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THE STELLATRON -- A STRONG-FOCUSING, HIGH-CURRENT BETATRON*

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SUMMARY

Electrons are accelerated in a betatron by the electric field induced by the changing magnetic flux linking the electron orbit. If the average magnetic field within the orbit is twice the value at the orbit, the radius will remain constant during the acceleration. In a radially decreasing field specified by $B_z = B_0(r_0/r)^n$ a particle will experience restoring forces toward an equilibrium orbit provided only that n lies between zero and unity. B_0 is the value of the magnetic field at the equilibrium orbit r_0. These are the principles upon which the first successful betatron[1] as well as the highest energy betatron[2] were designed. During the 1950's the FFAG accelerator concept added strong focusing fields to a betatron to achieve a configuration having large energy mismatch bandwidth but low current capability.

More recently, efforts have been made to extend the current-carrying capability of the betatron by the addition of a toroidal magnetic field. The resulting "modified betatron" configuration provides high-current capability, but is limited to the same energy mismatch bandwidth as a conventional betatron. Field errors due to the toroidal field and the beam self fields require the modified betatron to have a larger energy mismatch bandwidth than a conventional betatron.

By adding both a toroidal magnetic field and a strong focusing field to a betatron, we can achieve high current capability together will the required energy mismatch tolerance. The resulting configuration is a combination of stellarator and betatron (stellatron) fields, as shown in Figure 1. In this paper we will consider the behavior of an ℓ=2 stellatron, which provides quadrupole focusing and is analagous to an alternate-gradient strong focusing system in a toroidal field.

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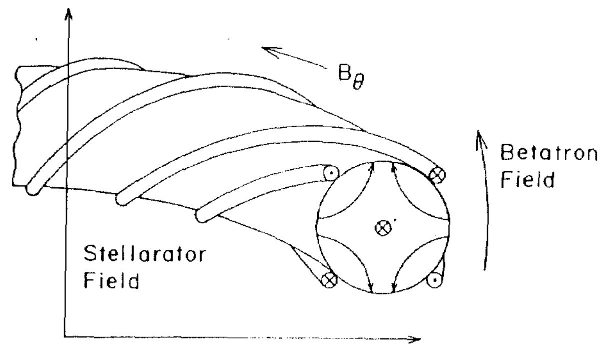

Figure 1
The Stellatron Configuration

ORBIT ANALYSIS

We have carried out two kinds of orbit analysis, an analytical and a numerical treatment. The analytical model follows the behavior of an intense electron beam in the stellatron by considering small departures from a "reference orbit", a circle located at the null point in the quadrupole field.

The numerical model, a single-particle code which integrates the relativisitic equations of motion for an electron in an applied magnetic field, has been utilized to study certain nonlinear aspects of the stellatron configuration. Unlike the analytical analysis, this model does not employ a paraxial approximation for the electron motion and does not use an expansion in the particle displacement from a reference orbit. Also, the applied field in this model includes toroidal corrections to first-order in the inverse aspect ratio.

This model has been utilized to investigate the single-particle bandwidth of the stellatron. Mismatch in excess of 50% can be tolerated for reasonable injection parameters.

Figure 2c shows typical stellatron orbits, projected on the minor cross-section, for a toroidal field of 5kG, with ±50% mismatch. The betatron field is 118G with a field index of 1/2. Figure 2b shows the orbit obtained for the same parameters, but without the helical field contribution, and shows that as little as ±3% mismatch is not tolerable. Likewise, Figure 2a shows that a conventional betatron (i.e., no helical or

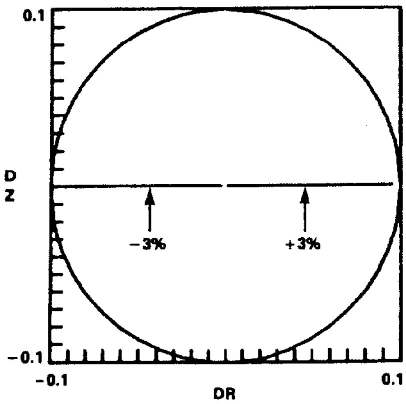

Figure 2a

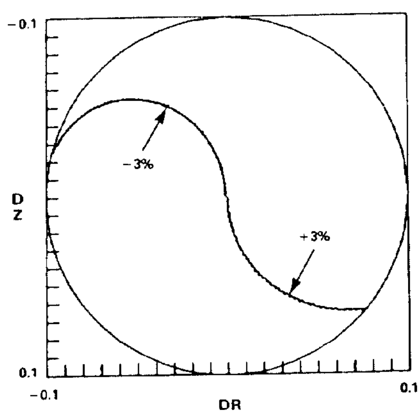

Figure 2b

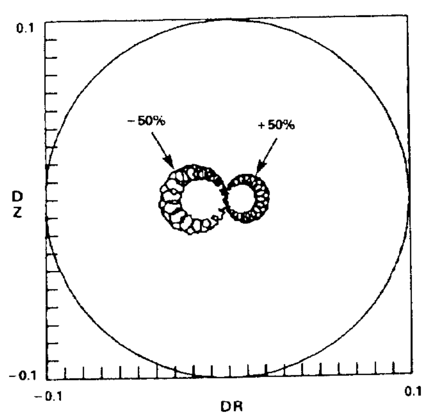

Figure 2c

Figure 2

Single-Particle Orbits
(a) Conventional Betatron
(b) Modified Betatron
(c) Stellatron

toroidal fields) is also unable to tolerate ±3% mismatch for these parameters. The bandwidth of the conventional and modified betatron configurations are in fact equal in this limit, with ±2.5% being the largest tolerable mismatch for these parameters.

