Higher Order Mode Studies in Accelerator Waveguides*

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While a study of the higher order modes of propagation in periodic structures has, perhaps, its own inherent interest, the motive for the investigation reported here was avoidance of beam excitation and interaction with higher order modes of a waveguide supporting an intended accelerating mode and the lessening of wake field effects by using the largest mean diameter in the waveguide consistent with the operating frequency. All the experimental data reported here was taken at 2856 mcs (10.5 cm) but clearly may be scaled.

Disk-loaded Waveguide. Examination of the dispersion diagrams for widely used disk-loaded periodic structures (i.e., having 3, 4 or 5 disks per free space wavelength) reveals that the velocity of light line intersects the HEM-11 mode in its dominant phase range, and suggests that operation with between 2 and 3 disks per wavelength will avoid that condition. This proves to be so, but then it is of interest whether the structure is useful for acceleration. The properties of such waveguides are summarized in Table I for several illustrative cases with Brillouin diagrams shown in Figure 1.

It can be seen that the $3\pi/4$-mode, for example, is not substantially inferior to any other mode and that the possibility of higher order mode excitation is slight, although it cannot be supposed that any further difficulties in the future do not exist. The only obvious objection to such modes as the $3\pi/4$ (or the $4\pi/5$) is the somewhat high attenuation coefficient and the additional complication in tuning the waveguide (for the $3\pi/4$-mode three times around the Smith chart has four 'dwell points'), but as an alternate, the waveguide can be tuned in, say, the $\pi/2$-mode at the appropriate frequency.

The reader may well be aware that the properties of the widely-used $2\pi/3$-mode are only marginally better than the $\pi/2$-mode; the shunt impedance function has a broad maximum near three disks per wavelength. The principal reason that the $2\pi/3$-mode was chosen for the SLAC machine was because the decision represented a 25 pc saving on the nominally 114,000 disks required to construct it in the $\pi/2$ mode. Additionally, the risk in transferring to a different mode of acceleration had been obviated by the prior demonstration at ENS, Orsay of the $2\pi/3$ step gradient design, although the possibility had already been investigated experimentally at the Stanford Microwave Laboratory.(1)

Table I

Disk aperture, 2a = 2.286 cm (0.900 in.)
Disk thickness, t = 0.584 cm (0.230 in.)
Phase velocity, $v_p/c = 1$ (2856 mcs)
Number of disks per free space wavelength = n

n	2b	v_g/c	r/Q	Q*	$a_0^2/\Sigma a_n^2$**
5	8.354 cm	0.0158	43 Ω/cm	7,300	0.90
4	8.280	0.0145	43	10,000	0.89
3	8.255	0.0123	42	13,200	0.84
2 2/3	8.230	0.0111	45	14,600	0.75
2 1/2	8.217	0.0087	44	15,000	0.68
2	8.166	0	29***	17,000	0.86

* Q estimated from Fig. 2 of ref. 2; Q = 17,000 - 3200 (n-2) as an approximation.
**Fraction of power in the fundamental space harmonic, based on the five lowest order terms.
*** Because of thestanding wave nature of the fields in this case the measured shunt impedance is reduced by a factor of two.

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Doubly Periodic Structures. The motive for operating an accelerating structure resonantly is, of course, maximum utilization of RF power; this technique is simpler than using RF feedback, particularly for super-conducting accelerators where the phase-shifter and power combiner would be in the cryostat.

If it is intended to operate a periodic structure resonantly in the longitudinal $\pi/2$ mode it is well-known that every other cavity is unexcited. This observation has led several investigators(3) to exploit the possibility of optimizing the characteristics of the excited set of cells and either removing the unexcited set of cells from the beam line (side-coupled structures) or at least to optimize the excited set at the expense of the unexcited set (in-line coupled). In the latter case, where optimization consists also of lengthening the excited set of cells, the term 'doubly periodic' is appropriate; the loading obstacles have two 'periodic lengths' but, of course, still have a basic, overall periodicity of one-half free space wavelength.

