© 1983 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.

IEEE Transactions on Nuclear Science, Vol. NS-30, No. 4, August 1983

## A Review of the Beam Breakup Problem in Linacs

W. J. Gallagher Boeing Aerospace Co., Seattle, Wash.

The phenomenon of beam breakup (or pulse shortening) was first observed about 1957 by the then principal industrial fabricators of microwave electron accelerators. For obvious commercial motives the effect and studies on its cause were somewhat confidential until about 1960 when a formal description appeared in the technical literature.(1)

The effect may be described as a progressive shortening of the output beam pulse compared to the injected pulse length with increasing beam current, the pulse length reduction occurring from the later end of the pulse. Transmitted beam current is shown as a function of pulse length in Fig. I for several machines to provide an idea of the magnitudes involved.<sup>(2)</sup>

To explain a mechanism capable of preventing a moderately high energy beam transiting the waveguide, scattering by residual gas and the consequences of ionization of that gas were re-examined with negative conclusions.<sup>(3)</sup> What seemed to be required was the existence of transverse fields which could impart to the beam a ratio of transverse to axial momentum of the order of magnitude of the ratio of the iris aperture to the waveguide length; for this reason space charge forces were ruled out also. The obvious remaining suggestion was higher order modes of the TM type. In the reference frame of the stationary electron a synchronous TM wave constitutes an electrostatic field; a TE wave becomes a magnetostatic field which cannot deflect a stationary electron. Further, it appeared reasonable that the deflection would require fields over many cavities, that is a wave near synchronism with the beam.

In retrospect, particular observations were clues to the nature of the process, although largely ignored at the time: (1) pulse shortening commenced from the end of the pulse indicating a build-up time, the length of the remaining pulse being the build up time. Thus in Fig. 1, reasonably good fits to the data are of the form  $i(1 - e^{-t/\tau}) = \text{Const.}$  where  $\tau$  is characteristic of the waveguide. (2) Several short beam pulses could be accelerated within the same RF pulse if sufficient spacing was maintained between them, indicating a decay of the deflecting fields. (3) A solenoidal field delayed on-set of the shortening process, and the missing segment of the pulse could be steered into the beam line, although the originally unaffected segment of the pulse was then lost. There were many other observations, the detailed recital of which is no longer interesting, such as placing gamma ray detectors along the length of the waveguide to learn where the beam crashed into the structure.

Specifically, the TM-11 like mode (later referred to as the hybrid electromagnetic or HEM-11 mode, during the development of RF particle separators) seemed to be a likely candidate although there were problems in an explication, principally that of explaining how charge bunches at the velocity of light (and one-wavelength spacing for the acceleration mode) could induce the TM-11 mode of different propagation constant. Also, the reasoning seemed circular; the mode exists because of beam deflection (excitation by an off-axis beam or misaligned waveguide) and the mode produces beam deflection. Such situations generally involve questions of stability and thresholds (below which nothing occurs). Similar problems had arisen in the analysis of backward wave oscillators, where it appeared that the bunched beam would collect at alternate field zeroes and not transfer power to the wave, but phase slip emerged as the applicable mechanism; the similarity of the two processes led R. L. Kyhl to an analysis, announced at the 1960 Amsterdam conference which confirmed the general opinion that pulse shortening was the result of higher order mode generation although no direct evidence was presented.<sup>(4)</sup> Shortly thereafter the Metrovik (Manchester) group provided some direct evidence that a higher order mode was indeed being generated by the beam;<sup>(5)</sup> similar observations were also made at Tokyo<sup>(6)</sup> and subsequently at Stanford.<sup>(7)</sup> This, of course, stimulated further investigation of higher order modes in disk-loaded guides<sup>(8)</sup>; studies had already been undertaken at Stanford and by other groups interested in employing the TM-11 mode for particle separators.<sup>(9)</sup>

Meanwhile in 1959, at the suggestion of R.F. Post on the basis of scaling arguments, Applied Radiation Corp. developed a prototype L-band accelerator with the improvement shown in Fig. 1. The scaling argument was essentially that (1) at lower frequencies the electrical length of a waveguide designed to provide a specified beam power conversion efficiency is less as well as that of the higher modes and (2) the characteristic build-up time for a resonant disturbance is substantially greater as a result of higher Q.

