LONGITUDINAL PHASE SPACE MATCHING BETWEEN MICROTRONS AT 185 MEV*

H. Takeda**
Argonne National Laboratory
Argonne, Illinois 60439

Summary

Electrons are accelerated to 185 MeV by a microtron. Then, they are injected into another microtron to boost the net energy up to a few GeV. Between the two microtrons both longitudinal and transverse phase space matching are required.

In this paper, we consider a longitudinal phase ellipse matching which utilizes triple left-right-left sector dipoles to induce a negative phase angle shear. This is accomplished because a high energy particle travels a shorter distance through the dipole system than a low energy particle.

Introduction

As with the transverse phase space ellipse, matched acceptance longitudinal phase space ellipses are defined by Twiss parameters at each point in a microtron characterized by an energy gain per turn and a synchronous angle. The energy width of the ellipse is proportional to the square root of energy gain per turn. Since the energy gain of 50 MeV by two linacs in the Double Sided Microtron (DSM) is eight times that of the Race Track Microtron (RTM), the energy width of the ellipse for the DSM is approximately three times greater than that of the RTM at the center of linac.

The beam is extracted at the return path of the RTM at 185 MeV, passes through the longitudinal and the transverse matching section, and is injected into the short straight section (SSS) of the DSM. Due to its symmetry, both acceptance ellipses are erect at the extraction point and at the injection point. But, they differ in phase-angle-width and energy-width for the same phase area.

The phase slip in the matching cavity and the longitudinal phase shear due to the velocity spread of the longitudinal ellipse at the matching section are negligible for the relativistic electron beam. Also the transverse matching quadrupole magnets are orthogonal to the longitudinal phase space. Hence, the longitudinal phase ellipse is matched to a first order only by dipoles for the phase angle shear and by a matching cavity for the energy shear.

It is generally considered that a sector dipole shears toward the positive phase direction for a positive energy error. If a matching system consists of only the positive phase angle shear element and an energy shear element, a strong energy shear toward the negative direction for a positive phase error is required. Thus, the longitudinal phase ellipse can be matched by a following positive phase angle shear. A disadvantage of this method is that the strong energy shear coupled with the phase angle jitter of the matching cavity induces a large energy smear from pulse to pulse.

However, with a negative phase angle shear system and a pre-positive phase angle shear in the beam upstream, only a weak matching cavity is required. So the phase ellipse can be better matched.

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**Current address is KMS Fusion, Inc., Ann Arbor, MI 48106

Right and Left Bend Dipole Matrix

Defining dT and dt as off-momentum and on-momentum differential trajectories, the path length difference between the trajectories in a system containing deflecting magnets is expressed as

$$\Delta\ell = \int_0^t dT - \int_0^t dt = \int_0^t x/\rho \; dt.$$

The transverse coordinate (x) on a bend plane is defined as positive if a particle is to the left of the reference trajectory. If the beam bends to the right, the radius of curvature ρ has a positive value. The reference trajectory path length (dt) and the bending angle ($d\theta$) are related as

$$dt = |\rho| d\theta \quad , \; d\theta > 0 \tag{1}$$

The transverse coordinate (x) of a particle along the trajectory is expressed as[1]

$$x(t) = C(t)\, x_0 + S(t)\, x_0' + D(t)\Delta p/p \tag{2}$$

Substituting Eq. 2 into Eq. 1, the path length difference may be expressed as

$$\Delta\ell = x_0 \int_0^t C(t)/\rho \; dt + x_0' \int_0^t S(t)/\rho \; dt + \Delta p/p \int_0^t D(t)/\rho \; dt \tag{3}$$

Using Eq. 3, the full matrices including the path length difference term in the deflecting plane x are obtained for a left bend sector dipole and a right bend sector dipole.

$$D_R = \begin{bmatrix} \cos r & R\sin r & 0 & R(1-\cos r) \\ (-1/R)\sin r & \cos r & 0 & \sin r \\ \sin r & R(1-\cos r) & 1 & R(r-\sin r) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \\ \Delta\ell \\ \Delta p/p \end{bmatrix},$$

$$D_L = \begin{bmatrix} \cos i & R\sin i & 0 & -R(1-\cos i) \\ (-1/R)\sin i & \cos i & 0 & -\sin i \\ -\sin i & -R(1-\cos i) & 1 & R(i-\sin i) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For a single magnet, a higher momentum particle travels over a longer path length since the ($\Delta\ell$, $\Delta p/p$) element is positive.

