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AN EQUIVALENT CIRCUIT MODEL OF THE GENERAL 3-DIMENSIONAL RFQ

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Summary

An equivalent circuit model has been developed, based on the description of the RFQ as a terminated waveguide. The RFQ is modeled as a length of inductively loaded square waveguide operating in "TE_{10}- and TE_{11}-like" modes in the dipole and quadrupole configurations respectively. This is described by 6 coupled first order differential equations using distributed values for the four quadrant shunt admittances and series impedances. The end terminations are represented by lumped terminating reactances. The first few orders of both dipole and quadrupole modes are described by the model. By relating the equivalent circuit values to physical dimensions, the effects on frequency and field distributions of vane movement, tube distortion, individual end tuner movement and vane strapping were determined. Agreement with measured properties is reasonable, and some new insights on RFQ operation have been gained.

Introduction

An equivalent circuit model has been developed that can be used to predict the three-dimensional rf properties of the radiofrequency quadrupole (RFQ), including the effect of end tuners, vane straps[7], and dimensional variations. Longitudinal and azimuthal field variations can be estimated for both the dipole and quadrupole field configurations of the first few resonator modes.

The equivalent circuit description follows from the practice traditionally used for describing waveguides with discontinuities: a length of waveguide is represented by a distributed circuit, while a discontinuity is represented by a lumped reactance inserted at a specific location in the distributed line. For the RFQ, the long section of tube is the distributed waveguide portion, while the end regions are regarded as terminating discontinuities.

The derivation of the equivalent circuit is broken into two parts: (a) finding a distributed circuit description of the waveguide section and (b) finding a lumped equivalent of the end regions. Following this, sample calculations are compared to measurements and known general RFQ properties.

A Distributed Equivalent Circuit for the Transverse Electric Modes of the RFQ Waveguide

The derivation of the equivalent circuit for the RFQ waveguide is explained in detail in reference 1. It is obtained by considering solutions of Maxwell's equations for the TE_{10} rectangular waveguide mode[2], and then extending the model, by induction, to the TE_{11} rectangular mode (Figs. 1 (a) and (b)). Finally the TE_{11} waveguide is loaded inductively (Fig. 1(c)) to reduce the frequency and produce the usual RFQ configuration.

The shunt capacitance per unit length, C_{SH}, of the usual TE_{11} mode (Fig. 1(b)) is given approximately by the capacitance per unit length between adjacent walls of the rectangular waveguide. The associated series inductance is related to the shunt capacitance by

$$4L_{series}C_{SH} = c^{-2} \qquad (1)$$

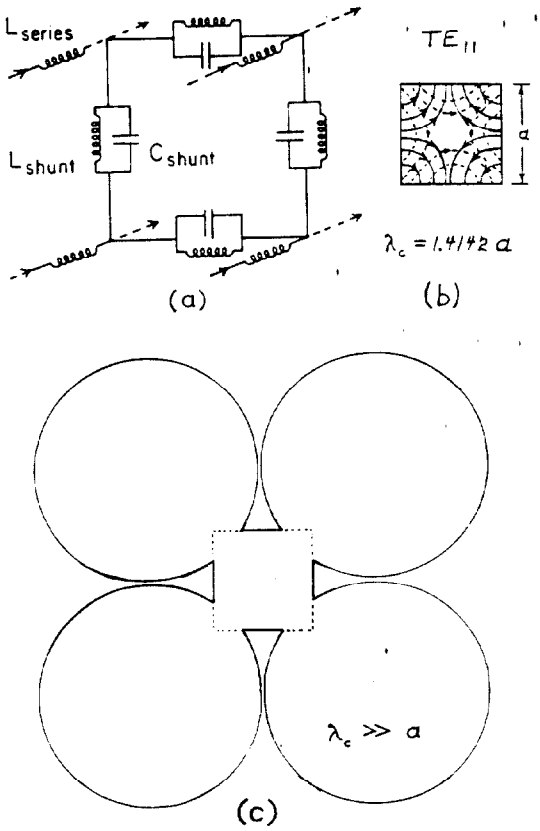

Fig. 1 (a) Distributed equivalent circuit for the TE_{11} rectangular waveguide mode,
(b) electric and magnetic field configurations for TE_{11} mode,
(c) inductively shunt loaded "TE_{11}-like" waveguide.

where c is the speed of light. To a good approximation, this relationship is independent of the shunt inductance.

