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# TRANSVERSE BEAM CONTAINMENT IN THE ANL 4-GEV MICROTRON\*

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#### Summary

Optical systems have been designed to contain the electrons during the acceleration from 0.185-4.0 GeV. These systems are located in the dispersive straight sections and maintain a matched dispersion free beam with  $\beta^* \approx 15.0$  m in the linac centers, and transverse beam waists in the centers of the dispersive straight sections. A thin-lens code has been developed to design the multi-energy system. Three versions of the focussing systems have been evolved: (i) Two quadrupole triplets for E < 1.62 GeV; (ii) A single triplet for 1.655 < E 2.215 GeV, and (iii) A pentaguad system for E  $\ge$ 2.250 GeV. For case (i) we step the exit edges for the 60° bending magnets so as to simulate a zero degree edge - this reduces vertical defocussing effects to an acceptable value. At the higher energies the exit edge angles are  $-60^{\circ}$ . The entrance angles are 15° on the linac sides of the dipoles. Energy behavior of the Twiss parameters and quadrupole strengths are presented.

## Introduction

The beam containment systems in the ANL sixsided microtron (hexatron) design must accommodate and transport electron beams to full energy with little loss in beam quality and intensity. The hexatron consists of three S-band linacs each separated by dispersive straight sections. The

hexatron accelerates the 0.185 GeV injected electrons to 4.0 GeV in 109 passes or 36-1/3 turns. A first order tracking procedure has been developed to simulate beam traversal through the system-this is justified for the predicted small emittance and energy spread inherent in such a machine.<sup>1</sup> The containment scheme we have developed involves solving the matched system for each 1/3 turn assuming constant energy; the energy is boosted 35 MeV between 1/3 turns. Figure 1 shows the hexatron layout for the 1/3 turn between linac centers (points A and E). The requirements are to maintain matched dispersion-free beam waists in the linac centers  $\beta_A^{\times} = \beta_E^{\times} = 15.0 \text{ m} - \text{this is effected}$  by incorporating a focussing sytem which is mirror symmetric about the point C. The active components assumed for the containment systems consist of two weak horizontal focussing quadrupoles ( $\rm Q_g$ ), two dipoles and the focussing system, all indicated in Fig. 1.

In Section II we discuss the focussing systems; optimization and study of the energy dependences of the beta and eta functions and quadrupole strengths are treated in Section III. Sensitivity to errors and aberrations are discussed in Section IV. Here we do not discuss injection and extraction from the hexatron; they are amply covered in Ref. 1.

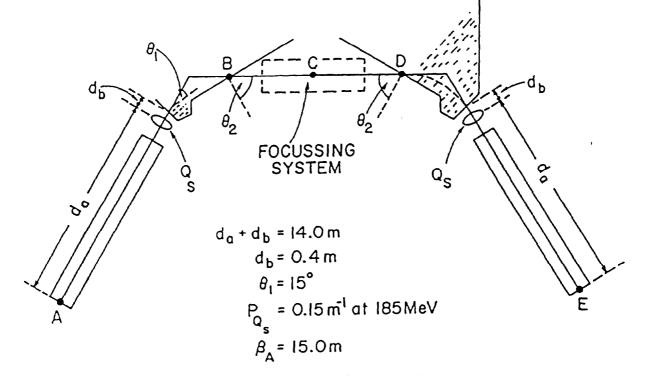


Fig. 1. Hexatron layout for one-third turn.

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## Focussing Systems

Focussing systems are evaluated by using a 2×2 matrix approach. Quadrupoles are treated as thin lenses. The dipoles are composed of edge-focussing thin lenses pre- and post multiplying the normal sector magnet transformation; they are assumed to bend the beams through  $\pi/3$  with an effective length  $\rho\pi/3$  where  $\rho$  is the radius of curvature. Real soft-edge fields modify the simple assumed behavior. We have studied several: (i) an Enge short-tail field<sup>2</sup> and (ii) a much sharper field behavior developed for the hexatron. The system was designed using the parametrization (i). The dipoles are taken with gap g = 0.06 m, and maximum field B = 1.015T. Thus the dipole edges correspond to thin lenses of power +tan  $\theta/\rho$  (def) in the radial (horizontal) plane. the vertical plane behavior was modified by the short-tail field behavior and can be represented by thin lenses of power  $-\tan\theta/\rho$  where the effective edge angles are represented by  $\theta = \theta - F_2 g/\rho$ . For a short-tail field we obtained  $F_2 = 0.4 + 0.577 \delta^2 + 0.29 \delta^4$ empirically from Enge's curves,<sup>2</sup> where  $\delta = \theta - 1.2*$ 0.414 \* g/p. We have also verified this parametrization using the program RAYTRACE.<sup>3</sup> We have chosen  $\theta_1 = +15^\circ$  at entrance; for the exit we use  $\theta_2 = 0^\circ$  for  $E \le 1.62$  GeV and  $\theta_2 = -60^\circ$  for E > 1.62 GeV. The stars at law entrance is the stars at law entrance is the stars at law entrance is the stars at law entrance. 1.62 GeV. The steps at low energy are required to reduce severe vertical defocussing.

