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# INTENSE BEAM RECIRCULATION

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The concept of racetrack-modified betatron recirculator-accelerator is introduced in order to recirculate intense-current electron beams under the same accelerating cavities several times (< 10) before extraction. Design criteria are obtained and the parameter space of operation is given.

## I. Introduction

The acceleration of intense current (I > 10 kA) electron beams from low voltage (V ≈ 1-2 MV) to high voltage (20 MV ≥ V ≥ 10 MV) has been accomplished during the last two decades. In particular, the penetration, of the 10-MV voltage technical barrier that exists in pulsed power acceleration of intense electron beams, has been accomplished by utilizing distributed accelerator concepts and modules from the area of low-current high-voltage traditional linear accelerators. The linear induction accelerator (Astron, ERA, ETA, ATA), the radial pulse line accelerator (LIU-10, Radlac), and the RF linear accelerator (Fermex) are examples of the technology that is being developed in order to accelerate intense electron beams well beyond the 10-MV barrier of pulse power technology. In utilizing the linear accelerator concept of distributed acceleration, exceedingly long systems result. The simplest approach to reducing the length of the system is to fold the accelerator by introducing a 180° bend in the path of the beam. The next step is to introduce a second 180° bend and close the system into a racetrack geometry, where acceleration occurs at the linear sections of the geometry and recirculation is accomplished via the two 180° bends, or toroidal sections.

The acceleration of intense electron beams in linear geometries in the low voltage range (50 MV ≥ V ≥ 2 MV) requires strong focusing which is provided in practice by strong, uniform, axial magnetic fields (B > 5 kG). In order to confine and bend the beam in the two toroidal sectors the axial magnetic field must be extended in the toroidal sectors, thereby forming a closed, nested series of axial flux tubes around the racetrack. The curvature of the flux tube and the associated transverse gradients of the axial magnetic field at the two toroidal sections require the superposition of additional magnetic fields in order to balance and cancel the curvature-driven drift of the beam away from radial confinement. The simplest magnetic field that can accomplish this is a vertical magnetic field at the toroidal sections as in the circular betatron accelerator. This basic geometry then, constitutes the longitudinal flux tube racetrack beam accelerator and recirculator.[1] Before we analyze the concept in some detail we present a brief survey of related concepts and accelerators.

The earliest example of a racetrack recirculator-accelerator is the modified synchrotron.[2] Acceleration is driven by rf cavities in the linear sections as well as by the time variation of the vertical betatron magnetic field in the toroidal sections. The advantage of the racetrack-modified synchrotron geometry over the circular synchrotron geometry is that it can accelerate to higher voltages.[2] The stability of orbits for betatron and synchrotron oscillations were investigated and stable regimes were obtained as a function of the betatron field index $n_\beta$ and the ratio L/R, where L is the length of each straight section and R the radius of each circular section.

The NBS linear induction accelerator[3] has been operated in a recirculation mode. The intense current electron beam (I ≤ 1 kA) enclosed by a drift tube is recirculated together with the drift tube, twice under the same cavity. The experimental point for the NBS high-current induction recirculator[4] is shown in Fig. 1. The limited success of recirculation is due to unoptimized focusing, low initial beam voltage, and subsequent low inductive acceleration. The recirculation of the drift tube itself, several times, under the accelerating gap may limit the number of passes the beam can make and may substantially complicate the focusing requirements, in the absence of gas focusing, for large beam currents.

Another area where there has been generated, over the last decade, a substantial experimental data base on the dynamics of intense low-voltage (V ≤ 2.0 MV) electron beams in circular and racetrack toroidal geometries is the area of utilization of intense electron beams to heat Tokamak plasmas.[5,6] These experiments demanded use of gas focusing because of the large beam current (I > 10 kA), low voltage (V < 2 MV), low axial magnetic field (B ≤ 5 kG), and large minor diameter of the tube surrounding the beam. Related analytical studies showed[5] that beam confinement increased drastically by superimposing a betatron magnetic field.