The fashion in "things to worry about" was then shifting from multipactoring to beam breakup and consequently numerous papers on the effect appeared<sup>(10)</sup> as well as schemes to avoid diaster.<sup>(11)</sup>

The completion of the 3 km accelerator at SLAC in 1965 occasioned a new and unanticipated aspect of beam breakup. Constant gradient waveguide, it was thought, by its design (varying dimensions) would mess up the propagation characteristics of higher order modes so as to preclude coherent interaction, but trapped resonances in the waveguide provided in each waveguide of a multi-section machine transverse thrusts for which an impulse approximation was presented by W. K. H. Panofsky.<sup>(12)</sup> On the Stanford Mark III the output beam pulse was often notably much shorter than the injected pulse but the effect was attributed to waveguide arcing or 'lateral deflections' and largely ignored, perhaps justifiably considering the then state of linac technology.<sup>(13)</sup> This beam instability had also been seen in the Kharkov machine about 1963, although the report was not widely advertised.<sup>(14)</sup>

This latter "cumulative" effect is distinguished from the earlier "regenerative" breakup by a different growth process, although both effects arise from excitation and interaction with the HEM-11 mode. In the regenerative case it is supposed that an HEM-11 wave exists in the waveguide, excited by an as yet unspecified mechanism; if the beam bunch packets are initially in phase with the transverse Efield component of the mode they will be deflected. The frequency component of the pass-band which represents a  $\pi$  phase shift or slip with respect to the beam bunches over the excited length of the waveguide will then be amplified because the deflected charge packets will eventually be in the axial E-field component of the mode. In addition, because the HME-11 mode is a backward wave for a diskloaded waveguide useful for acceleration in the TM-01 mode the RF power induced by the beam will travel oppositely to the electron motion, further strengthening the deflecting field.

A charged particle beam traversing a cavity or waveguide will see a shunt impedance corresponding to the field components along its trajectory of every mode that will fit in the structure (that is, satisfy the boundary conditions); consequently, it will deliver energy into every mode (consistent with transit time effect) which will then build up until the power delivered by it is equal to the mode losses in the structure. It is on this basis that P. Wilson calculated the threshold current for start of oscillation of the HEM-11 mode (TM branch) in an idealized (pure mode) of the disk-loaded waveguide with the result which may be stated<sup>(15)</sup> roughly as

 $I_{s} \stackrel{\circ}{=} \frac{V_{f}}{r} \frac{\beta \ell}{(\beta \ell)^{3}}$ 

where  $V_f$  is the final or extrant energy, r is the mode shunt impedance,  $\beta$  its propagation constant, 2b the cavity diameter and  $\ell$  the interaction length; it is assumed that the waveguide is an accelerator ( $V_f >> V_0$ ).

Owing to the multiplicity of possible modes in any structure it is fortuitous, and felicitous, that accelerator builders, concentrating upon an intended mode, did not get into greater difficulty than they have. In retrospect, they were of course saved by transit time effects and the inability to produce large beam currents.

The elegant formalism of Y. Garrault(16) describing the transverse force exerted by the field on a particle,

$$F_{n} = e \left[ (1 - \frac{V_{o}}{V_{pn}}) E_{nt} + j \frac{V_{o}}{\omega} \nabla_{t} E_{nz} \right] \exp(\omega t - \beta_{n} z)$$

where  $V_0$  is the particle velocity,  $V_{\rm PR}$  the phase velocity of the space harmonic  $(V_{\rm PR}$  =  $\omega$  / $\beta$ \_R) demonstrates why induced TE modes do not generally deflect particles; in the process of exciting such modes any transverse momentum of the particle will be spent in the energy transfer and thereby damped out.

In the cumulative case it is supposed that the same sort of HEM-11 excitation exists in each waveguide of a multisection accelerator, but in this case it is also supposed that the excitation is a trapped TM110-like mode either in a small length of constant gradient waveguide or over the whole length of a constant impedance waveguide. The bunch phase for maximum amplification is  $\pi/4$ , i.e., between the maximum of transverse and longitudinal field. The beam bunches not only receive a slight transverse "kick" in each section, but also transmits this excitation from one waveguide to the next so that eventually the displacement modulation grows until the beam is driven into the wall of the structure. In this case, whether the mode is backward wave is incidental and of no importance.

The model of cumulative beam breakup proposed by W. Panofsky, in which a small interaction arising in each waveguide of a long machine (each sections being treated as a singly resonant cavity) will when repeated over many sections result in a large overall gain, has been investigated further by R. Helm (SLAC).<sup>(17)</sup>

The proliferation of superconducting accelerators since about 1970 has led to some apprehension that in multisection machines or as a consequence of beam recirculation, breakup would be certain to occur since the displacement amplification ( $e^{F}$ ) scales as  $\sqrt{1Q}$ . This has of course resulted in several disquisitions<sup>(18)</sup>, from which it appears that there is many more possibilities for beam instability than in the ambient temperature case.

