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FREE ELECTRON LASER FOR A MILDLY RELATIVISTIC ELECTRON BEAM

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SUMMARY

The bandwidth and frequency enhancement of the free electron laser instability in a mildly relativistic ($\gamma \lesssim 1.15$) electron beam propagating through a dielectric loaded waveguide is presented. For an appropriate choice of the dielectric constant $\hat{\varepsilon}$ and the thickness of the dielectric material, it is shown that the instability bandwidth and frequency can be greatly enhanced for specified values of the beam energy and the wiggler wavelength.

One of the most basic instabilities that characterize a relativistic electron beam propagating through a helical (or undulator) wiggler magnetic field is the free electron laser instability.[1-7] In recent years, the free electron laser instability has been extensively investigated with particular emphasis on applications to high power microwave generation. In the previous theoretical studies of this instability, it appears that as a result of the relativistic Doppler effect, the frequency ω of the microwave radiation from the electron beam passing through a wiggler field with the axial wavelength $\lambda_0 = 2\pi/k_0$ is given by $\omega = (k + k_0)\beta c = \gamma^2(1 + \beta)\beta k_0 c$. Here k is the axial wavenumber of the radiation, $\gamma = (1 - \beta^2)^{-1/2}$ is the relativistic factor of the beam electrons, and c is the speed of light in vacuo. In this regard, in order to generate high frequency microwave radiation, high energy beams ($\gamma \gg 1$) are required. However, it is very undesirable to have a high γ value in a practical, compact microwave tube. I, therefore, develop a new idea to enhance the frequency upshift without making use of a high γ value. Moreover, I also present a new promising scheme for a broad bandwidth microwave amplifier.

The previous analysis[6,7] by the author for an electron beam in a perfectly conducting waveguide shows that the free electron laser instability is essentially a mode coupling between the electromagnetic and electrostatic modes, whose dispersion relations are expressed as

$$\frac{\omega^2}{c^2} - k^2 = \begin{cases} \alpha_{\ell n}^2/R_c^2 , & \text{TE mode,} \\ \beta_{\ell n}^2/R_c^2 , & \text{TM mode,} \end{cases} \tag{1}$$

and

$$\omega = (k + k_0)\beta c, \tag{2}$$

respectively, for a tenuous beam. Note that for a tenuous beam, the electrostatic mode can be approximated by the free streaming mode in Eq. (2). In Eq. (1), $\beta_{\ell n}$ and $\alpha_{\ell n}$ are the nth roots of the Bessel function $J_\ell(\beta_{\ell n}) = 0$ and its derivative $J_\ell'(\alpha_{\ell n}) = 0$, respectively, of order ℓ. R_c is the radius of a grounded conducting wall, and TE and TM represent the transverse electric and transverse magnetic modes, respectively. However, for present purposes, I assume a tenuous electron beam propagating through a cylindrical waveguide loaded with a dielectric material in the range $R_w < r < R_c$. Therefore, the radial profile of the dielectric constant is given by $\varepsilon(r) = 1$, for $0 \le r < R_w$, and $\varepsilon(r) = \hat{\varepsilon}$ for $R_w < r < R_c$. The permeability μ of the dielectric material differs from unity by only a few parts in 10^5, thereby approximating $\mu = 1$ in the subsequent analysis. Cylindrical polar coordinates (r, θ, z) are introduced. In the remainder of this article, properties of the mode coupling of the free electron laser in a dielectric loaded waveguide are investigated, in connection with enhancement of the frequency and bandwidth of the microwave radiation from a mildly relativistic electron beam.

