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A PLASMA WAVE ACCELERATOR - SURFATRON I

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The limitation on the total energy gain possible with recent plasma accelerator schemes such as the beat-wave accelerator of Tajima and Dawson is overcome by the Surfatron. By introducing a perpendicular magnetic field it is possible to keep particles in phase with the laser-induced plasma waves and hence accelerate them to arbitrarily high energy.

Recently there has been a great deal of interest in using laser-plasma interactions to accelerate particles to high energies more rapidly than the 20 MeV/m to which linear accelerators are currently limited.[1] The beat-wave accelerator is one scheme proposed by Dawson and Tajima[2] to excite large amplitude electrostatic plasma waves which can accelerate particles. Whereas particles in the beat-wave accelerator can gain only a finite amount of energy before they become out of phase with the beat wave, by introducing a perpendicular magnetic field the particles are deflected across the wave front thereby preventing them from outrunning the wave. The particles may be accelerated to arbitrarily high energy as they ride across the wave fronts like surfers cutting across the face of an ocean wave (see Fig. 1).

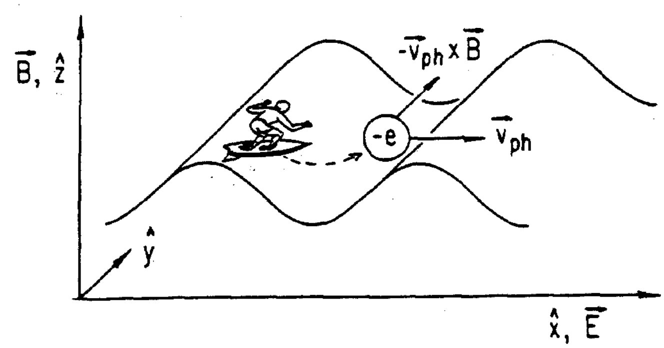

Fig. 1 An electron trapped by a potential trough moving at V_{ph} sees an electric field $\gamma_{ph}V_{ph}xB/c$ which accelerates it across the wave front.

Sugihara and Midzuno[3] and Dawson et al.[4] have shown that classical particles trapped by a perpendicularly propagating electrostatic wave are accelerated until they de-trap near the E x B velocity (cE/B). In this letter we consider the relativistic effects introduced when the E x B velocity is greater than the speed of light (i.e., E > B) and when the wave's phase velocity is not small compared to c.

We begin by giving a general treatment of the trapped particle motion analytically and numerically, followed by application of these results to the beat wave example. We consider a plane wave electric field and uniform magnetic field

$$\vec{E} = E_o \sin(kx - \omega t)\,\hat{x}$$

$$\vec{B} = B\hat{z}.$$

The equations of motion for a particle of charge q and rest mass m are given by

$$\frac{d}{dt}(\gamma V_x) = \frac{qE_o}{m}\sin(kx - \omega t) + \omega_c V_y \qquad (1)$$

$$\frac{d}{dt}(\gamma V_y) = -\omega_c V_x \qquad (2)$$

$$\gamma = (1 - V_x^2/c^2 - V_y^2/c^2)^{-1/2} \qquad (3)$$

where ω_c is the non-relativistic cyclotron frequency qB/mc and V_x and V_y are velocities in the x and y direction respectively. To solve for the particle's motion we assume it is trapped by the wave. The criterion for the particle to be trapped can be obtained by examining the x component of the force on the particle in the wave frame:

$$F_x = q(E_o + \gamma_{ph}V_yB/c)$$

where $\gamma_{ph} = (1 - V_{ph}^2/c^2)^{-1/2}$, $V_{ph} = \omega/k$. The first term of the Lorentz force is the trapping term and the second is the gyratory or de-trapping term. Therefore, an initially trapped particle can never de-trap if

$$\gamma_{ph}B < E_o \qquad (4)$$

For the zeroth order motion we assume that (4) is satisfied so that we may take $V_x = V_{ph}$. Integrating equation (2) and substituting from (3) gives

$$V_y = \frac{-\omega_c V_{ph} t}{\gamma_{ph}(1 + \omega_c^2 t^2 V_{ph}^2/c^2)^{1/2}} \qquad (5)$$

for the acceleration across the wavefront. Figures 2 and 3 show the velocity space trajectories obtained numerically for negatively charged particles trapped in low and high phase velocity waves, respectively. In both cases, the particles' total velocity asymptotes to the speed of light circle as predicted by equation (5).