The problem of excitation of the waveguide in a higher order mode by means of a beam having the microstructure (periodicity) of the acceleration frequency mode has been mentioned. The possibility of excitation by noise has been considered both theoretically and experimentally with somewhat inconclusive results. This has led to further examination of forced oscillation. The differential equation describing forced vibration is of the form

$$\ddot{y} + \frac{\omega_o}{Q} \dot{y} + \omega_o^2 y = f(t)$$
<sup>(1)</sup>

where the constants  $\omega_0$  and Q are the autoperiodic response and loss factor, respectively, of the system, derived in the free vibration case (f(t) = 0). If  $\omega_0/2Q = 0$  Eq.(1) defines a steady vibration; if  $\omega_0/2Q$  is positive the vibrations will damp down and if  $\omega_0/2Q$  is negative they will increase without limit.

A well-known method of fitting the solution of a differential equation to initial (or boundary) conditions is the Green's function technique. (Morse and Feshbach Ch 7; i, 791)

Thus, in the case of Eq. (1) with  $\dot{y}(o) = y(o) = 0$ 

$$y = \frac{1}{\omega'} \int_{0}^{t} e^{-\frac{\omega_{0}}{2Q} (t-t')} \sin \omega'(t-t') f(t') dt'$$
 (2)

is such a solution, where

$$\omega' = \omega_0 \sqrt{\frac{1}{\sqrt{1 - (\frac{1}{2Q})^2}}}; (2Q) > 1.$$

An extensive literature exists on solutions of Eq. (1). It is well-known that the response of a system to a periodic force-function is that of the forcing periodicity; the selfperiodic response of the system damps out quickly regardless of the loss factor. In the case where the forcing function is periodic and Fourier-analyzable ( $f(t) = \sum a_n \cos n$  wt) it is conceivable that a harmonic resonance in the higher order mode could induce an appreciable response. On the other hand, wavelength and propagation constants at the velocity of light have the relation

$$\frac{\lambda (\text{HEM}-11)}{\lambda (\text{TM}-01)} = \frac{\beta p (\text{TM}-01)}{\beta p (\text{HEM}-11)}$$
(3)

For example, when accelerating in the  $2\pi/3$ -mode at a specified frequency the velocity of light line will intersect the HEM-11,  $\pi$  mode; for  $\pi/2$  acceleration mode intersection occurs at the HEM-11,  $3\pi/4$  mode. In either case the induced frequency will be 3/2 the acceleration frequency, which was apparently noticed by J. Leiss.<sup>(19)</sup>

R. Cooper (LASL) has formulated a computer program for Eq. (1) where f(t) is a quasi-delta function of charges, radially off-set in a cavity (to couple to TM-mno modes) and finds that after a brief transient response (about 100 pulses) a substantial steady state response is obtained.<sup>(20)</sup> Physically it appears that a shock excitation from each pulse will induce a response in the cavity. Cooper's solution is done in the time domain, which thereby obscures the underlying mechanism which is supposedly that frequencies in the Fourier spectrum of the force function will excite the higer order modes in the structure. The interested reader will find a rather fulsome discussion of the present subject in Refs. (21).

The writer is pleased to acknowledge useful discussion with Dr. R. Kennedy of Boeing Radiation Effects Laboratory on the topic of beam breakup.

## References

- J. Boag et al., Proc. Second U.N. Int. Conf. Peaceful Uses of Atomic Energy, Geneva (1958) 14, 437 M. Kelliher et al., Nature, 187,1099 (1960).
- (2) Some additional data was provided by J. Haimson, IEEE Trans. Nuc. Sci., NS-12, 996 (1965).
- (3) K. Kerris et al., Studies to Improve Radiation Simulation Techniques, Contract NONR-3481(00) Hughes Aircraft Co., Fullerton, Calif. (1963) and Appendix A. Other unpublished analyses at Varian Assoc., Arco, Vicker's Research, Tokyo-Shibaura and Stanford Univ. also corroborated those results.
- (4) J. C. Nygard et al., Nuc. Instr. Meth. 11,126 (1961). The basis of the argument in this reference was given by Kyhl in confidential HVEC (Burlington, Mass.) reports during 1959; the writer has made frequent use of those reports here.
- (5) M. C. Crowley-Milling et al., Nature 191, 483 (1961).
- (6) H. Hirakawa, Japanese J. Appl. Physics 3, 27 (1964).
- (7) O. Altinmueller et al., Proc. Lin. Acc. Conf. (1966) LASL, Rep. LA-3609, 267.
- (8) G. Saxon et al., Proc. IEEE 110, 1365 (1963). Linear Electron Accelerator Studies, Status Report ML-581 (1959). P. Fardeau, Thesis; ENS, Orsay (1962).

