isopotential surfaces obtained from Eq. (1). The longitudinal profile of the horizontal vane tip is denoted by $x_p(z)$ and is found numerically from

$$\left(\frac{x_p}{r_0}\right)^2 + A\, I_0(kx_p) \cos kz = 1 \quad . \qquad (4)$$

The transverse radius of curvature at any x_p is

$$\rho_t = x_p\left(\frac{2x_p/r_0 + Q}{2x_p/r_0 - Q}\right) \quad , \qquad (5)$$

where

$$Q = A\, kr_0\, I_1(kx_p) \cos kz \quad .$$

Note that, at the center of the cell where $\cos kz = 0$, both x_p and ρ_t are equal to r_0. The property $x_p = r_0$ at $z = L/2$ is taken as the fundamental definition of r_0 for all vane-tip geometries discussed in this paper.

At any longitudinal position, the transverse cross sections of the isopotential surfaces from Eq. (1) are approximate hyperbolae. In practice, the vane tips cannot conform exactly to these shapes because adjacent vanes would approach each other asymptotically, and the surface electric field would increase indefinitely. Also, hyperbolic surfaces are more difficult to machine than circular cross sections. For these reasons, RFQ vane tips have been machined with circular arcs. The earlier versions machined at Los Alamos had longitudinal profiles agreeing with Eq. (4) and circular tips with radii given by Eq. (5).

Calculation of Fields for Arbitrary Geometries

For arbitrary vane-tip geometries, including the geometry described in the previous section, one needs a reliable method for calculating the surface electric field. Our approach to this problem has been to find a good approximation for the charge density induced on the vane-tip surfaces and then to derive the field information from the charge density. For example, the electric field at any point on the vane-tip surface is directly proportional to the charge density at that point; the intervane capacitance is proportional to the total charge on a vane tip. The potential near the vane tips can be calculated from the charge density and can be Fourier analyzed to determine the amplitude of any multipole component.

Written in terms of the surface charge density σ, the potential at any point $\vec{r}$ near the vane tips is

$$U(\vec{r}) = \int G(\vec{r};\vec{s})\, \sigma(\vec{s})\, dS \quad , \qquad (6)$$

where $G(\vec{r};\vec{s})$ is the potential produced at point $\vec{r}$ by a unit charge located at point $\vec{s}$ on a vane-tip surface; $\sigma(\vec{s})\, dS$ is the amount of charge in the infinitesimal

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area dS; and the integration is over all vane-tip surfaces. Any surface point, $\vec{s}$, can be derived from two independent variables, u and v. We assume that σ can be approximated by a bicubic spline function of u and v, where the locations of the knots of the spline, u_i and v_j, are specified; but the values of σ at the knots, σ_{ij}, are unknown and determined by minimizing

$$\int [V/2 - U(\vec{s})]^2 \, dS$$

with respect to the unknown σ_{ij}'s. The resultant system of equations appears extremely well conditioned and yields very satisfactory results. This allows us to analyze the properties of many types of vane-tip geometries.

Alternate Vane-Tip Geometries

Five vane-tip geometry types have been analyzed using the technique described above. The results are tabulated in Ref. 4. Types 1-3 have the same longitudinal vane-tip profile as specified by Eq. (4). Type 1 has a variable transverse radius of curvature given by Eq. (5); Types 2 and 3 have transverse radii of curvature equal to r_0 and 0.75 r_0, respectively. Types 4 and 5 also have r_0 and 0.75 r_0 for their transverse radii of curvature but have sinusoidal longitudinal profiles given by

$$x_p = r_0\left(1 - \frac{m-1}{m+1}\cos kz\right) \quad . \tag{7}$$

These geometry types are summarized in Table I.

For each type, the geometry is specified completely by two parameters, m and L/r_0. Field characteristics have been calculated for each type over a two-dimensional array of m and L/r_0. An interpolation procedure is used to find a particular characteristic at any value of m and L/r_0 within the range of the arrays.

Comparison for a Particular Example

To compare the properties of these five geometries, it is helpful to take a particular example for an RFQ linac and to see how various characteristics depend on the geometry. The characteristics include the field enhancement factor, the minimum aperture, the modulation parameter, the intervane capacitance per unit length, and the amplitudes of the various multipole components. For the particular example, we take the prototype design for FMIT, an 80-MHz RFQ that accelerates deuterons from 0.075 to 2.0 MeV.