Previous studies generally have been restricted to beam-line (excited) cavities nearly $\lambda/2$ long, presumably on the basis of relativistic transit time considerations; when the gap field is constant the electronic voltage gain,

$$V = \frac{\int V\,dt}{\int dt} = \frac{\int_{t-g/2c}^{t+g/2c} V_o \sin\omega t\,dt}{\int_{t-g/2c}^{t+g/2c} dt} = \frac{\sin\frac{\pi g}{\lambda}}{\frac{\pi g}{\lambda}} V_o \sin\omega t$$

where ωt is the phase at the center of the gap. This expression is maximized when g = 0 (and t = $\pi/2$), but that is useless. What is wanted is the highest value of sin X/X consistent with the largest gap voltage achievable; it is well-known that the highest energy gain in the simple TM-010 mode is gotten for a cavity of diameter/length ratio of 7/4, corresponding nearly at the velocity of light to the $7\pi/8$ mode which is nearly the intuitive value g = $\lambda/2$, T = 2/3. Several cases were investigated to explore this possibility, with 2a = 0.900 in. , t = 0.230 in. (in which case the $7\pi/8$ mode is not realizable);

MODES	r/Q	T	Q
$\pi/2 - \pi/2$	16 ohms/cm	0.84	10,000
$2\pi/3 - \pi/3$	24	0.83	13,000
$3\pi/4 - \pi/4$	29	0.76	14,600
$4\pi/5 - \pi/5$	31	0.76	15,000

The mode designation is acceleration-coupler cavity. The $2\pi/3 - \pi/3$ cavity case is shown in Fig. 2 with its dispersion diagram in Fig. 3; Q is estimated from Ref. 2. It has been noted frequently that machining tolerances are greatly relaxed in these so-called resonantly coupled (or doubly periodic) structures; this statement obviously cannot be applicable to the excited set of cavities, which observation is confirmed by the experience of J. McKeown and S. Hodge for the CRNL linac. (4)

Considering the precision with which the above measurements can be performed, the cases of $3\pi/4$ or $4\pi/5$ appear worth further investigation because the simple design and large aperture, particularly for beam energy recovery systems.

Large Aperture Studies. In their derivation of the wake field effect R. Cooper et.al.,(5) Fourier-analyze the wave-

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guide boundary (truncating the series with one or two terms) thereby representing the waveguide as a corrugated cylindrical surface of an average diameter. Insofar as that analysis is reasonable, which finds that transverse forces depend more or less inversely on the square of the average pipe radius, it would seem advisable for the purpose of avoiding wake field reaction on the beam to use the largest diameter beam aperture possible, consistent with the frequency of operation. This can be done at an unacceptable cost in lowered shunt impedance. But, observing that the series impedance (ratio of the fundamental space harmonic amplitude squared to the power flux)

$$Z_s = \frac{E_P^2}{P} = \frac{\omega}{V_g}\frac{r}{Q} \qquad (1)$$

from which, if the group (energy) velocity can be adjusted independently of the r/Q (which it clearly cannot), presumably a compensation can be found. What suggests itself is off-axis holes to provide magnetic coupling, cancelling some of the electric coupling owing to the larger beam hole, thereby lowering the energy velocity.[6] While it is known that this scheme is not expedient[7] what is wanted is a quantitative estimate. For this purpose a special case was examined; the disk-loaded waveguide ($2\pi/3$-mode traveling wave) with aperture 2a = 1.200 in having four additional coupling holes of 0.750 in diameter on a 2.470 in. BC (location of B_φ maximum) so as to reduce the group velocity. The experimental results are:

2a	r/Q	v_g/c	Q
1.200 in.	36 Ω/cm	0.034	14,000
1.200*	33	0.022	(13,500)
1.050	39	0.022	13,500

*Disks with off-axis holes; Q estimated from Ref. 2.

