IEEE Transactions on Nuclear Science, Vol. NS-30, No. 4, August 1983

RELATIVISTIC COLLECTIVE EFFECT ACCELERATOR TO REACH ULTRAHIGH ENERGIES

F. Winterberg
Desert Research Institute
University of Nevada System
Reno, Nevada 89507

Summary

A collective circular particle accelerator is proposed, with the potential to reach ultrahigh particle energies, including large luminosities. In the accelerator an electron cloud is produced within a large toroidal magnetic solenoid by inductive charge injection through the action of a travelling magnetic wave running around the torus. At the same time, the travelling magnetic wave accelerates the electron cloud to relativistic energies. By continuous inductive charge injection the cloud can be relativistically densified and the thusly densified cloud becomes the source of ultrastrong electric and magnetic fields, many times stronger than is possible using only externally applied fields. These large fields can then be used to keep ions in a circular orbit, and an accelerator with a circumference of 10 km could accelerate protons up to 10^3 TeV with luminosities up to 10^{33} $cm^{-2}sec^{-1}$.

Description of the Accelerator Concept

The principle of the idea is explained in Fig. 1.

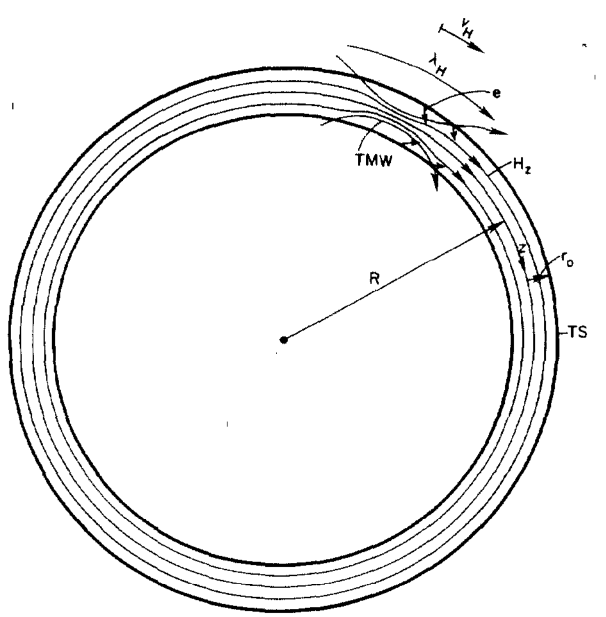

Fig. 1. Axial cross through accelerator. TS toroidal magnetic solenoid of major radius R and minor radius r_0; H_z solenoidal field along circular solenoid axis z; TMW travelling magnetic wave of width λ_H and moving along the toroidal ring axis with the velocity v_H; e electron injected into torus by inductive charge injection.

It shows a large toroidal magnetic solenoid, with the inside serving as the accelerator chamber. Its circumference can be many km long, depending on the particle energy which shall be reached. The magnetic field of this solenoid consists of a constant part, and a time dependent part of the travelling magnetic wave running around the circular axis of the solenoid. The constant magnetic field most simply can be produced by ordinary electromagnets, whereas the field of the travelling magnetic wave would have to be produced by the programmed magnetization of additional low inductance field coils. Furthermore, thermionic emitters are positioned along the entire length of the inner wall of the toroidal chamber. Then, as the rising magnetic field of the travelling wave passes by these emitters, electrons are first inductively injected into the torus, and thereafter accelerated by the magnetic mirror force of the wave. Because the thusly injected electron cloud acquires a large forward velocity along the circular torus axis, it becomes the source of a large selfmagnetic field. As a result of this selfmagnetic field, the electrons are pushed closer to the center of the toroidal chamber than without this selfmagnetic field, thereby making place for more electrons to be inductively injected at the periphery of the chamber by the rising magnetic field of the travelling wave. Since the travelling wave can in principle run around the torus many times, the electron cloud, not only can reach high energies, but at the same time can be also relativistically densified. This continuous inductive charge injection process is only limited by the synchrotron losses, which tend to decelerate the cloud. A limit is reached when the synchrotron losses exceed the energy gain from the travelling wave.