More recently there has been intense interest and analytical development of the modified betatron concept.[7,8] This concept is based on the traditional betatron with a vertical magnetic field $B_v$. A strong axial magnetic field $B_a$ is superimposed so that $B_a \gg B_v$, thereby allowing the initial injection and confinement of much larger beam current. Acceleration is obtained through induction by raising the vertical betatron magnetic field.

The racetrack beam recirculator-accelerator, discussed in this paper, is simply related to the modified betatron accelerator magnetic field geometry, in the sense that the two toroidal sections are half modified betatron sections. The acceleration, however, is obtained through radial pulse lines in the straight

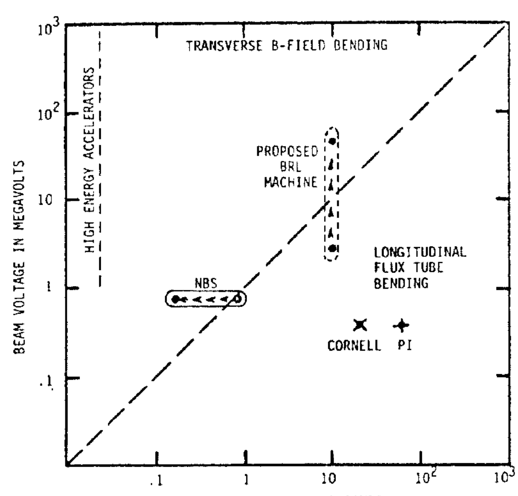


Figure 1. The available high-current beam recirculation data base.

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sections of the racetrack and the beam, extending axially over less than half the axial length of the geometry, will be recirculated, before extraction, only a very small number of times, say 10, compared to the $10^4$ times required for the modified betatron.

A recently proposed improvement to the magnetic field geometry of the modified betatron and the racetrack-modified betatron discussed here consists of superimposing stellarator fields,[1,9,10] (helical windings), to the modified betatron magnetic field geometry. The magnetic field produced by the helical windings is zero on the axis of the system, where the beam ideally resides, and increases radially away, thereby generating restoring forces on the beam particles that tend to keep them localized around and near the system axis. This property has been demonstrated by numerical solutions of the single particle equations of motion in the combined magnetic fields and by linearized first-order orbit theory and related orbital particle resonances.[9,10]

While there exists little doubt about the superior confinement and stability properties of the modified betatron geometry with superimposed helical windings, there is yet much practical interest in fully exploring the limits and in optimizing the performance of the modified betatron geometry without the helical windings for it is a much simpler and less expensive geometry. Figure 1 shows the parameter range for the proposed racetrack-modified betatron, as well as experimental points for the NBS linear induction accelerator, and the P.I. and Cornell beam injection and trapping experiments in toroidal geometries.

## II. Magnetic Field Structure

The racetrack geometry is naturally divided into straight sections and toroidal sections. The magnetic fields and the beam dynamics are treated separately in the two kinds of sections for simplicity, and they are matched in the transition region.

A cylindrical coordinate system r, θ, z is assumed in the straight section. The applied magnetic field is assumed uniform, $\vec{B} = i_z B_o$. The beam is assumed to have a flat radial density profile of radius $r_b$, density $n_b$, current $I_b$, and uniform velocity $U_b \simeq c$, and be surrounded by a conducting drift tube of radius $r_c$. At first the beam is assumed to be centered in the drift tube. The beam's self-fields are given by

$$E_r = \frac{1}{2\pi r_b}\frac{I_b}{\varepsilon_o U_b}\frac{r}{r_b} \; ; \quad B_\theta = \frac{U_b}{c^2} E_r \tag{1}$$

where $\varepsilon_o$ is the free space dielectric permittivity. In the presence of the applied magnetic field and the beam's self-fields the beam undergoes a rotation around the z-axis, given by the frequency $\omega_\theta$, which is found from the radial momentum balance equation to be

$$\omega_\theta = \frac{1}{2}\,|\omega_{cr}|\,\left[1 - \left[1 - 2\left(\frac{\omega_{br}}{\omega_{cr}}\right)^2\right]^{1/2}\right] \tag{2}$$

which requires for equilibrium that

$$\left(\frac{\omega_{cr}}{\omega_{br}}\right)^2 > 2 \quad \text{or} \quad B_o^2 > \frac{I_b}{2\pi\beta\gamma r_b}\,10^{-8} \tag{3}$$

$$\omega_{br}^2 = \frac{n_b q^2}{\varepsilon_o m_o \gamma^3} \quad ; \quad \omega_{cr} = \frac{qB_o}{\gamma m_o} \tag{4}$$