It is, therefore, required to derive the dispersion relation of the transverse electromagnetic mode in a dielectric loaded waveguide. In the analysis, a normal-mode approach is adopted in which all components of the electromagnetic field are assumed to vary according to $\delta\psi(\mathbf{x},t) = \hat{\psi}(r)\exp\{i(\ell\theta + kz - \omega t)\}$, where ℓ is the azimuthal harmonic number. The Maxwell equations for the electromagnetic field amplitudes can be expressed as

$$\nabla \times \hat{\mathbf{E}}(\mathbf{x}) = i(\omega/c)\hat{\mathbf{B}}(\mathbf{x}) \ , \ \nabla \times \hat{\mathbf{B}}(\mathbf{x}) = -i(\omega/c)\varepsilon(r)\hat{\mathbf{E}}(\mathbf{x}), \tag{3}$$

without including the influence of the beam presence. In Eq. (3), $\hat{\mathbf{E}}(\mathbf{x})$ and $\hat{\mathbf{B}}(\mathbf{x})$ are the electric and magnetic fields. Making use of Eq. (3), it is straightforward to show that

$$\hat{B}_\theta(r) = i\,\frac{\omega\varepsilon(r)}{cp^2}\frac{\partial}{\partial r}\hat{E}_z(r) - \frac{\ell k}{p^2 r}\hat{B}_z(r) \ , \tag{4}$$

$$\hat{E}_\theta(r) = -i\,\frac{\omega}{cp^2}\frac{\partial}{\partial r}\hat{B}_z(r) - \frac{\ell k}{p^2 r}\hat{E}_z(r) \ , \tag{5}$$

and

$$\left\{\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} - \frac{\ell^2}{r^2} + p^2\right\}\begin{Bmatrix}\hat{E}_z(r)\\ \hat{B}_z(r)\end{Bmatrix} = 0, \tag{6}$$

where $p^2 = \omega^2\varepsilon(r)/c^2 - k^2$, $\hat{E}_\theta(r)$ and $\hat{E}_z(r)$ are the azimuthal and axial components of the electric field and $\hat{B}_\theta(r)$ and $\hat{B}_z(r)$ are the azimuthal and axial components of the magnetic field.

The appropriate boundary conditions of $\hat{E}_z(r)$ and $\hat{B}_z(r)$ at $r = R_c$ are given by $\hat{E}_z(R_c) = 0$. Moreover, the fields $\hat{\mathbf{B}}(r)$, $\hat{E}_\theta(r)$, and $\hat{E}_z(r)$ are continuous across the boundary ($r = R_w$) of the dielectric material. Evidently, the solutions to Eq. (6) are given by a linear combination of the Bessel functions of the first kind $J_\ell(pr)$ and the second kind $N_\ell(pr)$ of order ℓ. After a tedious but straightforward algebra, it can be shown that the dispersion relation of the electromagnetic mode in a dielectric loaded waveguide is expressed as

$$D_T^E(\omega,k)D_T^M(\omega,k) = \frac{\ell^2(\eta^2 - \xi^2)(\eta^2 - \xi^2\varepsilon)}{\eta^4\xi^4} \tag{7}$$

where the TE and TM dielectric functions are defined by

$$D_T^E = \frac{1}{\eta}\frac{J_\ell'(\eta)N_\ell'(\zeta) - J_\ell'(\zeta)N_\ell'(\eta)}{J_\ell(\eta)N_\ell'(\zeta) - J_\ell'(\zeta)N_\ell(\eta)} - \frac{1}{\xi}\frac{J_\ell'(\xi)}{J_\ell(\xi)}, \tag{8}$$

and

$$D_T^M = \frac{\hat{\varepsilon}}{\eta}\frac{J_\ell'(\eta)N_\ell(\zeta) - J_\ell(\zeta)N_\ell'(\eta)}{J_\ell(\eta)N_\ell(\zeta) - J_\ell(\zeta)N_\ell(\eta)} - \frac{1}{\xi}\frac{J_\ell'(\xi)}{J_\ell(\xi)}, \qquad (9)$$

respectively, and the parameters ξ, ζ, and η are defined by

$$\omega^2/c^2 - k^2 = \xi^2/R_w^2\ , \quad \omega^2\hat{\varepsilon}/c^2 - k^2 = \zeta^2/R_c^2\ , \qquad (10)$$