The TE_{11} waveguide mode (Fig. 1(b)) has such a high cutoff frequency for waveguide dimensions equal to the gap spacing in an RFQ that it is not useful either in an RFQ device or as a model. By removing the inductive corners of the waveguide and replacing them with inductors of increased area (Fig. 1(c)), the shunt inductance is greatly increased and the cutoff frequency can be lowered to a practical value. In the process, the shunt capacitance will change a small amount, but the new series inductance will still be given approximately by equation 1. As the frequency drops, the value for C_{SH} becomes more accurate. Note that by removing the waveguide corners a four bar structure was formed, whose frequency could also have been lowered by adding lumped inductances at various points along the length, as is done with the Frankfurt RFQ[3]. Thus the standard four vane RFQ can be visualized as an inductively shunt loaded TE_{11} waveguide, whereas the four bar structure is a TEM transmission line with discreet lumped inductive loading.

The distributed equations for the standard four vane RFQ (Table 1) are derived from the equivalent circuit of Fig. 2, where opposite vane shunt reactance has been added for generality. The equations are equivalent to the telegrapher's equations for a loaded transmission line. Where there is no loading (i.e.,

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$$\frac{\partial i_1}{\partial z} = (-Y_1-Y_4-Y_5)\,V_1 + (-Y_4-Y_5)\,V_2 + (-Y_4)\,V_3$$

$$\frac{\partial i_2}{\partial z} = (Y_1)\,V_1 + (-Y_2-Y_6)\,V_2 + (-Y_6)\,V_3$$

$$\frac{\partial i_3}{\partial z} = (Y_5)\,V_1 + (Y_2+Y_5)\,V_2 + (-Y_3)\,V_3$$

$$\frac{\partial V_1}{\partial z} = (-Z_1)\,i_1 + (Z_2)\,i_2$$

$$\frac{\partial V_2}{\partial z} = (-Z_2)\,i_2 + Z_3\,i_3$$

$$\frac{\partial V_3}{\partial z} = (-Z_4)\,i_1 + (-Z_4)\,i_2 + (-Z_3-Z_4)\,i_3$$

$$V_4 = -V_1 - V_2 - V_3 \;;\quad i_4 = -i_1 - i_2 - i_3$$

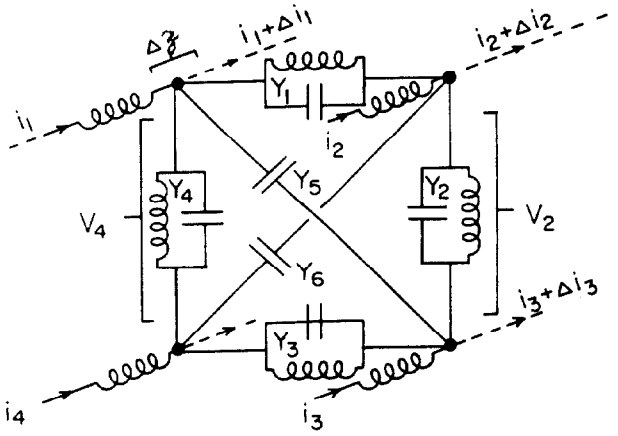

Fig. 2 Distributed equivalent circuit, including opposite vane capacitance. The system variables and constants are defined.

infinite shunt inductance), they describe the TEM mode operation. Note that the currents used in the equations are the longitudinal currents which flow axially, primarily along the vane tips. As there are only three independent voltages[1], there are only three transverse resonator modes.

The numerical values for the vane-to-adjacent-vane capacitance per unit length and the shunt inductance per unit length are easily derived from quasi-static formulae. The vane-to-opposite-vane capacitance is influenced by the shielding effect of adjacent vanes; a SUPERFISH[4] calculation of the dipole-quadrupole frequency separation is used to determine an approximate value.

A Lumped Equivalent Circuit Description of RFQ End Regions

For a standard RFQ end region configuration (Fig. 3), each tuner gap is modeled as a variable capacitor, and each path from the tuner around the end wall and back up to the vane tip, is modeled as an inductive loop. If the vane ends are cut back, the long "uniform" TE waveguide part of the RFQ is considered to end at approximately the mean position of the cutback. Thus the lumped equivalent circuit for the end region must include the vane-to-vane capacitance for the length between the mean cutback position and the end tuner.

Calculation of the numerical values of the end tuner gap and vane-to-vane capacitances is straightforward, with some uncertainty introduced by fringing fields - an error of 20% might be expected. In contrast, the calculated numerical value of the end loop inductance is uncertain by at least a factor of two, primarily because the geometry is non-standard. Thus a calibrating constant in the end loop inductance formula was determined by fitting a complete RFQ calculation to measured properties.