Another constraint on the injected beam current and voltage that must be satisfied is the limiting current, given by

$$I_b \ll I_L = \left(\gamma_o^{2/3} - 1\right)\frac{8.5 \times 10^3}{\ln(r_c/r_b)} \tag{5}$$

and $\gamma_o$ is the relativistic mass at injection. It is important therefore that system parameters $r_b$, $r_c$, $I_b$, $\gamma_o$ at injection are such that the constraints of Eqs. (3) and (5) are satisfied.

When initially injecting the beam into the racetrack geometry it is possible, in fact it might be unavoidable, that the beam center is not centered on the axis of the drift tube. Let Δr be the radial location of the beam center. The displacement of the beam from the axis gives rise to wall image electric and magnetic fields at the beam location. At the displaced beam center these fields are given, for the case of small displacement, by

$$\vec{E}_d = \frac{qn_b}{2\varepsilon_o}\left(\frac{r_b}{r_c}\right)^2(\Delta r)\hat{i}_r \; ; \quad \vec{B}_d = \frac{U_b}{c^2}\,\hat{i}_z \times \vec{E}_d \tag{6}$$

The total force resulting on the beam center is

$$\vec{F}_d = q(\vec{E}_d + \vec{U}_b \times \vec{B}_d) = \frac{q^2 n_b}{2\varepsilon_o \gamma^2}\left(\frac{r_b}{r_c}\right)^2(\Delta r)\hat{i}_r \tag{7}$$

which in the absence of the axial magnetic field would drive the beam to the tube wall. In the presence of the axial magnetic field $B_o$, the equation of motion for the beam center in the transverse r–θ plane can be solved to find the slow rotation $\dot{\theta}$ to be

$$\dot{\theta} \simeq \frac{1}{2}\,\omega_{cr}\left(\frac{\omega_{br}}{\omega_{cr}}\right)^2\left(\frac{r_b}{r_c}\right)^2 \tag{8}$$

which is valid when

$$2\left(\frac{\omega_{br}}{\omega_{cr}}\right)^2\left(\frac{r_b}{r_c}\right)^2 < 1 \tag{9}$$

which is well satisfied on account of the more stringent requirement of Eq. (3).

In concluding the straight-beam section it is noted that two major, well-known equilibrium and stability constraints have been pointed out, i.e., Eqs. (3) and (5). It has also been shown that the beam rotates around itself, Eq. (2), and that when displaced from the tube axis, the beam center rotates around the tube axis with a slower angular velocity given by Eq. (8).

The procedure that has been carried out for the straight sections now needs to be repeated for the toroidal sections, albeit in the presence of more complicated magnetic and electric fields. Utilizing a cylindrical coordinate system (r,ϕ,z) with the z-axis along the major axis of the torus, the principal axial magnetic field is given by

$$B_\phi = B_{o\phi}\left[1 - (r - r_o)/r_o\right] \tag{10}$$

where $r_o$ is the major radius of the toroidal section and defines the axis of the toroidal tube. The vertical betatron magnetic field is given by its two components

$$B_z(r,t) = B_{oz}(t)\left[1 - n_\beta(r - r_o)/r_o\right] \tag{11}$$

$$B_r(r,t) = -\,B_{oz}(t)\,n_\beta z/r_o \tag{12}$$

The electric and magnetic fields, due to induced charges and currents on the inner wall of the toroidal tube, assumed of infinite conductivity, as well as the additional components due to the possible displacement of $(\Delta r, \Delta z)$ of the beam center from the toroidal tube axis, are given by