Negative Phase Angle Shear by a Left+Right+Left Bend(LRL) System

If the beam transport section, which includes the negative phase angle shear system, is maintained near achromatic, the first and second terms in Eq. 3 for the transport section may be made negligible compared to the third term.

This dispersion dependent term is evaluated by multiplying the three sector dipole matrices for equal bend radii (R) and equal right and left bend angles (r).

$$(\Delta\ell \; , \; \Delta p/p) = f(r)*R$$

where

$$f(r) = \sin r(1-\cos r)(3*\cos r-5)+\sin^3 r+3(r-\sin r) \tag{4}$$

As Figure 1 shows, the f(r) becomes negative near

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$r = 50^{0}$ and reaches a minimum near $r = 120^{0}$. Then it increases rapidly and becomes positive.

When we separate the dipole magnets by equal drift (d), it contributes to the geometrical function f(r) by

$$f_{dr} = (d/R)\sin^{2} r\{4\cos r - 4 - (d/R)\sin r\} \quad (5)$$

The relation between angle of phase shear and difference in path length is

$$\Delta\phi = + 2\pi/(\lambda\beta) * \Delta\ell = 2\pi/(\lambda\beta) * R * f * \Delta p/p \quad (6)$$

where λ and β are linac wave length and particle velocity in speed of light units. The negative geometrical function implies a negative phase shear.

Longitudinal Phase Space Matching

Upstream of the matching cavity and dipole system, we induce a large phase angle shear. This is done by displacing the return path at 185 MeV in the RTM so that the electrons, after the 180^{0} bend through the dipole, run parallel to the linac. Then the beam is once more positively phase angle sheared by another dipole followed by extraction. As will be explained, this pre-phase angle shear further reduces the required energy shear of the matching cavity.

Here, we formulate the matching procedures and obtain the matching conditions. In general, any dipole system can be expressed as a matrix

$$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$$

which operates on a vector consisting of a phase angle error and an energy error.

$$\begin{pmatrix} \Delta\phi \\ \Delta W \end{pmatrix}$$

A non-phase slip linac cavity is idealized as

$$\begin{pmatrix} 1 & 0 \\ L_{lin} & 1 \end{pmatrix}$$

Let us define D_1 as a two 180^{0} bend system and D_2 as a negative phase shear system. With matching cavity strength (L_{lin}), the longitudinal phase ellipse transformation matrix from RTM through the negative bend system is given as

$$T = D_2 L_{lin} D_1 = \begin{pmatrix} 1 & k_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ L_{lin} & 1 \end{pmatrix}\begin{pmatrix} 1 & k_1 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+k_2 L_{lin} & k_1+k_2+k_1 k_2 L_{lin} \\ L_{lin} & 1+k_1 L_{lin} \end{pmatrix}$$

Since the longitudinal ellipse at the RTM return path is errect, the ellipse is expressed as

$$(\delta\phi)^2 + (\beta_0 \delta W)^2 = \beta_0 \varepsilon_0$$

where

$$\sigma_0 = \begin{pmatrix} \beta_0 & 0 \\ 0 & \gamma_0 \end{pmatrix}, \quad \gamma_0 = 1/\beta_0$$

From the transverse phase space analogy, the longitudinal ellipse at the DSM is obtained from

$$\begin{pmatrix} \beta_2 & -\alpha_2 \\ -\alpha_2 & \gamma_2 \end{pmatrix} = T\sigma_0 T^{T}$$

Thus the ellipse parameters at SSS of the DSM is expressed as

$$\beta_2 = (1+k_2 L_{lin})^2 \beta_0 + (k_1 + k_2 + k_1 k_2 L_{lin})^2 \gamma_0, \gamma_0 = 1/\beta_0$$

$$\gamma_2 = L_{lin}^2 \beta_0 + (1 + k_1 L_{lin})^2 \gamma_0 \quad (7)$$

$$\alpha_2 = -(1 + k_2 L_{lin})\beta_0 L_{lin} - (k_1 + k_2 + k_1 k_2 L_{lin})\gamma_0 * (1 + k_1 L_{lin})$$

The ellipse parameters immediately after the cavity are

$$\beta_i = \beta_0 + k_1^2 \gamma_0$$

$$\gamma_i = L_{lin}^2 \beta_0 + (1 + k_1 L_{lin})^2 \gamma_0 \quad (8)$$

$$\alpha_i = -\beta_0 L_{lin} - k_1 \gamma_0 (1 + k_1 L_{lin})$$

The phase ellipse is given by

$$(\delta\phi)^2 + (\alpha_t \delta\phi + \beta_t \delta W)^2 = \beta_t \varepsilon$$

Since the dipole system shears only the phase angle, the half energy width of the ellipse must match the DSM (δW_g).

$$\gamma_i = \delta W_g^2/\varepsilon$$

Let us solve the second equation in (8) for L

$$L_{lin} = \frac{-k_1/\beta_0 + \sqrt{(k_1/\beta_0)^2 - (\beta_0 + k_1^2/\beta_0)(\gamma_0 - \gamma_i)}}{\beta_0 + k_1^2/\beta_0}$$

A positive sign is chosen for a positive energy kick. The relation $L_{lin} \sim 1/k_1$ implies that the larger shear by the two 180^{0} dipoles, the smaller the matching cavity strength.