RFQ configurations in which a vane extends to the end plate are easily modeled by making the appropriate capacitance very large and inductance very small - thus shorting the vane tip to the end wall.

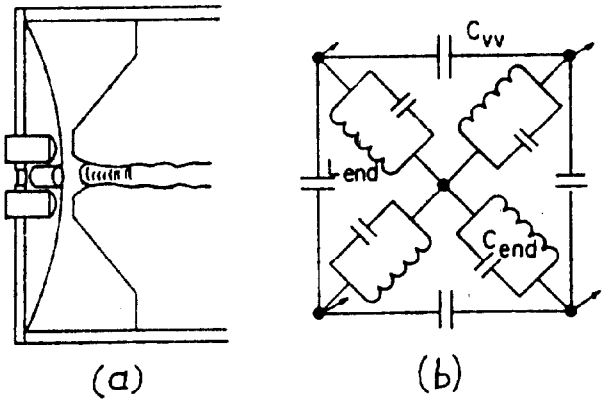

Fig. 3 (a) The end region of a standard four-vane RFQ, showing an end tuner and a cutback vane.
(b) The lumped equivalent circuit model of an RFQ end region.

Comparison with the RFQ Low Power Measurements of Nakanishi et al.[5]

The report of the Japanese measurements[5] contains an extensive set of data including longitudinal and azimuthal distributions of field for symmetric and displaced vanes, the effect of end tuner variations on both longitudinal and azimuthal distributions, higher order mode spacings, and quadrupole-dipole mode frequency differences. Comparison with these measurements constitutes a good test of the equivalent circuit model.

The model was found to reproduce the following basic RFQ properties demonstrated by the data.

(1) If, starting at a withdrawn position, all end tuners are gradually inserted, the longitudinal vane voltage distribution is first convex to the axis, then flat (at "cutoff") and then concave - the frequency progressively decreasing.
(2) Increasing end tuner capacitance on only one end causes the relative vane voltage to increase at that end.
(3) The dipole mode frequencies are lower than the quadrupole mode frequency.
(4) The spacing between the two lowest quadrupole modes was measured to be 33.0 ± 0.2 MHz and predicted to be 32 MHz.

The quadrupole-dipole frequency separation of 3.5 MHz was used to determine the equivalent circuit values for the end inductances and cutback calibration.

The Japanese report shows data (reproduced as lines on Fig. 4) for a case in which a single vane was moved transversely by 0.25 mm (Fig. 4(a)). The field imbalance was then corrected by inserting only one set of end tuners (Fig. 4(b)). The equivalent circuit model predictions are shown as individual points on Fig. 4; the agreement is good.

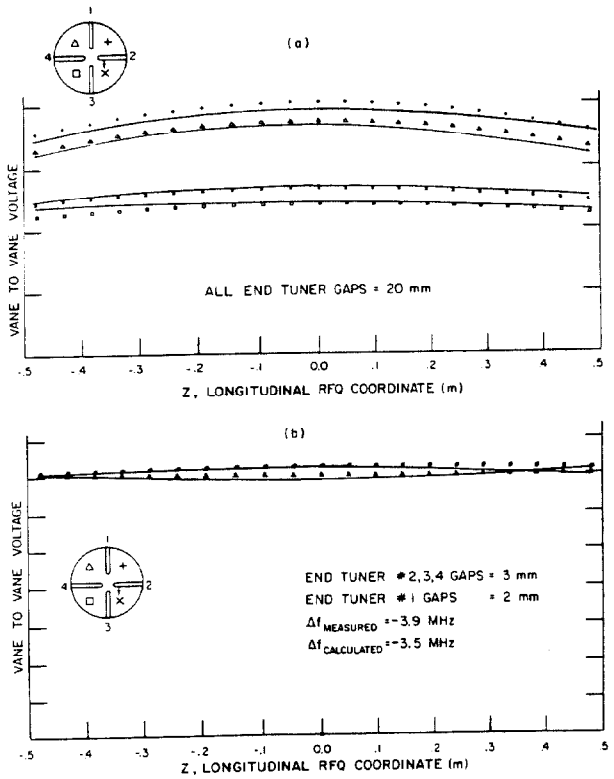

Fig. 4 Comparison of measured (solid lines) and calculated (symbols) vane-to-vane voltages for (a) a 0.25 mm transverse displacement of one vane, (b) end tuners inserted as shown, to balance the fields.