To accelerate ions to ultrahigh energies, they must be held in the orbit by action of the strong beam field, and which requires that the ions move in the opposite direction of the electrons. An additional axial electric field, which is needed to accelerate the ions does not decelerate the electrons because their charge is opposite to the charge of the ions.

Colliding beams could be realized, either by two interesting toruses or by replacing the torus with an intersecting figure eight configuration.

This concept has some similarity to an old concept by Budker[1], but differs from it that it uses a travelling magnetic wave to densify an electron beam in vacuum rather than in a plasma, a problem unsolved by Budker.

The concept has also some relation to a linear relativistic collective effect accelerator previously proposed[2,3,4].

Magnetic Confinement of the Electron Cloud

The electron cloud which is produced by inductive charge injection can be confined by an external magnetic field H_z, directed along the circular torus axis z, provided

$$E_r < H_z \quad , \quad (1)$$

where

$$E_r = 2\pi ner \quad , \quad (2)$$

is the electric field produced by the electron cloud and directed along the minor torus of radius $r = r_0$. In eq. (2) n is the electron number density and e the electron charge. Inequality (1) is the usual magnetic insulation condition.

If the electron cloud is set into motion with the velocity v along the circular torus axis z, it produces an azimuthal magnetic field given by ($\beta \equiv v/c$, c velocity of light)

$$H_\phi = 2\pi nerv/c = \beta E_r \quad . \quad (3)$$

With this selfmagnetic field, the magnetic insulation

0018-9499/83/0800-3238$01.00© 1983 IEEE

condition (1) becomes

$$E_r^2 < H_z^2 + H_\phi^2 \quad , \quad (4)$$

or by inserting eq. (3) into inequality (4) and putting $\gamma \equiv (1 - \beta^2)^{-1/2}$

$$E_r < \gamma H_z \quad . \quad (5)$$

In a more formal way this result can be also obtained by a Lorentz transformation between the frame S at rest with the accelerator, and the frame S' moving with the electron cloud. One there has the following relations

$$\left.\begin{array}{ll} n = \gamma n' & (a) \\ E_r = \gamma E_r' & (b) \\ E_z = E_z' & (c) \\ H_z = H_z' & (d) \\ H_\phi = \beta\gamma E_r' = \beta E_r & (e) \end{array}\right\} \quad . \quad (6)$$

In the comoving system S', where $H_\phi' = 0$, one has $E_r' < H_z' = H_z$, but in a system at rest rather $E_r < \gamma H_z$. For this reason the density of the electron cloud, as seen in the accelerator rest frame, can be γ times larger than would be possible for a cloud a rest and confined by the same external field H_z. The selffields of the cloud, that is E_r and H_ϕ, can therefore become about γ times larger than the confining magnetic field. As a consequence, ions held in orbit by these fields can acquire energies γ times larger than those which could be reached without using the relativistic magnetic insulation effect, expressed by inequality (5).

Prior to its being set into motion, the maximum electron number density n_0 in the electron cloud is given by

$$n_0 = E_r/2\pi er \simeq H_z/2\pi er \quad . \quad (7)$$

After being set into motion, the density in the comoving system remains the same. We therefore have to put $n' = n_0$, with the maximum density in the accelerator rest frame given by

$$n = \gamma n' = \gamma n_0 \quad . \quad (8)$$

Let us take the following example $H_z = 3 \times 10^4$ G, and $r = 1$ cm, for which eq. (7) gives $n_0 \simeq 10^{13}$ cm^{-3}. If $\gamma = 100$, the maximum electron density in the accelerator rest frame would be $n \simeq 10^{15}$ cm^{-3}, and for $\gamma = 10^3$ even the respectable $n \simeq 10^{16}$ cm^{-3}.

