A CYCLOTRON RESONANCE LASER ACCELERATOR*

P. Sprangle, L. Vlahos** and C. M. Tang
U.S. Naval Research Laboratory
Washington, D.C. 20375

Summary

A laser acceleration mechanism which utilizes a strong static, almost uniform, magnetic field together with an intense laser pulse is analyzed. The interaction and acceleration mechanism relies on a self resonance effect. Since the laser field is assumed to be diffraction limited, the magnetic field must be spatially varied to maintain resonance. The effective accelerating gradient is shown to scale like $1/\sqrt{E_b}$, where E_b is the electron energy. For a numerical illustration we consider a 1×10^{13} W/cm^2, CO_2 laser and show that electrons can be accelerated to more than 500 MeV in a distance of 15 m (approximately two Rayleigh lengths).

Introduction

At a recent laser accelerator workshop held at LANL a number of potential candidates for high-gradiant accelerators were discussed. Details of the various proposed schemes can be found in ref (1). Although the list of concepts considered at the workshop was extensive one mechanism which may have some interesting features was not discussed in detail. The purpose of the present paper is to analyze and derive some of the scaling relations associated with this acceleration scheme, which we will call the cyclotron resonance laser (CRL) accelerator.[2,3] The CRL acceleration mechanism utilizes a resonance effect between a beam of gyrating electrons and a high power laser beam. The basic physical configuration for the CRL accelerator is similar to that of the inverse free electron laser accelerator,[4-8] except that the wiggler field is replaced by a longitudinal magnetic field. The electrons gyrate about and stream along an external magnetic field, see Fig. 1, and are continually accelerated. Cyclotron resonance between the electrons and radiation field occurs when $\omega = k v_z + \Omega_o/\gamma$ resulting in an exchange of energy. In an ideal situation, i.e., no dispersion in the radiation field, $\omega = ck$, the resonance condition becomes $\gamma(1 - \beta_z) = \Omega_o/\omega$, where $v_z = c\beta_z$ is the axial electron velocity and $\gamma = (1 - \beta^2 - \beta_z^2)^{-1/2}$. It can be shown that the quantity $\gamma(1 - \beta_z)$ is an exact constant of the motion, even in the presence of a large amplitude radiation field and uniform magnetic field. Hence, synchronism between the electrons and laser field can be continually maintained even as the electrons are accelerated. This maintenance of synchronism is called "self resonance". In a more realistic environment the resonance condition can be violated by such non-ideal effects as dispersion, radiation depletion, etc. The following fully non-linear analysis will attempt to include some of these non-ideal effects.

Model and Analysis

We will assume that the laser field is a circularly polarized Gaussian (lowest order) radiation beam and that the maximum radial extent of the electron beam is small compared to the laser spot size. Therefore, only the representation of the radiation field on axis is needed and is given by the vector potential

$$\underset{\sim}{A}(z,t) = A(z)(\sin \phi(z,t)\hat{e}_x + \cos \phi(z,t)\hat{e}_y), \quad (1)$$

where $A(z) = A_o/(1 + (z/z_R)^2)^{1/2}$, $\phi(z,t) = \int_o^z k(z')dz' - \omega t$ is the phase, $k(z) = \omega/c + z_R^{-1}(1 + z^2/z_R^2)^{-1}$ is the wavenumber, $z_R = \pi r_o^2/\lambda$ is the Rayleigh length, $\lambda = 2\pi c/\omega$, r_o is the minimum spot size and ω is the laser frequency. The radiation field in Eq. (1) is not self-consistently evaluated, in the sense that depletion and phase shift effects due to the presence of the electron beam are neglected.

Because the axial phase velocity of the laser field in Eq. (1) is not equal to c, but in fact varies with z, it is anticipated that to provide for a means of maintaining resonance between the particles and laser field it will be necessary to vary slightly the applied magnetic field. If the applied magnetic field varies gradually we may represent it by

$$\underset{\sim}{B}_o(x,y,z) = -(1/2)(\partial B_o(z)/\partial z)(x\,\hat{e}_x + y\,\hat{e}_y) + B_o(z)\hat{e}_z, \quad (2)$$

where $x\,B_o^{-1}\,\partial B_o/\partial z$ and $y\,B_o^{-1}\partial B_o/\partial z$ are small compared to unity.

The electron trajectories in the presense of Eqs. (1) and (2) may be written as the sum of a slowly varying guiding center contribution and a rapidly varying cyclotron contribution. The electron's transverse momentum and position are represented as

$$(P_x, P_y) = (P_{gx}, P_{gy}) + P_\perp(\cos\theta, \sin\theta), \quad (3a)$$

$$(x,y) = (x_g, y_g) + r(\sin\theta, -\cos\theta), \quad (3b)$$

where (P_{gx}, P_{gy}) and (x_g, y_g) denote the transverse components of the electron's guiding center momenta and coordinates, $P_\perp$ is the magnitude of the gyrating part of the momentum, r is the Larmor radius and θ is the electron's phase angle. We now assume that x_g, y_g, P_{gx},

* Work supported by U.S. Department of Energy.
**Present address: University of Maryland, College Park, MD 20742.