The waist locations relative to the dipole exit (B) are given by, e.g.,  $\beta_B \alpha_B / (1 + \alpha_B^2)$  where the Twiss parameters  $\beta_B$  and  $\alpha_B$  are evaluated at point B. The following optical systems must transform the waists to point C (see Fig. 1); the distance  $d_{BC} = 14.015 - \rho * 0.866$ . Three quadrupole focussing systems were developed to achieve the desired conditions - Fig. 2 displays the thin-lens equivalents along with the dispersion ray trajectory. Focussing lenses are horizontal focussing (HF). Points B, C, and D correspond to those in Fig. 1. At points B and D we have  $n_{\rm R}$  =  $\rho/2$ ; the slope  $n'_B = \sin(\pi/3)$  for system I and  $n'_B = 0$  for systems II and III. At lower energies the horizontal waist occurs upstream of Ql  $(x_w < d_1)$  in system I; thus strong focussing is required to bring the beeam to another waist at C. We observe  $x_w$  increases with energy - the triplet system II can be used if  $d_2 < x_w < d_{BC}$  and if this waist can be shifted to point C. These conditions exist for 1.655 < E < 3.195 GeV; however, extraction dictates a maximum energy of 2.215 GeV for the triplet. The pentaquad system is used for E > 2.250 GeV; the solution to the quadrupole strengths are not unique, however. For  $2.25 \le E \le 3.545$  GeV we use a "weak" pentaquad system with P<sub>1</sub>  $\le 0.5$  m<sup>-1</sup> at all energies. A "strong" system III with P<sub>1</sub> > 1.1 m<sup>-1</sup> is used for E > 3.580 GeV.

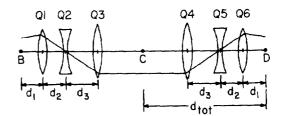
The x transfer matrix for the half-optical system is given by

$$M_{BC} = \begin{pmatrix} a & b \\ c & l \end{pmatrix}$$
(1)

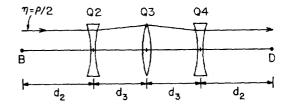
with c=0 and d=1/a for systems II and III. For system I, the dispersion ray is zero a distance s =  $n_B/n_B^t = \rho/(2 \sin \pi/3)$  upstream of point B at e.g., point B' (within the dipole). Then the transformation is point to parallel.

$$M_{B'C} = M_{BC} \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} e & f \\ \frac{1}{f} & 0 \end{pmatrix}$$
(2)

E ≤ 1620 MeV



II)  $1655 \le E \le 2215 \text{ MeV}$ 



Ⅲ) E ≥ 2250 MeV

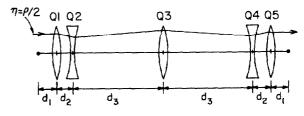


Fig. 2. Thin-lens representations for the focussing systems used in the hexatron for beam containment. The trace of the dispersion ray is indicated.

In the vertical plane the transformation is (1). The net x or y transfer matrix for the dispersed straight section is obtained by matrix multiplication of (1) and its mirror image

$$M_{BD} = \begin{pmatrix} ad+bc & 2bd \\ \\ M_{BD} & 2ac & ad+bd \end{pmatrix}$$
(3)

#### Optimization

Three conditions must be satisfied at point C:  $\alpha_x = \alpha_y = n' = 0$ . These requirements are met by solving a number of equations involving quadrupole strengths and locations. By thin-lens optics we specify some quadrupole strengths - for system I we have  $P_1 = (s+d_1)^{-1} + (d_2)^{-1}$  and  $P_3 = 1/d_2$ , where  $P_4$  is the focussing strength of the i<sup>th</sup> quadrupole. For system II the requirement of c = 0implies  $P_3 = 2P_2/(1+P_2d_3)$ . Similarly, we have  $P_3 = [2P_2(1-P_1d_2) - 2P_1]/[1-P_1(d_2+d_3) + P_2d_3(1-P_1d_2)]$ for system III. A horizontal waist occurs at point C for