- (9) H. Hahn, R.S.I. 34, 1094 (1963). K. Brown et al., Proc. Int. Conf. High Energy Acc. BNL (1961) p. 79. M. Bell et al., Nature 198, 277 (1913). Y. Garrault, C. R. (Paris) 254, 843; 254, 1391 (1962); 255, 2920 (1962); 256, 3268 (1963).
- (10) G. Chang, M.Sc. Thesis U.C. Berkeley (1963) summarized in an appendix to Ref. (5). J. Bjorkhom et al., IEEE Trans. Nuc. Dev. ED-12, 281 (1965). E. Chu, SLAC TN-66-17 (1966). M. Bander, SLAC TN-66-28 (1966). E. Farinholt et al., Proc. to the Int. Conf. High Energy Acc., Cambridge, Mass. CEAL-2000 p.90 (1967). R. Gluckstern, Proc. MURA Conf. Linear Acc., Stroughton, Wisc. (1964) and Int. Rep. AADD-38, BNL (1964). R. GLuckstern et. al., Proc. Lin. Acc. Conf. LASL Rep. LA-3609 p.281 (1966). P. Wilson, HEPL Report 297 Stanford Univ. (1963) and Revision A.
- J. Haimson, Ch. 3.2, Linear Accelerators (Eds. P. Lapostolle and A. Septier) Amsterdam (1970) p. 415 et seq. W. Bertozzi et al., IEEE Trans. Nuc. Sci., NS-14, 191 (1967).
- (12) W.K.H. Panofsky, SLAC TN-66-27 (1966), RSI 39,206 (1968) R. Neal et al., Science, 152, 1353 (1966).
- (13) M. Chodorow et al., Stanford High-energy Linear Electron Accelerator, ML Report 258 (1955) and RSI 26, 134 (1955) p.202.
- (14) A. Valter et al., Fifth Int. Conf. High Energy Acc. Frascati p. 233 (1965).
- (15) P.B. Wilson, HEPL-297 (1963), HEPL-TN-67-6 (1967) Stanford Univ.
- (16) Y. Garrault, Hybrid EH Guided Waves, Advances in Microwaves, 5 (ed. L. Young) Academic Press (1970).
- (17) R. Helm, Computer Study of Wave Propagation, Beam Loading and Beam Blowup in the SLAC Accelerator, Proc. Lin. Acc. Conf. LASL Report LA-3609 p. 254 (1966).
- (18) R. Helm, Preliminary Estimate of Beam Blowup for a Superconducting Electron Linac, SLAC-TN-67-6/HEPL-TN-67-3 Stanford (1967). P. Wilson, Beam Breakup in Superconducting Linacs, HEPL-TN-67-2 Stanford Univ. (1967). K. Mittag et al., Beam Breakup in a Superconducting Electron Accelerator, HEPL-685 (1972) Stanford and Proton Lin. Acc. Conf., Los Alamos (1972). R. E. Rand et al., HEPL-889 (1981) Stanford Univ. and refs. therein.
- (19) J. Leiss, Minutes Conf. Proton Linear Acc. Yale 1963, p. 74.
- (20) R. K. Cooper (LASL), private communication (1982). See also V. Neil et al., Further Theoretical Studies of the Beam Breakup Instability, Particle Accelerators 9, 213 (1979).
- R. Helm, G. Loew and WKH Panofsky ch 7, p. 163. The Stanford Two-Mile Accelerator, W. Benjamin Inc., N.Y. (1968). R. Helm and G. Loew Ch B.I Eds. P. Lapostolle A. Grivet, North Holland Pub. Amsterdam (1966) p. 173.

DESIGNATION	L	-	¥g/c	1	1	SOURCE
H, Hughes Aircraft Co.	0.9=	47 <b>Β</b> Ω/m		.3 n/m	2856(17/2)	(1)
¥. Varian Assoc.	1.6	50	.01	.3	2856(217/3)	Unpub.
T, Takya Univ.	1.1	33			2856(1/2)	(2)
Al. Applied Rad, Co.	1.21	39	.0025	.07	1300(2m/3)	Unpub.
A2, Applied Rad. Co.	3	40			2856(17/2)	Unpub.

 Kerris, et al., Studies to Improve Radiation Simulation Techniques, ONR Report Contr. HOYR-3481(00) Hughes Aircraft Co., Fullerton, CA.

(2) H. Hirakawa, Jap. Journ. Applied Physics 3, 27(1964).



SINGLE SECTION REGENERATIVE BEAM EREAKUP CHARACTERISTICS

## 2660