For comparison, the case of 2a = 1.200 in. is shown without such holes and the case 2a = 1.050 in., where the group velocity is the same as in the case with holes. Such a cost in shunt impedance may be acceptable.

Because the degree of magnetic coupling depends on the location, number, size and shape of the holes, which constitute too many variables to examine systematically, only round holes were investigated but holes with higher polarizability (kidney-shaped) and the least number of holes would be advantageous.

It is also obvious that aperture enlargement could be effected and the group velocity controlled by means of the disk thickness but, even accepting the loss of shunt impedance, the scheme is ultimately self-defeating because the larger diameter aperture would exist over a greater length.

Energy gain and optimization. The microwave voltage developed across a resonant waveguide section can be calculated as if the guide were a singly resonant circuit, the internal periodic structure then being a means of providing an optimum transit time factor. The available incident power (P_i) will result in an input power to the structure (accelerating cavities),

$$P_o = P_i \frac{4\beta'}{(1+\beta')^2} \qquad (2)$$

where the beam loaded coupling coefficient β' is given by

$$\beta' = \frac{\beta V(1+\beta)}{V(1+\beta) + i\, r_o L} \qquad (3)$$

in which β is the open circuit coupling coefficient, r_o the open circuit or no-load shunt impedance per unit length and V is the beam energy gain in the structure. When the microwave voltage developed across the structure is diminished by the transit time factor (T) and the beam power (unavailable to sustain the cavity fields) is taken into account, the peak electronic voltage gain,

$$V^2 = \frac{2 r_o L T^2}{(1+\beta)} (P_o - iV) \qquad (4)$$

from which, including the foregoing remarks, it follows that the electronic energy gain[8]

$$V = \sqrt{\frac{8\beta}{(1+\beta)^3} P_i r_o L T^2} - \frac{\sqrt{2}}{(1+\beta)^2}\, i r_o L T \qquad (5)$$

It was formerly held that traveling-wave operation was more efficient than resonant operation for the reason that in resonant operation half the power was traveling in an opposite direction to the beam and therefore was useless for acceleration. Moreover, it is pointless to operate resonant waveguide with misadjusted coupling, so that if the beam power conversion efficiency ($\eta = iV/P_i$) is maximized with respect to the coupling coefficient, the condition is

$$\frac{i^2 r_o L}{P_i} = \frac{(1+\beta)(2\beta-1)^2}{4\beta} \qquad (6)$$

Then the beam loaded energy gain,

$$\frac{V}{\sqrt{P_i r_o L T^2}} = \sqrt{\frac{8\beta}{(1+\beta)^3}} - \frac{(2\beta-1)}{\sqrt{2\beta(1+\beta)^3}} \qquad (7)$$

Maximizing the energy gain with respect to the coupling coefficient results in precisely the same condition, Eq. (6), shown in Fig. 4, from which it can be seen that similar performance specifications can be obtained as in traveling-wave operation.

References

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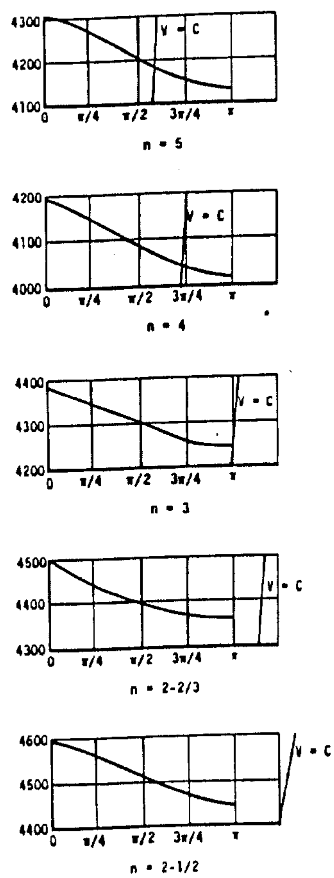