The moving cloud produces a current, which if expressed in Ampere is given by $I_z = 5rH_\phi$, and by using eq. (5) is $I_z \leq 5r\gamma H_z$. For the given example $I_z \leq 1.5 \times 10^7$ [A] if $\gamma = 100$, and $I_z \leq 1.5 \times 10^8$ [A] if $\gamma = 10^3$. The magnetic field at the cloud radius $r = 1$ cm is $H_\phi \lesssim \gamma H_z$ which for $\gamma = 100$ is $H_\phi \leq 3 \times 10^6$ G and for $\gamma = 10^3$, $H_\phi \lesssim 3 \times 10^7$ G. The radial electric field is $E_r \lesssim \gamma H_z$ and is for $\gamma = 100$, $E_r \lesssim 10^9$ V/cm and for $\gamma = 10^3$, $E_r \lesssim 10^{10}$ V/cm. Since the maximum electric field at the inner wall of the accelerator tube must be less than 10^8 V/cm, the limit for field ion emission, the wall radius must be larger than 10 cm for $\gamma = 100$ and larger than 1 meter for $\gamma = 10^3$.

To keep the cloud stable the condition

$$\omega_p < \omega_c \quad , \quad (9)$$

has to be satisfied, where $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ is the plasma frequency and $\omega_c = eH_z/mc$ the electron cyclotron frequency. For $n_0 = 10^{13}$ cm^{-3} and $H_z = 3 \times 10^4$ G one finds $\omega_p = 1.8 \times 10^{11}$ sec^{-1} and $\omega_c = 5.3 \times 10^{11}$ sec^{-1}. These numbers barely satisfy condition (9), but if necessary, it would be no problem to make ω_p still smaller than ω_c, simply by increasing the radius of the electron cloud.

Relativistic Densification of the Electron Cloud

After it is set into motion, an electron cloud of initial density $n = n_0$, can be relativistically densified, if either during its acceleration more electrons are added to it, or the cloud radius shrinks.

To analyse the inductive charge injection process for accelerated electrons clouds, we obtain from curl $\underline{E} = -(1/c)\partial\underline{H}/\partial t$,

$$E_\phi = -(r/2c)\dot{H}_z \quad , \quad (10)$$

$$E_z = (r/2c)\dot{H}_\phi \quad . \quad (11)$$

These induced electrical fields lead to the radial drift motion

$$v_r = dr/dt = -r/2[(H_z\dot{H}_z + H_\phi\dot{H}_\phi)/(H_z^2 + H_\phi^2)] \quad . \quad (12)$$

or in integrated form:

$$r/r_0 = [H(0)/H]^{1/2} \quad , \quad (13)$$

where $H^2 = H_z^2 + H_\phi^2$ and $H(0)$ the initial magnetic field strength at the injection radius $r = r_0$.

The maximum field of the travelling wave maybe about twice as large than the constant field, and if the circumference of the torus is large compared to the length of the travelling wave, then $\overline{H_z} \simeq H_z(0)$. How short the travelling wave can be made depends on how fast one can turn on a large magnetic field. With the length λ_H defined as the width over which the wave field is strong, one has

$$\lambda_H = 2v_H\tau_H \simeq 2c\tau_H \quad , \quad (14)$$

where $v_H \simeq c$ is velocity of the travelling wave and τ_H the rise time for the magnetic field. In practice $\tau_H = 10^{-6}$ sec is possible and with some difficulty $\tau_H = 10^{-7}$ sec. One finds that 60 m $< \lambda_H <$ 600 m.

For $\gamma >> 1$, $H \simeq \gamma H_z$, and hence

$$r/r_0 \simeq [\gamma(0)/\gamma]^{1/2} \quad , \quad (15)$$

where $\gamma = \gamma(t)$.