P_{gy}, r, $P_\perp$ and $\theta + \phi$ are slowly vary functions of z, i.e., change slightly during a cyclotron period. With these assumptions together with Lorentz force equation , the following set of fully relativistic non-linear orbit equations are obtained

$$\dot{U}_\perp = -\alpha\,\omega\,(1 - n\,\beta_z)\cos\psi + \frac{c\beta_z U_\perp}{2}\,\frac{\Omega_o'}{\Omega_o}, \qquad (4a)$$

$$\dot{U}_z = -\alpha\,n\,\omega\,\frac{U_\perp}{\gamma}\cos\psi - \frac{cU_\perp^2}{2\gamma}\,\frac{\Omega_o'}{\Omega_o}, \qquad (4b)$$

$$\dot{\psi} = \Omega_o/\gamma - \omega(1 - n\beta_z) + \alpha\,\omega\,\frac{(1 - n\beta_z)}{U_\perp}\sin\psi, \qquad (4c)$$

$$\dot{\gamma} = -\alpha\,\omega\,\frac{U_\perp}{\gamma}\cos\psi, \qquad (4d)$$

where $\alpha = |e|A(z)/m_o c^2$, $U_\perp = P_\perp/m_o c$, $U_z = P_z/m_o c$, $\beta_z = U_z/\gamma$, $\gamma = (1 + \underset{\sim}{U}\cdot\underset{\sim}{U})^{1/2}$, $n = 1 + (1 + z^2/z_R^2)^{-1}(cz_R/\omega)^{-1}$ is the effective index of refraction of the medium, $\psi = \theta + \phi$, $\Omega_o = |e|B_o(z)/m_o c$ is the non-relativistic cyclotron frequency and $\Omega_o' = \partial\Omega_o/\partial z$. These non-linear orbit equations in normalized form become

$$\frac{\partial U_\perp}{\partial\xi} = -\alpha\left(\frac{nb - \gamma\Delta}{\gamma - b}\right)\cos\psi + \frac{U_\perp}{2}\,\frac{\partial b/\partial\xi}{b} \qquad (5a)$$

$$\frac{\partial\psi}{\partial\xi} = \Delta + \alpha\left(\frac{nb - \gamma\Delta}{\gamma - b}\right)\frac{\sin\psi}{U_\perp} \qquad (5b)$$

$$\frac{\partial\Delta}{\partial\xi} = \frac{\partial b/\partial\xi}{b}\left[b/U_z - \frac{(n - \Delta)}{2}\,\frac{U_\perp^2}{U_z^2}\right] + \alpha\,(1 - n^2 + n\Delta)\,\frac{U_\perp}{U_z^2}\cos\psi \qquad (5c)$$

$$\frac{\partial\gamma}{\partial\xi} = -\alpha\,U_\perp\left(\frac{n - \Delta}{\gamma - b}\right)\cos\psi \qquad (5d)$$

where $\xi = z\omega/c$ is the normalized axial distance, $b = \Omega_o(z)/\omega$, $U_z = (\gamma - b)/(n - \Delta)$, $\Delta = \Delta\omega/\omega$ and $\Delta\omega = (\Omega_o/\gamma - \omega(1 - n\beta_z))/\beta_z$ is the frequency mismatch. For a constant magnetic field, refractive index and laser amplitude, Eqs. (5) have constants of the motion C_1 and C_2 which are given by

$$f(\gamma) + \alpha\,U_\perp\sin\psi = C_1 \qquad (6a)$$

$$\gamma(n - \beta_z) = C_2 \qquad (6b)$$

where $f(\gamma) = \gamma(U_{oz}\,\Delta_o - (1 + n^2)(\gamma/2 - \gamma_o))$, the subscript zero denotes the quanitites initial value and β_o, n and α are constant. It can be shown that as the particles are accelerated they are also bunched around the resonance phase $\psi_R = \pi$. To obtain the scaling of the effective accelerating gradient we consider the case where the initial transverse momentum is zero, i.e., $U_{o\perp} = 0$ and $\gamma_o(1-\beta_o) \approx 1/2\,\gamma_o$. The spatial rate of change of γ is therefore

$$\frac{\partial\gamma}{\partial z} = \frac{\alpha}{\gamma}\,\frac{\omega}{c}\,(\gamma/\gamma_o - 1)^{1/2} \qquad (7)$$

where n = 1 and α is the constant. Integrating (7) and assuming that the final gamma, γ_f, is much greater than γ_o, gives

$$\gamma_f/\gamma_o = \gamma_o^{-4/3}(3\pi\,\alpha\,L/\lambda)^{2/3} \qquad (8)$$

where L is the interaction length and λ is the laser wavelength. From (7) and (8) we conclude that the accelerating gradient is proportional to $E_b^{-1/2}$ where E_b is the electron energy and that the electron energy is proportional to $L^{2/3}$. If n and α are not uniform the magnetic field must be contoured to maintain cyclotron resonance. The optimum variation of the magnetic field is found by setting $\partial\Delta/\partial\xi = \Delta = 0$ and solving for b in (5c).