At the current state of the art, phase angles down to 1^{0} can be controlled for an s-band linac. The energy jitter due to phase jitter is

$$\varepsilon_{jit}(\mathrm{MeV}) = \Delta W_{kick} \Delta\phi\,(o) = \pi/180 * L_{lin}(\mathrm{MeV})$$

Since energy jitter is proportional to L_{lin}, the smaller matching cavity creates less phase jitter.

The strength (k_2) of the triple dipole system is obtained from solving the third of Eq. (7) setting $\alpha_2 = 0$.

$$k_2 = \frac{-k_1\gamma_0(1 + k_1 L_{lin}) - \beta_0 L_{lin}}{\beta_0 L_{lin}^2 + \gamma_0(1 + k_1 L_{lin})^2}$$

From Eq. (6), $\delta W/W \simeq \Delta p/p$ and the definition $k_2 = \delta\phi_{shear}/\delta W$, the bending radius (R) of the triple dipole is given as

$$R = k_2 \frac{\lambda W}{2\pi f}$$

Example

For a system specified in Table 1, we calculate the parameters which match the longitudinal phase ellipses. They are shown in Table 2.

Table 1. Set-up Parameters.

ϕ_s = 20 (Deg) : RTM Synchronous Angle

β_0 = 0.4532 (Rad/MeV)

$\pi\varepsilon$ = 30π (KeV Deg)

$k_1 = 2\pi/6(\text{MeV}) = 1.047$ (Rad/MeV)
$\delta W_g = 88.9$ (KeV)
$r = 80$ (Deg)

Table 2. Calculated Results.

$L_{lin} = 1.4612$ (MeV)
$k_2 = -0.4312$ (Rad/MeV)
$\varepsilon_{jit} = 25.5$ (KeV)

R (cm)	d/R	B (kG)
50	0.4702	12.34
60	0.3372	10.29
70	0.2365	8.83

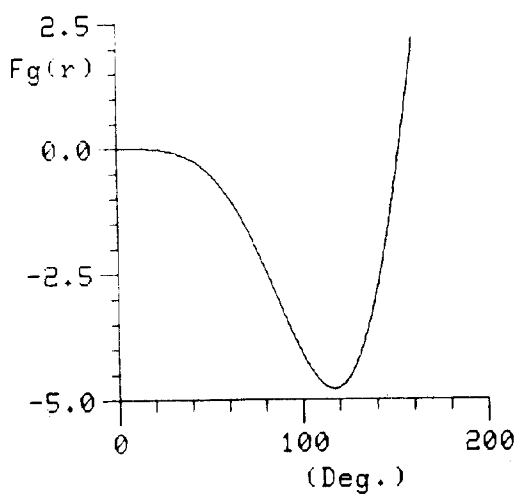

Fig. 1. Geometrical Factor (No Drift)

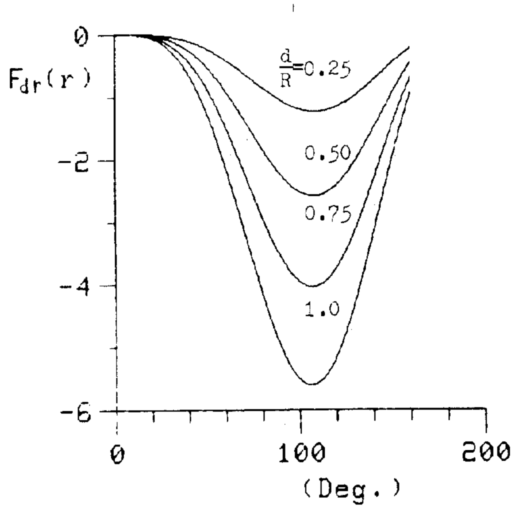

Fig. 2. Drift Geometrical Factor.

References

1. Klaus G. Steffen, High Energy Beam Optics, John Wiley & Sons, New York (1965).