The axial equation of motion for the electrons accelerated by the mirror force of the travelling magnetic wave is

$$m\,d/dt(\gamma v) = -\mu dH_z/dz \quad , \quad (16)$$

where $\mu = \gamma m v_\perp^2/2H_z$ is the orbital magnetic moment of the electron trajectory in the r-ϕ plane, and which is an adiabatic invariant. To obtain an upper value for $v_\perp$ and hence for μ, we equate the centrifugal and centripedal forces acting on an electron. For $eH_z r > mc^2$ one has $v_\perp \simeq c$, and hence $\gamma m v_\perp^2/r \simeq \gamma mc^2/r \simeq eH_z$, therefore $\mu \lesssim er/2$. For $\gamma >> 1$ we obtain from eq. (16)

$$mc\,d\gamma/dt \leq (er/2)dH_z/dz \quad , \quad (17)$$

and with the approximations $dH_z/dz \simeq -2H_z/\lambda_H$, and $d\gamma/dt \simeq c\,d\gamma/dz$, we find for the acceleration length z to reach γ:

$$z \gtrsim (mc^2/erH_z)\gamma\lambda_H \quad . \quad (18)$$

For $r = 1$ cm, $H_z \simeq 3 \times 10^4$ G, one has $erH_z \simeq 10^7$ eV, and hence $z \gtrsim 5 \times 10^{-2}\lambda_H\gamma$. To reach $\gamma \simeq 100$, with $\lambda_H = 100$ m gives $z \gtrsim 500$ m, and for $\gamma = 10^3$, $z \gtrsim 5$ km.

The accumulation of electric charge by inductive injection is determined by

$$\partial n/\partial t + (1/r)\partial/\partial r(rnv_r) = 0 \quad . \quad (19)$$

Because

$$v_r = dr/dt = -(r/2)(\dot{\gamma}/\gamma) \quad , \quad (20)$$

this is

$$\partial n/\partial t - (1/2r)(\dot{\gamma}/\gamma)\partial/\partial r(nr^2) = 0 \quad . \quad (21)$$

Eq. (21) has the general solution

$$n(r,t) = \gamma f(r^2\gamma) \quad , \quad (22)$$

where f is an arbitrary function. If $r^2\gamma$ = const., it follows that f = const., and hence

$$n(r,t) = \gamma(t)n_0(r) \quad , \quad (23)$$

that is a relativistically densified cloud.

Synchrotron Losses of the Electrons

The synchrotron losses of an electron moving in a circular orbit of radius R are given by $I = (2/3)e^2c\gamma^4/R^2$, and to obtain the total losses, this number has to be multiplied with the number of electrons given by $N_e = 2\pi R\pi r^2 n = 2\pi^2 Rr^2\gamma n_0 \simeq (\pi/e)Rr\gamma H_z$. The synchrotron radiation losses are therefore

$$P = IN_e = (2\pi ec/3)(r/R)\gamma^5 H_z \quad . \quad (24)$$

For the above given example, r = 1 cm, R = 1.6 km, (2πR = 10 km) $H_z = 3 \times 10^4$ G, one finds that $P \simeq 1.9\gamma^5$ erg/sec. For $\gamma = 10^2$ one has $P \simeq 2 \times 10^3$ Watt, but for $\gamma = 10^3$ already $P \simeq 2 \times 10^8$ Watt.