TABLE I

CHARACTERISTICS OF VANE-TIP GEOMETRIES

Type	x_p	ρ_t	Symbol
1	TTP[a]	TTP[a]	□
2	TTP[a]	r_0	○
3	TTP[a]	0.75 r_0	Δ
4	sinusoidal	r_0	+
5	sinusoidal	0.75 r_0	x

[a]Two-term potential function.

The following procedure was used:

1. The RFQ design was based strictly on the ideal two-term potential function, resulting in a cell-by-cell specification of r_0 and A.
2. For each geometry type, the modulation parameter, m, was found at each cell that maintained the design value of A. Cell lengths and r_0 were not changed.
3. Using the tables from Ref. 4, the various characteristics were calculated at each cell.

The results for the five geometries are presented in Figs. 1, 2, and 3.

Figure 1 shows the field enhancement factor, the minimum aperture, the modulation parameter, and the intervane capacitance, all calculated at each cell and plotted against longitudinal distance along the RFQ. We define an enhancement factor, κ, for each cell by

$$\hat{E}_s = \kappa \, V/r_0 \ ,$$

where $\hat{E}_s$ is the cell's peak surface field. Note in Fig. 1 that κ can be reduced ~10% using $\rho_t = 0.75\ r_0$. Also, the capacitance between vane tips is relatively constant for the geometries having ρ_t proportional to r_0.

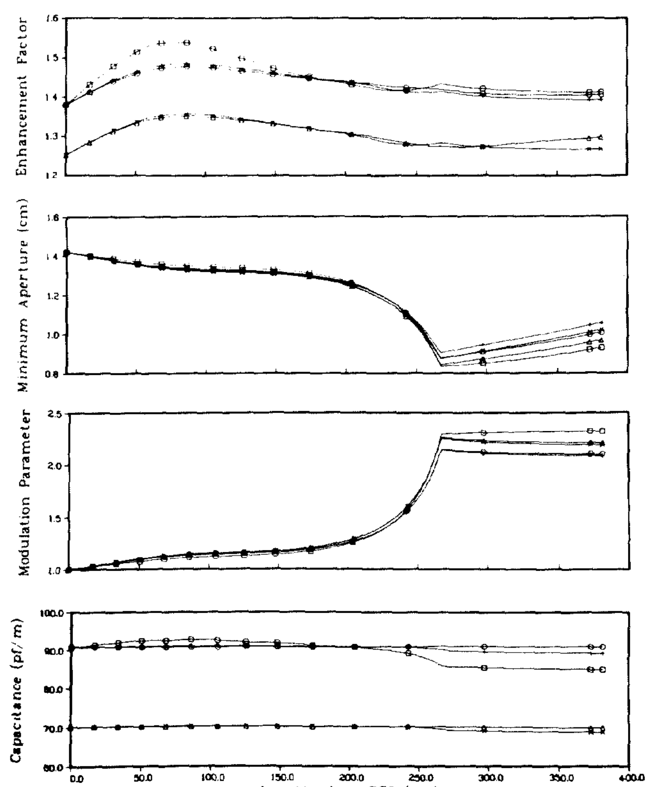

Fig. 1. Enhancement factor, minimum aperture, modulation parameter, and intervane capacitance plotted versus distance along the RFQ. Symbols are plotted at every 10th cell.