INJECTION

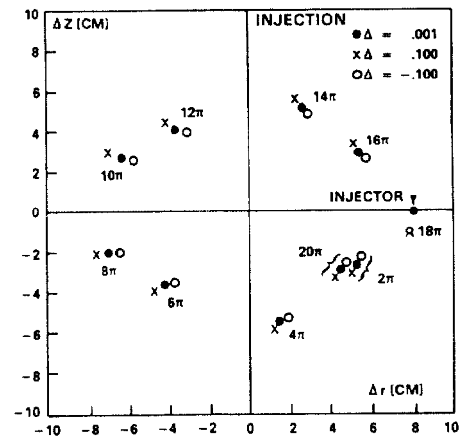

Figure 3
Injection into a Stellatron

The problem of injection into this device has been addressed using the solution to the linearized, paraxial equations. We have found injection parameters which would allow the beam to miss the injector during the first ten or so revolutions, by which time the external fields can be changed to ensure trapping. Figure 3 shows the results of a calculation for injection of a 10 kA beam, matched at $\gamma=7$, with a vertical field index of 1/2, including a toroidal field of 5 kG and a helical quadrupole field with amplitude, 2.5 kG, and having 10 field periods around the torus. The figure represents the location of the beam center on successive transits of the accelerator. The beam misses the injector (located at $\Delta r=8$cm, $\Delta z=0$) for nine revolutions. The results for beams which are mismatched by ±10% are also shown on the figure, to demonstrate the insensitivity of the results to this parameter.

The helical field in a stellatron offers greater flexibility for injection than is possible in a modified betatron. The twisted helix imposes an externally-controlled orbit rotational transform which carries the incoming charge away from the injector, as seen in Figure 3. In the modified betatron, the injected change executes a slow poloidal drift, whose frequency is determined by the beam current and energy and by the toroidal magnetic field. The bandwidth of the stellatron provides a larger margin for field or energy mismatch at injection. Also, the separatrix associated with the quadrupole field may allow injection along magnetic field lines from an external accelerator.

RESONANCES

The introduction of fixed toroidal and helical fields to the betatron causes the betatron wavelengths to depend on energy, resulting in resonant instabilities driven by field errors during acceleration. If the toroidal field is sufficiently large, the betatron wavelengths will be insensitive to beam current. Such instabilities may be avoided by holding all the fields in constant ratio during acceleration. Alternatively, the effect of the instabilities may be minimized if the energy gain per revolution is large enough to rapidly pass through each resonance.

NEGATIVE-MASS INSTABILITY THRESHOLD

The negative-mass instability causes the longitudinal bunching of charge which can occur when the angular frequency is a decreasing function of the particle energy, i.e., $d\dot{\theta}/d\gamma<0$. For weak-focusing accelerators, such as the betatron, this inequality is always satisfied, and the negative-mass instability is always active. Strong-focusing accelerators, on the other hand, have a regime of low-energy operation, $\gamma<\gamma_T$, where $d\dot{\theta}/d\gamma>0$, and the negative-mass instability is absent. The stellatron shares this feature of strong-focusing accelerators.

For a stellatron configuration, the transition energy, γ_T, is given by

$$\gamma_T \approx \left[\frac{1}{2} + \frac{\mu^2}{m^2 + mb - \frac{1}{2}}\right]^{1/2},$$

where m is the toroidal mode number of the stellerator field, $b \equiv B_{\theta 0}/B_{z0}$ is the ratio of toroidal-to-vertical fields, and μ is given by $\mu \equiv \epsilon mb/2$, with ϵ a dimensionless measure of the helical field amplitude. Typically, $\epsilon\sim 1$.

Without the helical field components, i.e., $\mu\equiv 0$, this expression reduces to $\gamma_T = 1/\sqrt{2}$, which is unphysical. For typical stellatron parameters (i.e., $\epsilon=1$, m=20, b=50), a transition energy, $\gamma_T=13$ (or 6.1 MeV), is obtained. This feature of the stellatron may allow injection to proceed without the involvement of the negative-mass instability.

CONCLUSIONS

The superposition of twisted quadrupole, toroidal, and conventional betatron magnetic fields appears to offer significant practical advantages for the confinement and acceleration of large electron currents (10's of kiloamperes) to moderate energies (100's of MeV). Foremost among these advantages is the greatly improved energy bandwidth over that of a weak focusing device. The large bandwidth of the stellatron relaxes the requirements for monoenergetic injection, for a uniform (within a few percent) magnetic field configuration and for a rigid mechanical design. Injection should not be any more difficult than for other high-current accelerator concepts, and is facilitated by the externally-applied rotational transform of the stellerator field. The orbits should remain stable from injection up to the highest energies achievable by conventional inductive acceleration.

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