Acceleration of Ions in the Relativistically Densified Cloud

The electric and magnetic fields of the relativistically densified cloud are given by $E_r \simeq H_\phi \simeq \gamma H_z$, and the radial force acting on an ion of velocity v_i is

$$F_r = e(E_r - \beta_i H_\phi) \simeq e\gamma H_z(1 - \beta_i) \ , \ \beta_i = v_i/c \quad . \quad (25)$$

The ions move in a direction opposite to those of the electrons, and one has $\beta_i \simeq -1$, and therefore $F_r \simeq 2e\gamma H_z$. To keep the ions in orbit, the centrifugal force $F_c = \gamma_i Mc^2/R$, $(\gamma_i >> 1)$, must be balanced by F_r, and one finds $\gamma_i = (2e/Mc^2)\gamma H_z R$. The maximum attainable ion energy $\varepsilon_i \simeq \gamma_i Mc^2$ is therefore

$$\varepsilon_i \simeq 2e\gamma H_z R = 600 \ H_z R \quad [\mathrm{eV}] \quad . \quad (26)$$

Taking the example R = 1.6 km, $H_z = 3 \times 10^4$ G and $\gamma = 100$, one would have $\gamma_i/\gamma = 2 \times 10^3$. For $\gamma = 10^2$ one has $\varepsilon_i \simeq 3 \times 10^{14}$ [eV], and for $\gamma = 10^3$, even $\varepsilon_i \simeq 3 \times 10^{15}$ [eV]. For colliding beams the center of mass energy is twice as large. The acceleration of the ions in the guiding field of the electron cloud can be done by standard techniques.

Synchrotron Losses by the Ions and the Attainable Luminosity

The circulating electron ring is loaded with ions by the relative fraction $f_i = N_i/N_e << 1$. The synchrotron losses of the ions are again given by eq. (24), if γ is replaced by γ_i. The total losses by the ions are therefore related to the electron synchrotron losses P, by

$$P_i = (\gamma_i/\gamma)^4 f_i P \quad . \quad (27)$$

For $\gamma_i/\gamma = 2 \times 10^3$, and using the above given example, one finds $P_i \simeq 3 \times 10^6\gamma^5 f_i$ [Watt]. A practical upper limit for P_i is about 3×10^9 Watt, which would require that $f_i \sim 10^3/\gamma^5$. For $\gamma = 10^2$ one finds $f_i \sim 10^{-7}$, and for $\gamma = 10^3$ one finds $f_i \sim 10^{-12}$. The total number of ions are $N_i \simeq 6 \times 10^{14}$ for $\gamma = 10^2$, and $N_i \simeq 6 \times 10^{10}$ for $\gamma = 10^3$.

The luminosity L of colliding beams is

$$L = N_i^2 \nu/\pi r_b^2 \quad , \quad \nu = c/2\pi R \quad . \quad (28)$$

Optimistically, $r_b \simeq 10^{-4}$ cm may be possible. For R = 1.6 km one has $\nu = 3 \times 10^4$ sec^{-1}. Then, if $\gamma = 10^2$, with $N_i = 6 \times 10^{14}$, one has

$$L \simeq 4 \times 10^{33}/r_b^2 \ \ \mathrm{cm^{-2}sec^{-1}} \quad , \quad (\varepsilon_{CM} = 600 \ \mathrm{TeV}) \quad , \quad (29)$$

and for $\gamma = 10^3$, with $N_i \simeq 6 \times 10^{10}$,

$$L \simeq 4 \times 10^{25}/r_b^2 \ \ \mathrm{cm^{-2}sec^{-1}} \quad , \quad (\varepsilon_{CM} = 6000 \ \mathrm{TeV}) \quad , \quad (30)$$

In the first example, (ε_{CM} = 600 TeV), $L \simeq 10^{33}/\mathrm{cm^2 sec}$ can be easily reached by equating r_b with the radius $r \simeq 1$ cm of the electron beam. In the second example (ε_{CM} = 6000 TeV), $L \simeq 10^{33}$ cm^{-2}sec^{-1} could be only reached if in the collision region $r_b \simeq 10^{-4}$ cm.

