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Subharmonic Beam-loading in Electron Linear Accelerators

W. J. Gallagher Boeing Aerospace Co., Seattle, Wash. 98124

The intention of operating an electron linear accelerator subharmonically beam loaded for free electron laser (FEL) application requires justification of the beam-loaded energy gain equation. The mode of operation typically planned is 5-10 nanocoulombs single RF cycle pulses at 25-50 nanosecond This inquiry investigates details of that sort of intervals. beam loading and the performance achievable.

Pulse train beam loading is necessarily of course subharmonic, but two distinguishable cases arise, depending on whether there is one or more beam pulses transiting the waveguide at the same time. The energy gain of single short pulses, less than the fill time of the waveguide, where the pulse transit time through the accelerator waveguide is short compared to the pulse length and where pulse spacing is greater than one RF fill time has been treated in a previous paper (Ref. 1).

For constant gradient waveguide,

$$V = V_0 - ir I_0 L v_{got}; V_0 = \sqrt{2l_0 Pr L}$$

where $V_{\mathbf{0}}$ is the no-load energy gain, i the pulse current, rthe shunt impedance per unit length, Io the initial attenuation coefficient, L the waveguide length, v_{g0} the initial group velocity and ζ the interval of pulse duration. It was also shown in Ref. (1) that the above expression is equivalent to

$$V(t) = V_0 - \frac{\omega}{2} \frac{rL}{Q} q(t)$$
 (1)

where q is the charge in the pulse.

Several other investigations of single bunch beam loading have been undertaken, notably at SLAC, where it has been found experimentally that the beam-loading varies directly as the bunch charge and independently of its energy; that investigation also included radiation effects of the wake field and losses owing to parasitic effects of higher order modes.⁽²⁾ Similar investigations have been made at Osaka Univ.⁽³⁾ and NBS (Gaithersburg).⁽⁴⁾

In the case of beam loading where there are multiple pulses transiting at the same time, spaced far enough apart that significant RF power is introduced between pulses, the energy gain may be calculated by dividing the waveguide into a number of segments, each equal in length to the integral of

the interpulse time (T) and the local group velocity ($v_g(z)$). For constant gradient waveguide, where τ is the filltime.

$$T = \int \frac{dZ}{v_{g}(Z)} = \frac{1}{v_{go}} \int \frac{dZ}{(1-2I_{o}Z)} = -\frac{Q}{\omega} - n(1-2I_{o}Z) \bigg] \frac{z_{n+1}}{z_{n}}$$

or

$$Z_{n} = \frac{e^{-n\omega T/Q}(e^{\omega T/Q}-1)}{2I_{0}} \qquad n=1,2,3...T/T \quad (2)$$
$$\sum Z_{n}=L$$



The net energy gain in the steady state is then the sum of the energy gains in all the segments. The energy gain of each segment, from Eq. (1), is

$$V_{n} = (\mathbf{E}_{o,n} - \frac{\omega}{2} \mathbf{q} \mathbf{q}) \mathbf{Z}_{n}$$
⁽³⁾

where $E_{o,n}$ is the initial field intensity in each segment (gotten from the energy gain equation itself)

$$E_{o,n} = E_{o,n-1} - \frac{\omega}{2} \frac{r}{Q} q = E_{o} - (n-1) \frac{\omega}{2} \frac{r}{Q} q$$
(4)

By induction it can be seen that

$$V_{n} = (E_{0} - (n-1)\frac{\omega}{2}\frac{r}{Q}q)Z_{n} - \frac{\omega}{2}\frac{r}{Q}qZ_{n}$$
⁽⁵⁾

and

$$V = \sum V_n = \sum (E_0 - n \frac{\omega}{2} \frac{r}{Q} q) Z_n$$

$$= E_0 L - \frac{\omega}{2} \frac{r}{Q} q \cdot \sum n Z_n$$
(6)

The number of segments is evidently τ /T. As an example, at 2856 mcs. a 2.7m disk-loaded, constant gradient waveguide having the following properties;

Loss factor, Q = 13,200Shunt impedance r = 53 megohms/m Initial attenuation coeff. $I_0 = 0.114$ nep/m Initial group velocity, $v_{g0}/c = 0.019$ Fill-time, $\tau = 0.712 \ \mu s$

will with interpulse times (T) of 100 nsec have a "beam loading factor" $\sum_{n} nZ_n = 9.327m$, calculated by observing T/T = 7 and from Eq. (2) that:

It may be supposed on energetic arguments that the energy gain in the subharmonic case may be calculated on the basis of the equivalent beam current, $i = \omega q/2\pi$, using the conventional beam loaded, steady state energy gain equation, (3)

$$V = E_{o}L - \frac{ir}{4I_{o}} \left[2I_{o}L + (1-2I_{o}L) \ln(1-2I_{o}L) \right]$$
(7)

This is, of course, shown to be justified quite closely. Moreover, in the limiting case we may calculate the energy gain in the steady state using Eq. (7) and a specified beam current or by Eq. (6), using the corresponding charge per cycle. For the example given earlier and 100 ma beam (corresponding to 0.035 nC/cycle) by Eq. (7) the energy gain is 26.80 MeV; for the 2034 terms arising in Eq. (6), $\sum n Z_n = 2329m$ and the energy gain is 26.74 MeV.(6)

It will not, presumably, surprise the reader that Eq. (6) does not devolve into Eq. (7) for the fundamental case, but approximations in the derivation, while tantamount to a very close analysis of the model is, nevertheless, not exact.

