IEEE Transactions on Nuclear Science, Vol. NS-30, No. 4, August 1983

THE DESIGN OF PERIODIC BEAM TRANSPORT SYSTEMS*

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Summary

Periodic beam-transport systems have several advantages including insensitivity to errors, minimum magnet apertures, and use of standardized components. A simple procedure is given for the design of modules, with and without bending magnets, that have the same matched beam properties. These modules can be combined in certain ways to produce systems that are achromatic and require a minimum number of matching elements.

Introduction

In many cases, an achromatic beam transport system composed of sets of periodic focusing cells has significant advantages over the more traditional systems,[1,2] particularly if the beam must be transported a considerable distance. Such a system will have reduced magnet costs because the aperture requirements are minimized, tolerances are relaxed, and there is a limited number of different magnet types. The system also is simple to operate because there are few parameters that need to be varied by an operator. In this paper, a procedure is presented for designing periodic beam-transport cells that can be combined into achromatic bending systems in a variety of configurations.

Beam Transport Formalism

The beam envelope in a periodic focusing system can be described by the Twiss parameters, α, β, γ, and the phase shift, μ.[3] The beam radius then is $\sqrt{\beta\varepsilon}$ where ε is the emittance.

The beam transport system in the horizontal plane x is described by its transfer matrix R_x which relates the coordinates (x, x') of a ray at Position 1 to that at Position 2 by

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = R_x \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} \quad ,$$

where

$$R_x = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} .$$

There is a similar relation for the (y, y') plane. Associated with a periodic focusing system is a matched beam with periodic Twiss parameters. These are easily determined from the transfer matrix for one cell of the system which can be written as[3]

$$R = \begin{pmatrix} \cos\mu + \alpha \sin\mu & \beta \sin\mu \\ -\gamma \sin\mu & \cos\mu - \alpha \sin\mu \end{pmatrix} \quad . \qquad (1)$$

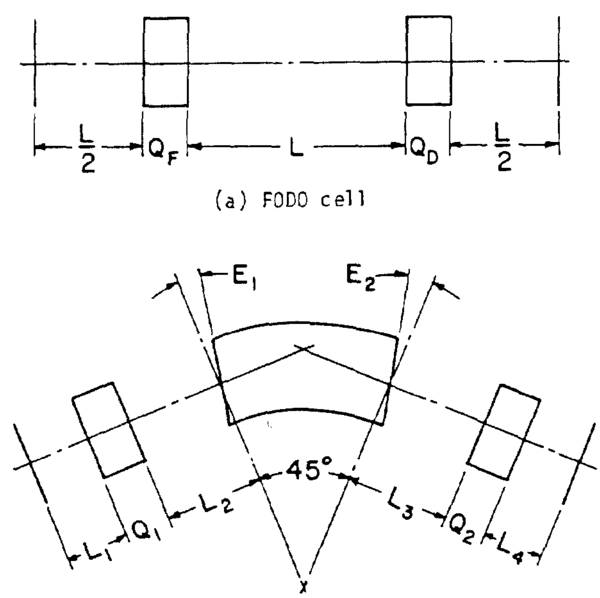
Periodic focusing cells can be designed easily with the computer code TRANSPORT,[4] which has the capability of fitting to a desired phase shift. Then, the matched beam parameters can be determined from Eq. (1). The maximum emittance that can be transported is

$$\varepsilon_m = \frac{a^2}{\beta_m} \quad , \qquad (2)$$

where a is the radius of the beam pipe, and β_m is the maximum value of β for the matched beam (which occurs in the focusing quadrupole). If the beam in the system is mismatched, less emittance can be transported.

General Design Principles

The simplest form of periodic focusing cell is a FODO array consisting of a focusing quadrupole and a defocusing quadrupole of equal strength as shown in Fig. 1a. It is possible to design a cell containing a bending magnet so that the matched beam Twiss parameters are identical to those of the FODO cell in both planes. An example containing a 45° bend is shown in Fig. 1b. A transport system made up of a series of these cells will be achromatic if, for each bending cell, there is a second identical cell located at a phase shift of 180° with respect to the first. When this is done, the dispersion of the second cell will cancel that from the first cell. The intervening cells may be either bending or FODO cells. The major advantage of this arrangement is that it is necessary only to match the beam into the system once at the beginning. Another advantage is that such a system has no second order geometric aberrations and, in many cases, the second-order chromatic aberrations can be eliminated also by the introduction of two families of



(a) FODO cell

(b) 45° bend cell

Fig. 1. A FODO cell and a matching cell containing a 45° bend. The bend cell was designed with equal edge angles (E_1 = E_2) and equal spacings of the quadrupoles from the bend (L_2 = L_3).