Adding a Tenuous Plasma

If a tenuous plasma with a density $n_p = fn$, is added, a new situation arises, because the radial force on the beam electrons is now

$$F = e[(1 - f)E_r - \beta H_\phi] = [1/\gamma^2 - f]eE_r \quad . \quad (31)$$

For $\gamma >> 1$, the electric and magnetic fields are practically unchanged and the ions can be kept in their orbit with almost the same force. However, for $f > 1/\gamma^2$, the radial force becomes negative and the beam contracts. In case the beam contracts it can be also relativistically densified without the addition of new electrons, provided $r/r_0 = 1/\sqrt{\gamma}$, where r_0 is the initial and r the contracted beam radius.

The addition of a plasma leads to Coulomb collisions, by which the beam is heated. Per electron this heating rate is given by

$$dE/dt = 4\pi nf(e^4/mc)\ln\Lambda \quad , \quad (32)$$

where $\Lambda = r/r_e$, $r_e = e^2/mc^2$. Besides being heated, the beam is also cooled by a second kind of synchrotron losses resulting from electron oscillations transverse to the beam. These losses are given by

$$P_e = (2/3)(e^2\overline{\dot{v}_\perp^2}/c^3)\gamma^4 \quad , \quad (33)$$

where $\overline{\dot{v}_\perp^2}$ is computed from the radial equation of motion

$$\gamma m\ddot{r} = -feE_r = 2\pi ne^2 fr \quad . \quad (34)$$

One obtains $\overline{\dot{v}_\perp^2} = (1/3)(2\pi ne^2 f/\gamma m)^2 r^2$, and hence

$$P_e = (8\pi^2/9)(n^2e^6f^2r^2\gamma^2)/m^2c^3 \quad . \quad (35)$$

For the beam to contract $P_e > dE/dt$, which leads to

$$eE_r f\gamma^2 > (9\ln\Lambda)mc^2 \quad . \quad (36)$$

Putting $f = k/\gamma^2$, $k > 1$; and $E_r \simeq \gamma H_z$ this condition is:

$$r\gamma > (9\ln\Lambda)mc^2/keH_z \simeq 4.5 \times 10^5/kH_z \quad , \quad (37)$$

$$\text{or} \qquad \gamma \gtrsim 2 \times 10^{11} \ (kr_0H_z)^{-2} \quad . \quad (38)$$

For the above given example and k = 2, we find that the beam contracts if $\gamma > 50$. If for example $\gamma = 100$ the final beam radius would be $r = r_0/\sqrt{\gamma} = 0.1$ cm. This 10-fold focussing densifies the beam 100-fold. A tenfold reduction in the beam radius permits to reduce the radius of the accelerator tube also 10-fold, and still keeping the radial electric field below the limit for field ion emission.

The addition of a plasma leads to more synchrotron losses and to keep these losses down, one may wish that $P_e < I$, where $I = (2/3)e^2c\gamma^4/R^2$ is the orbital synchrotron loss rate. By a simple calculation one finds that $P_e < I$ implies

$$\gamma > 3^{-1/4}(eH_zRk/mc^2)^{1/2} \simeq 1.8 \times 10^{-2}(H_zRk)^{1/2} \quad . \quad (39)$$

For the above given example one finds that $\gamma > 1400$. Therefore, unless the γ-value of the electron cloud is large, a price in more power to drive the beam has to be paid.

References

1) H.J. Budker, CERN Symposium 1956, reprinted in "Collective Methods of Acceleration", ed. N. Rostoker and M. Reiser, Hardwood Academic Publishers, Chur, 1979, p.1. ff.
2) F. Winterberg, Phys. Rev. 25 619 (1982).
3) F. Winterberg, Physics Today 34 No.9, p.9 (1981).
4) F. Winterberg, Proceedings of the ECFA-RAL meeting "The Challenge of Ultrahigh Energies", Oxford Sept. 27 -30, 1982.
5) G.S. Janes, R.H. Levy, H.A. Bethe, B.T. Feld, Phys. Rev. 145 925 (1966).
6) N. Rostoker, W. Clark, P. Korn and A. Mondelli, Ann. N.Y. Acad. Sci. 251 568 (1975).