$$\vec{E}_d = \frac{qn_b r_o}{2\epsilon_o}\left\{\left[\left(\frac{r_b}{r_c}\right)^2 \frac{\Delta r}{r_o} + \frac{1}{2}\left(\frac{r_b}{r_o}\right)^2 \ln \frac{r_c}{r_b}\right]\hat{i}_r + \left(\frac{r_b}{r_c}\right)^2 \frac{\Delta z}{r_o}\hat{i}_z\right\} \quad (13)$$

$$\vec{B}_d = \frac{U_b}{c^2}\hat{i}_\phi \times \vec{E}_d + \frac{qn_b r_o U_b}{2\epsilon_o c^2}\frac{3}{2}\left(\frac{r_b}{r_o}\right)^2\left[1 + \frac{1}{2}\ln\frac{r_c}{r_b}\right]\hat{i}_z \quad (14)$$

Solving the equation of motion of the beam center for the applied and self-fields of Eqs. (10-14), it has been found by Kapetanakos et al.,[11] that the rotation frequency of the beam center around the minor axis, $r = r_o$, of the toroidal section is given by

$$\dot{\theta}^2 = \left(\frac{B_{oz}}{B_{o\phi}}\right)^2 \omega_{oz}^2\left[n\xi^2 - n_\beta\xi - n_s\left(\frac{r_b}{r_c}\right)^2\right]\left[n_\beta\xi - n_s\left(\frac{r_b}{r_c}\right)^2\right] \quad (15)$$

where,

$$\omega_{oz} = \frac{qB_{oz}}{m_o\gamma_o}, \quad n_s = \frac{1}{2}\left(\frac{\omega_{br}}{\omega_{oz}}\right)^2$$

$$\eta = 1 - \frac{\nu}{\gamma_o}\ln\left(\frac{r_c}{r_b}\right), \quad \xi^{-1} = \left[1 + \frac{2\nu}{\gamma_o}\left(1 + \ln\left(\frac{r_c}{r_o}\right)\right)\right] \quad (16)$$

When,

$$n_\beta = \frac{1}{2}\eta\xi \quad \text{and} \quad n_s\left(\frac{r_b}{r_c}\right)^2 < \frac{1}{2}\eta\xi \quad (17)$$

then $\dot{\theta}^2$ is positive definite and the orbit of the beam center is closed, i.e., stable in the r-z plane. The rotation frequency then reduces to

$$\dot{\theta} = \left(\frac{B_{oz}}{B_{o\phi}}\right)\omega_{oz}\left[n_\beta\xi - n_s\left(\frac{r_b}{r_c}\right)^2\right] \quad (18)$$

and the condition for the stability of the orbit, given in Eq. (17), yields the following constraint on the beam current

$$I_b \leq 4.26 \times 10^3\left[1 - \frac{\nu}{\gamma_o}\ln\left(\frac{r_c}{r_b}\right)\right]\left(\frac{r_c}{r_o}\right)^2\gamma_o^3 \quad (19)$$

which is more stringent than the critical current constraint of Eq. (3) and the limiting current constraint of Eq. (5) for the straight section. There are two additional equilibrium adjustments resulting from toroidal effects and finite (large) beam current. The first is the adjustment in the required value of the betatron magnetic field needed to confine the rotating beam at a specific major radius $r_o$. The second equilibrium adjustment is the increase in the radial displacement, $\Delta r_o$, of the beam orbit for fixed energy mismatch, $\delta\gamma_o$, and initial major radius, $r_o$. These adjustments are discussed in Ref. 11.

The racetrack geometry under consideration here has certain advantages over the pure toroidal modified betatron geometry in the sense that the vertical betatron fields in the two toroidal sections can be separately adjusted to compensate for weakly unstable beam center orbits and displacements resulting from energy mismatch. The axial-toroidal magnetic field can be programmed to increase in time, because of the limited number of passes, so that weakly unstable orbits can be confined. The presence of the straight sections allows the opportunity for even stronger focusing and recentering of the beam before re-entry to the subsequent toroidal section.

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