and $\eta = \zeta R_w/R_c$, and the prime denotes $J_\ell'(x) = dJ_\ell/dx$ and $N_\ell'(x) = dN_\ell/dx$. Several points are noteworthy from Eqs. (7) - (10). First, the dispersion relations of the TE and TM modes are decoupled for $\ell = 0$. Second, in the limit of $\hat{\varepsilon} \to 1$ or $R_w/R_c \to 1$, the dispersion relation in Eq. (7) can be simplified as Eq. (10) with $\xi/R_w = \alpha_{\ell n}/R_c$ for the TE mode and with $\xi/R_w = \beta_{\ell n}/R_c$ for the TM mode. Third, for a completely filled dielectric waveguide ($R_w \to 0$), Eqs. (7) - (9) can be also reduced to Eq. (10) with $\zeta = \alpha_{\ell n}$ for the TE mode and with $\zeta = \beta_{\ell n}$ for the TM mode.

For given values of the dielectric constant $\hat{\varepsilon}$, ξ is determined from Eqs. (7) - (9) in terms of ζ. The oscillation frequency ω and axial wavenumber k in a dielectric loaded waveguide are obtained from the simultaneous solution of Eq. (10) for specified ξ and ζ. Figure 1 is a plot of the dielectric dispersion relation in the parameter space (ω,k) for $\ell = 1$, $R_w/R_c = 0.8$ and several values of the dielectric constant $\hat{\varepsilon}$. The straight lines in Fig. 1 represent the free streaming mode in Eq. (2) for $\gamma = 1.107$ and several values of the normalized wiggler wavenumber k_0R_c. The dispersion curves in Fig. 1 correspond to the lowest radial mode number. In a range of physical parameters, the free streaming mode $\omega = (k + k_0)\beta c$ intersects the dielectric dispersion curve of the electromagnetic mode, thereby indicating the free electron laser instability. The mode coupling occurs at $k = k_p$, distinguishing two cases; (a) the short helical wavelength (SHW) mode corresponding to the normalized mode coupling wavenumber $k_pR_c = 9.3$ for $\hat{\varepsilon} = 2$ and $k_0R_c = 9$ in Fig. 1 and (b) the long helical wavelength (LHW) mode corresponding to $k_pR_c = 12.3$ for $\hat{\varepsilon} = 6$ and $k_0R_c = 1.5$ in Fig. 1.

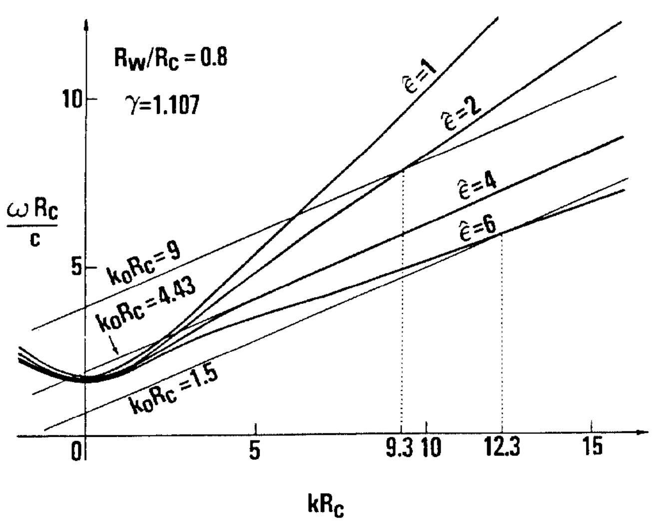

FIGURE 1 PLOT OF THE DIELECTRIC DISPERSION RELATION IN THE PARAMETER SPACE (ω,k) FOR $\ell = 1$, $R_w/R_c = 0.8$ AND SEVERAL VALUES OF THE DIELECTRIC CONSTANT $\hat{\varepsilon}$ FOR $\gamma = 1.107$ AND SEVERAL VALUES OF k_0R_c. THE STRAIGHT LINES REPRESENT $\omega = (k+k_0)\beta c$