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In the case of transient operation of a linear accelerator, such that there is only one bunch in the waveguide during bunch transit and where there is no significant input RF power during the pulse length of the packet, the energy gain of the first and last particle of the bunch is independent of the bunch shape and depends only on the stored energy in the waveguide and the total charge transiting the structure.⁽¹⁾

$$\frac{V_{i}}{V_{f}}^{2} = 1 + \frac{W_{B}}{W_{s}}$$
(8)

where $V_{\rm i}$ is the energy gain of the initial particle and $V_{\rm f}$ that of the final particle; $W_{\rm B}$ is the energy in the beam pulse,

$$W_{\rm B} = \int iV \, dt$$
 (9)

and W_s is the energy stored in the waveguide at the end of the pulse. By assumption, the stored energy at the beginning of the pulse (W_o),

$$W_{O} = W_{S} + W_{B} \tag{10}$$

which can be calculated from the power input and a description of the waveguide.

Derivation of Eq. (8) is lengthy, but a transcending one is provided by the observation that the energy gain of a particle is proportional to the electric field in the waveguide and that the stored energy is proportional to the square of the field intensity in the structure,

$$\left(\frac{v_i}{v_f}\right)^2 = \frac{w_o}{w_s}$$

from which, noting Eq. (10), Eq. (8) follows.

For example, if 10 MW is to accelerate 5 nC to 30 MeV with 0.01 energy spread, from Eq. (9) the beam energy $W_B = 0.15$ joules; from Eq. (8) the final stored energy must be $W_S = 7.39$ joules, so that the initial stored energy must be $W_0 = 7.54$ joules, by which a partial description of the requisite waveguide is derived. The above considerations do not

address the question of achieving the intended energy gain which involves an extended discussion, omitted for sake of brevity.

In the case of subharmonic loading, where there are several pulses transiting the waveguide at the same time, the energy spread in a single bunch may be calculated as the difference of energy gain of the first and last particle of the same bunch. Using the segment model above and the "fat electron" approach

$$V_{ni} = (E_{o} - \eta \frac{\omega}{2} \frac{r}{Q} q) Z_{n}$$
(11)

$$V_{nf} = (E_o - n \frac{\omega}{2} \frac{r}{Q}q)Z_n - \frac{\omega}{2} \frac{r}{Q} q Z_n$$

Hence,

$$V_{i} = \Sigma V_{ni} = E_{o}L - \frac{\omega r}{2} \frac{r}{Q} q \Sigma nZ_{n}$$
(12)

$$V_{f} = \Sigma V_{nf} = E_{0}L - \frac{\omega r}{2} \frac{q}{Q} q - \Sigma (n+1)Z_{n}$$

and the fractional energy spread is therefore

$$\frac{v}{v} = \frac{\sum_{z} \sqrt{qL}}{v_{o} - \frac{1}{4} \sqrt{q} \sqrt{z} (n+2)Z_{p}}$$
(13)

This calculation is ideal; it assumes an infinitesimally small bunch (vanishingly small phase extent, although that is somewhat inconsistent), ignores space charge forces and injection phase and is, therefore, a limiting case; the spectrum width cannot be better than that indicated (without programming injection phase).

In general, the energy gain of a particle in a short pulse is t

$$V(t) = E_{o}L \cos \phi(t) - \frac{\omega}{2} \frac{r}{Q} L \int_{0}^{t} q(t) dt \qquad (14)$$

Hence to produce a constant energy gain (V) the injection phase as a function of charge injected is, of course $\label{eq:constant}$

$$\cos\phi(t) = \frac{V}{V_0} + \frac{\omega}{2} \frac{r}{QE_0} \int_0^L q(t) dt \qquad (15)$$

If the last particle in the pulse transits on the wave crest $(\phi=0)$ for economical reasons, then noting that the stored energy per unit length in constant gradient waveguide is everywhere the same, Eq. (14) may be written

$$V \approx \frac{\omega r L}{Q} \left(\frac{P L}{v_{go}} - V q_{\star} \right) - \frac{\omega}{2} \frac{r}{Q} L q_{o}$$
(16)

where \boldsymbol{q}_{0} is the total charge in the pulse, which solving for the dimensionless ratio

$$\frac{\omega}{2} \frac{rL}{Q} \frac{q_0}{V} = \sqrt{3 - \frac{\omega rPL^2}{Q v_{go} V^2}} - 2$$
(17)

from which it is evident that such a phase programming scheme will require an extravagant amount of stored energy in the waveguide.

Obviously, all the above arguments may be derived for constant impedance accelerator waveguide but are omitted for want of space.

References

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- The form of the equation given here differs from that of R. B. Neal, Theory of the Constant Gradient Linear Electron Accelerator, Report ML-513, WW Hansen Lab. Stanford Univ. (1958) but is equivalent. See W. J. Gallagher IEEE Trans. Nucl. Sci. NS-28, 3513 (1981).
- 6. Here the calculator must resort to an "analytic engine", which turns itself in a more stately manner upon such arithmetic processes than other rhabdological agencies; computers don't make mistakes and they don't get bored.