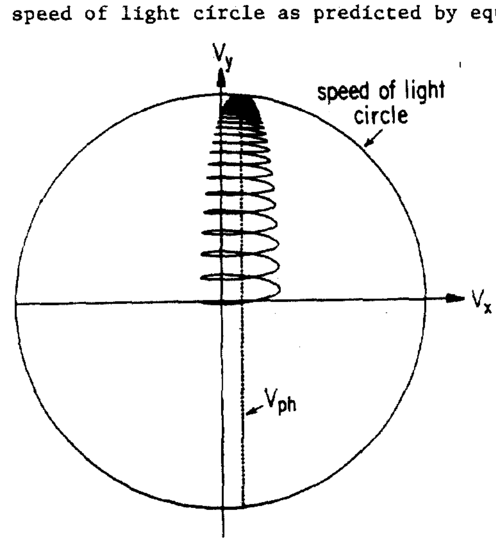

Fig. 2 Velocity-space trajectory of a particle in a low phase velocity wave (V_{ph} = .1 c, $E_o/B = 1.5$, $\omega/\omega_c = 2$).

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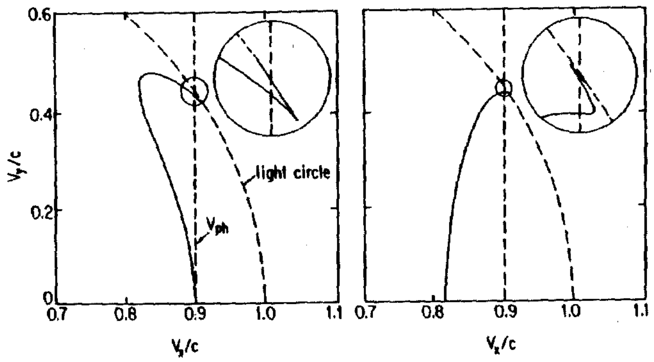

Fig. 3 Velocity-space trajectories of particles in a high phase velocity wave (V_{ph} = .9 c, E_o/B = 2.5, ω/ω_c = 9) for initial velocities (a) equal and (b) slightly below the phase velocity.

The higher order motion observed in Figures 2 and 3 can be represented by the first order expression for equation (1):

$$\ddot{x}_1 + \frac{\omega_B^2}{\gamma} x_1 \simeq -\left(\frac{\gamma_{ph}^2}{\gamma^2}\right) \omega_c^2 V_{ph} t \qquad (6)$$

where $x_1 = x - V_{ph}t$ and $\omega_B = (eE_o k/m)^{1/2}$ is the non-relativistic bounce frequency. This driven oscillator equation describes the bounce motion of a particle in the potential trough of the wave and its shift out of the bottom of the potential well due to the relativistic mass increase and the $V_y \times B$ force. From the decreasing bounce frequency and adiabatic invariance of the x motion we obtain the following expression for the bounce amplitude in velocity space:

$$\Delta V = \Delta V_o (1 - V_y^2/c^2)^{3/8} \left[1 - \frac{1}{8}\left(\frac{\gamma_{ph}^2 V_y B}{cE_o}\right)^2\right]$$

where V_o is the initial velocity bounce amplitude and $(\gamma_{ph}^2 V_y B/cE_o)^2 \ll 1$. This accurately describes the bounce amplitude observed in Fig. 2. In the high phase velocity examples of Fig. 3 the acceleration is so rapid that only after the particles have neared their asymptotic values does a slow bounce motion appear. However, an initial velocity shift is visible in Fig. 3(a) as the particle falls behind the wave due to its relativistic mass increase and can be shown from equation (6) to be roughly

$$V_{x1} \simeq \frac{-V_{ph}^2/c}{(\omega/\omega_c)\ (E_o/\gamma_{ph}B)} .$$

For the parameters of Fig. 3(a) V_{x1} = -.08c in agreement with the figure. For particles which start out slightly slower than the wave the acceleration is more monotonic as shown by Fig. 3(b).

The total energy gained by the particles as a function of distance traversed across the wave front can be found by integrating (5) and eliminating t in favor of y in the expression (3) for γ. Thus,

$$\gamma(y) = \gamma_{ph}^2 y \omega_c V_{ph}/c^2 + \gamma_{ph} \qquad (7a)$$

Alternatively, in terms of distance in the direction of the wave

$$\gamma(x) = \gamma_{ph}(1 + \omega_c^2 x^2/c^2)^{1/2} \qquad (7b)$$

These are plotted in Fig. 4 along with the numerical results corresponding to the particle of Fig. 3(a). The total velocity asymptotes to c while the energy continues to increase indefinitely. It is clear from equation (7) that a high phase velocity wave is advantageous for rapidly accelerating particles in addition to minimizing the damping of the trapping wave by the thermal plasma.