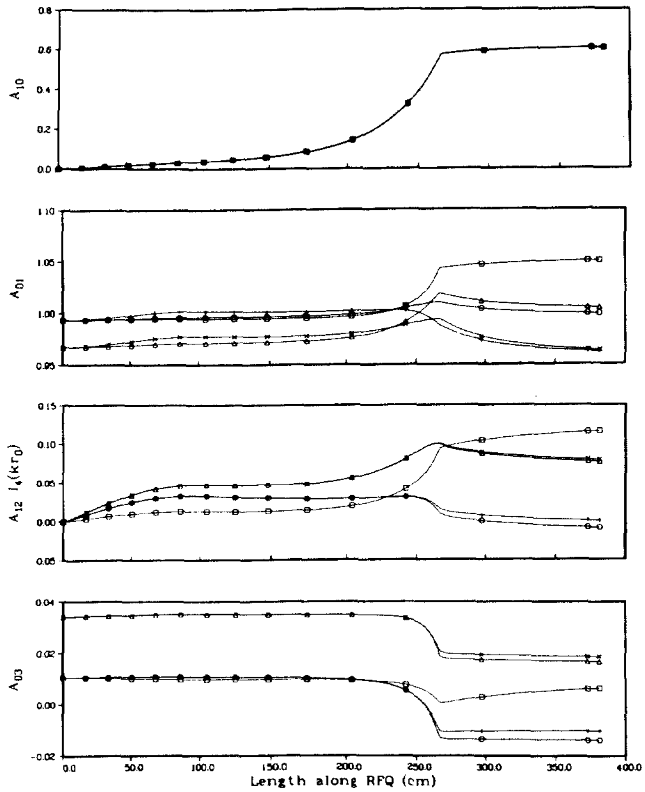

Fig. 2. Multipole coefficients versus distance along RFQ. Symbols are plotted at every 10th cell.

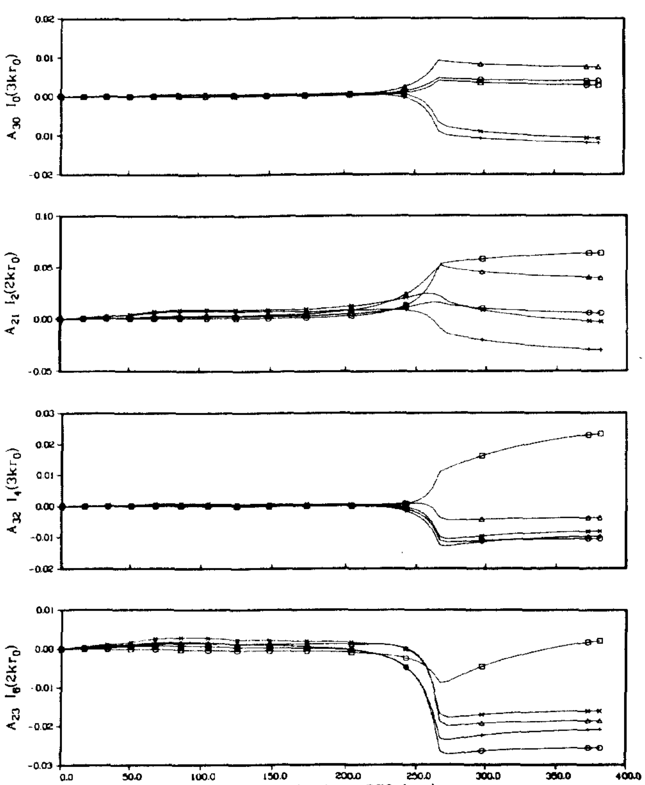

Fig. 3. Multipole coefficients versus distance along RFQ. Symbols are plotted at every 10th cell.

The complete multipole expansion for the potential is

$$U(r,\theta,z) = \frac{V}{2}\left[\sum_{m=1}^{\infty} A_{0m}\left(\frac{r}{r_0}\right)^{2m}\cos 2m\theta + \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} A_{nm} I_{2m}(nkr)\cos 2m\theta \cos nkz\right] . \quad (8)$$

In the first sum, the even m coefficients are zero because of symmetry; in the second sum, the coefficients are zero when m and n are both even or both odd. The eight lowest order, nonzero coefficients are shown in Figs. 2 and 3. Because of the procedure that was used, A_{10} is the same for all geometries. The quadrupole coefficient, A_{01}, deviates by a few per cent from its ideal value of unity. The quadrupole gradient could have been maintained at its design level by changing r_0 slightly to compensate for A_{01}.

Conclusions

We conclude that vane-tips having ρ_t proportional to r_0 can offer the advantages of a reduced peak surface electric field and an intervane capacitance independent of longitudinal position. The latter property should be beneficial in the tuning process. The higher order multipoles will not have serious effects for small beam radii. Our latest RFQ design for FMIT uses geometry Type 3 (Table I). According to a study[5] using a multiparticle program, the beam quality produced by this RFQ was unaffected by the higher order multipoles.

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