*Work supported by the US Department of Energy

sextupoles located at the quadrupoles.[5] The number of cells is minimized if the phase shift per cell is 90°. Figure 2 shows some examples of achromatic systems made up of the cells shown in Fig. 1. An arbitrary number FODO cells could be added to the beginning or end of each system without changing its optical properties. Bends can be either in the horizontal or vertical plane, or both, as long as the bends are used in pairs, separated by 180° in phase.

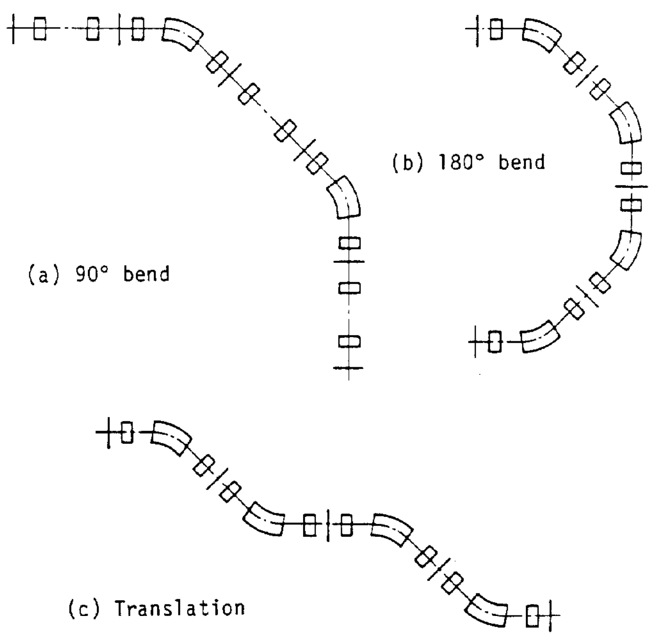


Fig. 2. Three possible achromatic transport systems made from the modules shown in Fig. 1. The 90° system also contains an additional FODO cell at each end. An arbitrary number of these can be added to either end of each system without changing the optical properties.

Design Procedure

The procedure for designing these systems with TRANSPORT is simple. First design a FODO cell of the desired length by varying Q_F and Q_D (shown in Fig. 1a) together, fitting to the desired phase shift (normally 90°). The matched beam then can be calculated from the transfer matrix of the cell. Next, the bend cell is designed. The fitting conditions are the diagonal elements of R_x and one of the off-diagonal elements, say R_{11}, R_{22}, and R_{21} for the x-plane and the corresponding elements for the y-plane (R_{33}, R_{44}, and R_{43} in TRANSPORT notation). There are six fitting conditions; therefore, there must be six free parameters available. These can be chosen from the two quadrupole strengths Q_1 and Q_2; the four lengths, ℓ_1, ℓ_2, ℓ_3, and ℓ_4; and the bending-magnet edge angles, E_1 and E_2--although not all combinations are permissible. In particular, the length of the system and the positions of some of the focusing elements must be allowed to vary. This is apparent when we realize that we are trying to design a focusing system consisting of quadrupoles and a bending magnet, perhaps with edge focusing, that has the same focal length and the same separation between principal planes as the FODO cell. For the system shown in Fig. 1a, the edge angles E_1 and E_2 were varied together as were the spacings between the quadrupoles and the bend, ℓ_2 and ℓ_3. This achieves maximum symmetry in the system and minimum beam size. The quadrupole strengths Q_1 and Q_2 and the other lengths, ℓ_1 and ℓ_4 are not quite equal. Alternatively, it is possible to vary all four lengths and require that the quadrupole strengths be equal. A wedge bending magnet could be used if all lengths and quadrupoles are varied. A limitation on these systems is that the cell length of the FODO system must be sufficiently short that the focusing strengths of the quadrupoles are greater than that of the bending magnet, otherwise a fit is not possible.

After both cells are designed, the system then can be assembled with no further fitting required other than to design a matching system. If the source of beam is itself a periodic focusing system such as a linear accelerator, matching is easily accomplished with a quarter-wave or half-wave section as described by Brown.[6]

Conclusions

A procedure is given for designing achromatic beam-transport systems that are composed of a few standard elements, have well-defined aperture requirements, few variables for the operator, and low aberrations. Such systems are suited particularly for transporting beams to and from accelerators that are themselves periodic focusing structures because the matching procedures are also straightforward.

Acknowledgment

The author is indebted to K. L. Brown, SLAC, for many fruitful discussions.

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