$$P_{2} = \frac{x_{w}^{2} - 2d_{BC}}{d_{3}(d_{2} - x_{w})}$$
(4)

in system II and for

$${}^{d}_{3}{}^{P}_{2} \approx [UVP_{1} + 2\gamma_{B}{}^{d}_{BC} - \alpha_{B}] / [\alpha_{B} - \gamma_{B}(d_{1} + d_{2}) - d_{2}VP_{1}]$$
(5)

in system III where  $U = d_2+d_3$  and  $V = \alpha_B-\gamma_Bd_1$ . In Eq. (5) the Twiss parameters  $\alpha_B$  and  $\gamma_B$  represent the horizontal plane behavior at the dipole exit B. For system I, the treatment leads to a quadratic equation in P<sub>2</sub>; we choose the solution which results in the minimum value of P<sub>2</sub>.

A one-dimensional optimizer is used. For system I we minimize  $|\alpha_x|$ , and we minimize  $|\alpha_y|$  for systems II and III. The positions of Q3 and Q2 are the varied parameters for systems I and II, respectively. For system III we vary P1 for fixed lens positions. Figure 3 shows the obtained energy dependence of the quadrupole strengths in the focussing systems - the abscissa reads 1/3 turns corresponding to  $\Delta E = 35$  MeV. The discontinuities occur at the transition points between focussing systems. The betatron tunes  $v_x$  and  $v_y$  have been calculated also - we define  $v_1 = 3[\sqrt{3}/\beta_1]/2\pi$  where the detoeration is over 1/2 turn for advantage. the integration is over 1/3 turn. for a given focussing system the  $v_j$  are relatively constant with energy. The  $v_x$  are 5.3, 2.35, and 2.40, and the  $v_y$  span 1.15 - 1.30, 1.2 - 1.8, and 1.1 - 1.2 (2.5 - 2.6), for the systems I, II, and III weak (strong), respectively. A tracking code has been utilized to follow the evolution of the beta functions through the system. We started with  $\beta_A^{\pi}$  = 15.0 m in both transverse planes. The distributions are shown in Fig. 4 for three "fixed" points in 1/6 turn: the dipole end, Q2, and in Q3. At low energies the  $\beta_x$  start out large in Q3 > 120 m) but come down quickly. The  $\beta_v$  reach maximum values in Q2 of near 150 m near 1600 MeV.

## Sensitivity to Perturbations

The beam centroids are sensitive to magnet drifts and misalignments. This subject has been discussed in detail in the Argonne proposal and active damping systems have been designed for correction. Here we mention how the design beta functions respond to randomly generated errors in the quadrupole strength. We tracked the beta functions when the quadrupole strengths were allowed to deviate by up to  $\pm 0.2\%$  from their design values. No differences could be observed in the  $\beta_{\chi}$ behavior but the  $\beta_{\chi}$  did exhibit some changes, especially at the transition region between the focussing systems I and II. The motion was stable with beam containment observed, however.

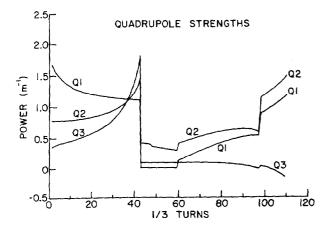


Fig. 3. Quadrupole powers obtained from optimization.

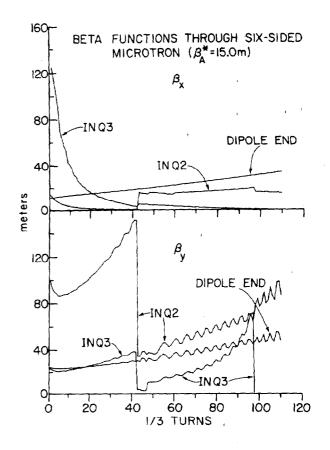


Fig. 4. Monoenergetic beta functions at the indicated locations.

The present hexatron design allows for transport of off-energy bunches with  $\Delta E/E$  values up to  $\pm 0.5\%$  at 0.22 GeV. It might be expected that the present system introduces chromatic aberrations. In fact, we have performed thick-lens studies to second order at 0.22 GeV with the program TRANSPORT.<sup>4</sup> We do observe a small mismatch in the betatron functions and a small shift by 0.2 mm in the linacs of the beam centroid. However, we note that strategic placement of five very weak sextupoles ensure no chromatic effects in the transverse plane amplitudes or centroids in the linac.

## Future Work

Pending approval of the hexatron project, a further tracking analysis should be performed which simulates both the longitudinal and transverse phase space behavior and considers the rf defocussing of the linac cavities.

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