FIG. 1

BRILLOUIN DIAGRAM FOR VARIOUS DISC LOADING

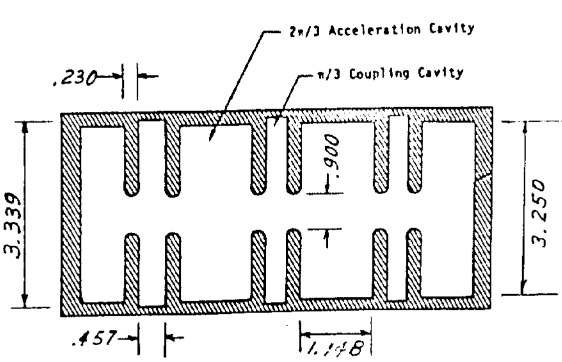

FIG. 2. 2π/3 DOUBLY PERIODIC GUIDE.

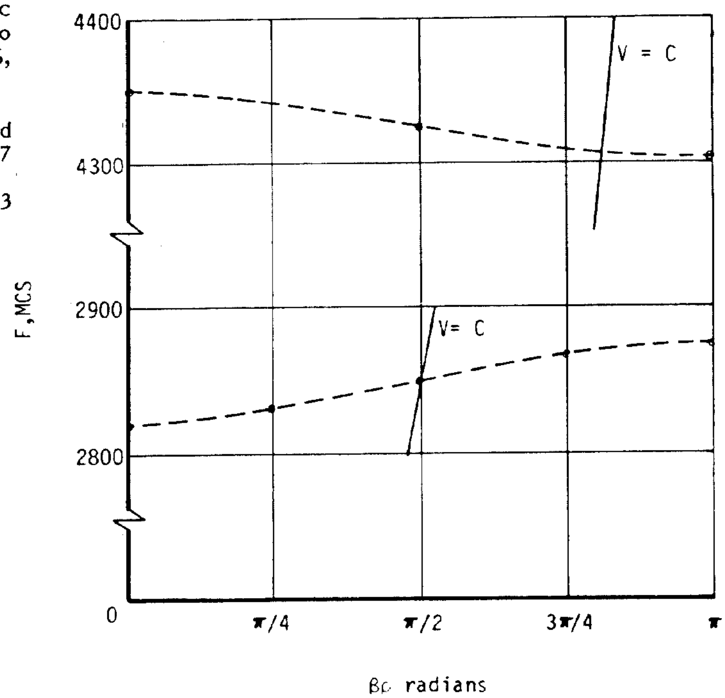

FIG. 3 2 π/3 DOUBLY PERIODIC; DISCRETE RESONANCES OF HEM-11 MODE (2 CAVITIES)

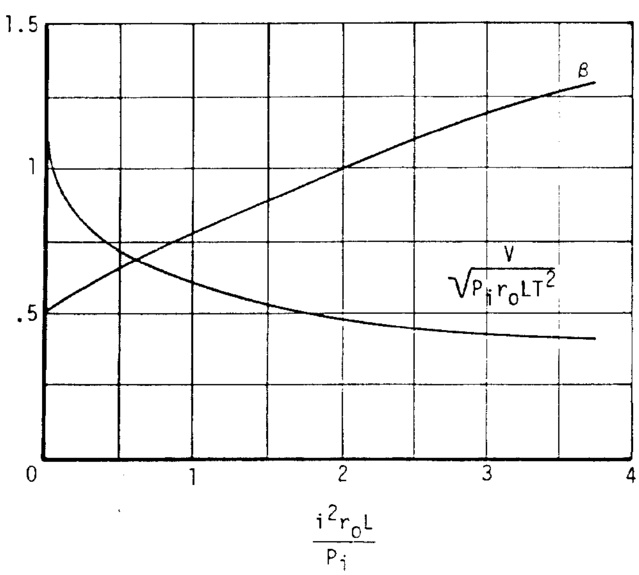

FIG. 4 GRAPH OF EQS. (6) AND (7).