Sensitivity to a Systematic Linear Variation in Mean Bore Radius

J. Potter has suggested an L^2 dependence of the longitudinal voltage tilt on systematic longitudinal variations[6]. This is important for it suggests a practical limiting length (of an unstrapped RFQ) which is dictated by achievable machining and alignment tolerances.

The present model was applied to 270 MHz RFQ's of various lengths, with approximately 0.46 cm beam hole radius, 0.35 cm vane tip radius, and end regions adjusted to produce an essentially flat longitudinal field. A ± 0.5% linear longitudinal variation in each vane-to-vane distributed shunt capacitance was then introduced for each length, L, as given by the equation on Fig. 5. This is equivalent to a 0.075 mm (1.6%) change in mean bore radius along each length. The frequencies were essentially the same for all lengths. The dashed line on Fig. 5 shows a rapid increase of longitudinal field tilt with RFQ length, consistent with an L^2 dependence. This demonstrates how one may quantitatively determine a maximum practical RFQ length, given machining and assembly tolerances.

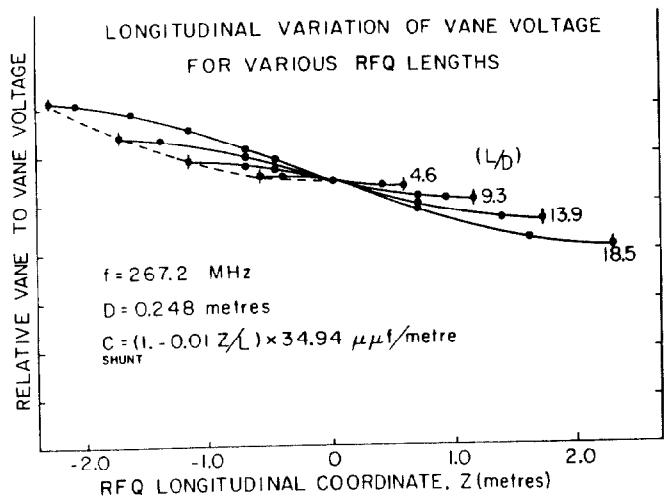

Fig. 5 The longitudinal variation in vane voltage (normalized at the centre) for different RFQ lengths, produced by a 1% longitudinal variation in vane capacitance.

Calculation of the Properties of Vane Strapping

Howard and Lancaster[7] have introduced vane strapping as a method of stabilizing the RFQ fields against mechanical errors. The straps consist of semi-circular metal hoops joining opposite vanes by looping through a hole in adjacent vanes.

The straps influence the fields in two ways:

(1) opposite vanes are connected to each other through a small inductance, so only a small voltage difference will exist between them, even with opposite vane imbalances.

(2) the strap introduces a significant increase in nearest neighbour vaneto-vane shunt capacitance.

The latter effect produces a very significant decrease in the RFQ frequency (∿ 1 MHz per ring pair), and, because of the local increase in capacitance, causes the vane voltage to peak at the strap. This effect can be compensated by introducing a local decrease in tube inductance. As an example of the stabilizing effect of compensated straps, a calculation was done for a 270 MHz, 1.6 metre long RFQ with a dielectric perturbation at the left end of one quadrant. The straps in the middle obviously stabilize the azimuthal field (Fig. 6) and appear to divide the system into two essentially independent (as far as tolerances are concerned) RFQ units.

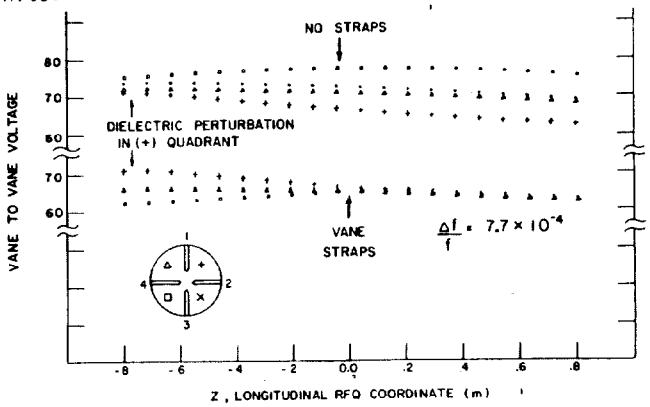

Fig. 6 The calculated vane voltage distributions caused by a small dielectric perturbation at the left end of #1 quadrant, without any straps (upper) and with a compensated strap in the middle (lower).

Conclusions

The ability of the present equivalent circuit model to predict basic RFQ properties and give approximate quantitative predictions of mode frequencies, tuning plunger properties, perturbed field distributions, and strap characteristics make it a very useful aid in understanding and designing RFQ structures.

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