As a numerical illustration we will consider a CO_2 laser with an energy flux of 1 x 10^{13} W/cm^2 and a spot size of r_o = 0.5 cm. The Rayleigh length is 7.8 m and the peak laser electric field is E_L= 60 MeV/cm. For this example α = 0.02. Taking the external magnetic field to be initially 100 kG requires an injected electron beam of 25 MeV. Figure 2 shows the electron energy as a function of interaction distance with a uniform magnetic field and an optimally contoured magnetic field. The electron energy reaches ~ 500 MeV in a distance of $2z_R$ = 15.5 m, for a contoured magnetic field. The contoured magnetic field was increased approximately by 15%. The phase of the electrons is shown in Fig. 3 during the initial stage of the acceleration. The initial uniform distribution, from 0 to 2 π, of electron phases rapidly bunch around the stationary phase $\psi_R = \pi$.

References

1. Laser Acceleration of Particles, AIP Conf. Proceedings No. 91, ed. Paul J. Channell, American Institute of Physics, New York, 1982.
2. A. A. Kolomenskii, and A. N. Lebedev Soviet Physics, Dokl., 7, 745 (1963).
3. V. B. Krasovitski and V. I. Kurilko, Sov. Phys.-Techn. Phys., 11, 1953 (1967).
4. A. A. Kolomenskii and A. N. Lebedev. Soviet Physics JETP 23, 733 (1966).
5. R. B. Palmer, J. Applied Physics 43, 3014 (1972).
6. H. Motz, Comtemporary Physics 20, 547 (1979).
7. P. Sprangle and C. M. Tang, IEEE Trans. Nucl. Sci. NS-28, No. 3, 3346 (1981).
8. C. Pellegrini, Proceeding of the Conf. of Challenge of Ultra-High Energies, Oxford, England, 1982.

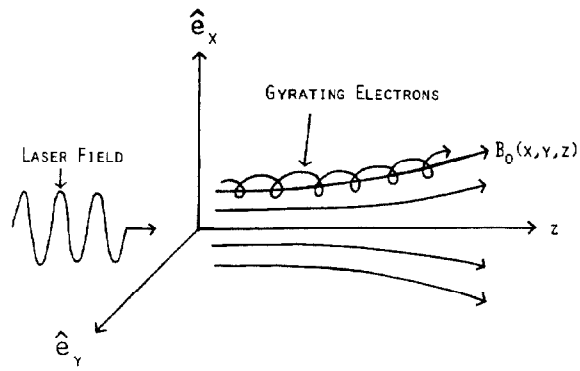

Fig. 1. Illustration of the CRL acceleration process in which electrons are continually energized via a self-resonance effect.

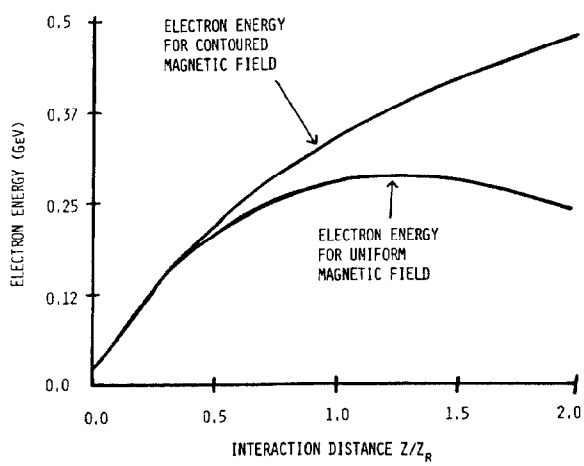

Fig. 2. Electron energy as a function of the interaction distance.

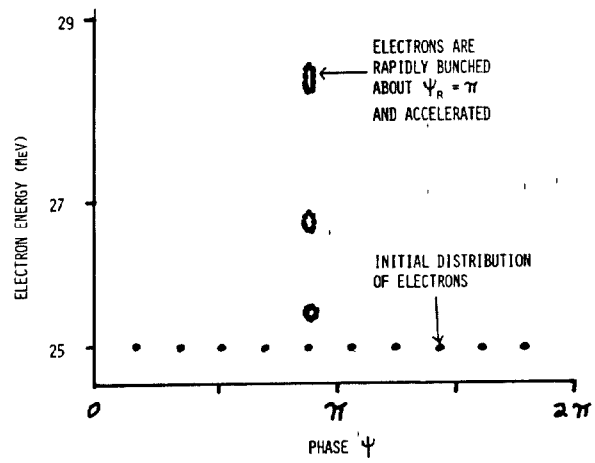

Fig. 3. The phases of the electrons during the initial stage of the acceleration.