SHORT HELICAL WAVELENGTH MODE

The SHW mode is the high frequency operation of the free electron laser instability. The normalized radiation frequency $\omega/k_0\beta c = (k_p + k_0)/k_0$ versus the dielectric constant $\hat{\varepsilon}$ is plotted in Fig. 2(a) for the SHW mode, $\gamma = 1.1$, several values of k_0R_c, and parameters otherwise identical to Fig. 1. The electromagnetic dispersion relation of a short axial wavelength mode satisfying $kR_c >> 1$ can be approximated by $\omega \simeq kc/\hat{\varepsilon}^{1/2}$, thereby giving the normalized radiation frequency

$$\omega/k_0\beta c \simeq (1 - \beta\hat{\varepsilon}^{1/2})^{-1}. \qquad (11)$$

Shown also in Fig. 2(a) is plot of $\omega/k_0\beta c$ in Eq. (11). Obviously from Eq. (11) and Fig. 2(a), the normalized radiation frequency $\omega/k_0\beta c$ increases rapidly as the dielectric constant $\hat{\varepsilon}$ increases from unity to $\hat{\varepsilon} = 1/\beta^2$. In this regard, it is important to emphasize that the submillimeter microwave radiation can be easily produced by this scheme even for a moderate electron energy ($\gamma \lesssim 1.15$). The limitation of the radiation frequency is the availability of the proper dielectric material in the present time.

LONG HELICAL WAVELENGTH MODE

After a careful examination of Fig. 1, it is noted that the LHW mode coupling can occur only for the dielectric loaded waveguide. Figure 2(b) is plots of the normalized radiation frequency $\omega/k_0\beta c = (k_p + k_0)/k_0$ versus $\hat{\varepsilon}$ for the LHW mode, $\gamma = 1.15$, several values of k_0R_c, and parameters otherwise identical to Fig. 1. Note that the normalized wiggler wavenumber k_0R_c for the LHW mode is much smaller than that for the SHW mode. However, by an appropriate choice of the dielectric material, the radiation frequency ω for the LHW mode can be many times of the wiggler frequency $k_0\beta c$.

WIDE BANDWIDTH AMPLIFIER

An outstanding microwave amplification requires a broad instability bandwidth. As shown in Fig. 1, the dispersion curves of the free streaming and dielectric waveguide modes for $k_0R_c = 4.43$ and $\hat{\varepsilon} = 4$ coincide practically in the range $4.5 < kR_c < \infty$, thereby indicating possibilities of wide bandwidth amplifier. In general, for a specified beam energy γ, proper choice of the dielectric constant $\hat{\varepsilon}$ and the wiggler wavenumber k_0 gives a wide band free electron laser amplifier. The instability bandwidth can be easily more than fifty percent.

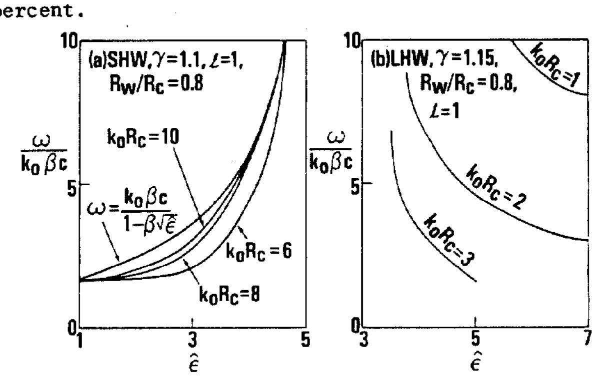

Fig. 2 Plot of normalized radiation frequency $\omega/k_0\beta c$ vs. $\hat{\varepsilon}$ for (a) the SHW mode, $\gamma = 1.1$, (b) the LHW mode, $\gamma = 1.15$, several values of k_0R_c, and parameters otherwise identical to Fig. 1.

Finally, I conclude this article by pointing out that the instability growth rate for large wave-number perturbations ($kR_c >> 1$) is substantially reduced by the axial momentum spread of the beam electrons,[6,7] limiting the enhancement of the band-width and radiation frequency. However, the axial momentum spread of an electron beam for the free electron laser instability can be much less than that for other microwave tubes such as the gyrotron. The growth rate and bandwidth of the free electron laser instability are currently under investigation by the author for a broad range of physical parameters, including the influence of the axial momentum spread on stability behavior.

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