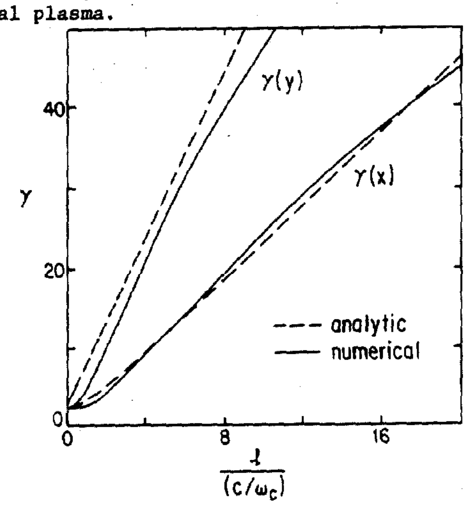

Fig. 4 Total particle energy (γmc^2) versus distance travelled (ℓ) in the direction of the wave (x) or across the wave front (y).

We now apply our acceleration results to the example of the fast electrostatic (upper hybrid) wave which may be created by the beat wave technique[2] or by forward Raman scattering of a single incident laser.[5] In this case, the phase velocity of the electrostatic wave is the group velocity of the incident wave; namely,

$$V_{ph} = c(1 - \omega_p^2/\omega_o^2)^{1/2} \simeq \omega_{UH}/k \simeq \omega_p/k \qquad (8)$$

$$\gamma_{ph} = \omega_o/\omega_p \qquad (9)$$

where ω_o is the angular frequency of the incident laser and ω_p is the plasma frequency ($\omega_p^2 = 4\pi n_o e^2/m_e$, e is the electronic charge, $\omega_{UH}^2 = \omega_p^2 + \omega_{ce}^2$, n_o is the plasma density, and the subscript e denotes electron quantities). Since V_{ph} is nearly c, a trapped particle's velocity is primarily in the direction of the wave while its acceleration is primarily perpendicular to the wave. This has the advantage that the power radiated will be less by a factor of γ^2 than that of a linear device for the same acceleration.

The trapping condition (4) can be written in the form

$$\omega_p/\omega_{ce} > \gamma_{ph} \tag{10}$$

where we have taken E_o to be the cold wavebreaking imit[6] $(4\pi e n_o/k)$ since V_{ph} is much greater than the thermal velocity. Inequality (10) justifies approximating the frequency of the upper hybrid wave as ω_p in equation (8). Substituting from (9) for γ_{ph} we may put the trapping condition in the form of a handy formula:

$$\frac{B_{kG}}{n_{16}\,\lambda_\mu} < 1 \tag{11}$$

where B_{kG} is the magnetic field in units of kilogauss, n_{16} is plasma density in $10^{16}/cm^3$ and λ_μ is the wavelength of the incident laser in microns. Thus, the magnetic field must be fairly modest for typical laboratory parameters or the particles will not be trapped.

Finally, substituting equations (8) and (9) in (7) yields for the change in γ per unit distance

$$\frac{\Delta\gamma}{\Delta y} = \frac{\omega_o^2\omega_c}{\omega_p^2 c}$$

$$\frac{\Delta\gamma}{\Delta x} \simeq \frac{\omega_o\omega_c}{\omega_p c}$$

where the latter expression is valid for $\omega_c t >> 1$. These provide the following handy formulae for the rate of energy gain of either electrons or protons:

$$\frac{\Delta U}{\Delta y} = 30\text{GeV/cm}\ \left(\frac{B_{KG}}{n_{16}\,\lambda_\mu}\right)\frac{1}{\lambda_\mu}$$

$$\frac{\Delta U}{\Delta x} = .1\text{GeV/cm}\ \left(\frac{B_{KG}}{n_{16}\,\lambda_\mu}\right)\sqrt{n_{16}}$$

Together equations (11) and (12) summarize the rate at which particles can gain energy. Equation (12a) suggests that a short wavelength laser is desirable to minimize the width of the plane wave front needed, while equation (12b) shows that a high density plasma is desirable to minimize the overall length requirement of the device.

The Surfatron accelerator appears to have great promise for parameters within the realm of current technology. For example, with a .3 micron laser, a plasma density of $10^{19}/cm^3$ and 300 kG magnetic field, 1 TeV electrons or protons might be produced in a device only 10 cm wide and 4 meters long. Theoretically, arbitrarily higher energies can be reached by merely extending the device. In contrast, the beat-wave accelerator without a magnetic field would require 100 meters to reach 1 TeV energies with the .3 micron laser, and acceleration rates decrease when the total energy requirement is raised.

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