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THEORY OF BUNCHED BEAM STOCHASTIC COOLING

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Abstract

A theoretical description of the time-evolution of a bunched beam subjected to stochastic cooling is provided.

1. Introduction

Bunched beam stochastic cooling differs non-trivially from continuous coasting beam cooling[1,2,5]. A primary manifestation of the differences is a qualitatively distinctive frequency-space structure of the spectrum of incoherent and collectively modulated Schottky signals derived from, and experienced by the particles in a bunch. We start then with a study of the bunched beam Schottky signal. In the following, $x_j = A_j \cos(Q\omega_0 t + \phi_j{}^0)$ represents the linear betatron oscillation and

$$\theta_j = \omega_0 t + a_j \sin\left[\omega_s(a_j)t + \psi_j{}^0\right] = \omega_0\left[t - \tau_j(t)\right]$$

represents the quasilinear synchrotron oscillation, with the usual meaning of the symbols. The particle index j in the argument of a function f(j) will denote a part or the whole of the complete set of action-angle variables of particle j : $(I_j;\psi_j) = (I_x{}^j, I_z{}^j, J^j;\ \phi_x{}^j, \phi_z{}^j, \psi^j)$ with $I_{x,z} = (\frac{1}{2})A^2_{x,z}$ and $J = (\frac{1}{2})a^2$. The dependence of synchrotron oscillation frequency on amplitude for non-linear oscillations is expressed by the function $\omega_s(a_j)$.

2. Spectral properties of bunched beam Schottky signals

Schottky signals derived from a distribution of particles in a bunch on repeated traversals through a localized PU at azimuth $\theta = \theta_p$, have the following spectral representations[1,2]:

$$d(t) = qf_0 \sum_{j=1,(m,\mu)=(-\infty),(\pm)}^{N,(+\infty)} (A_j/2)\, J_\mu\left[(m \pm Q)a_j - Q(\xi/\eta)a_j\right] \times e^{i\Omega^{(\pm)}_{m,\mu}(j)t \pm i\phi^0_j + i\mu\psi^0_j - im\theta_p} \tag{1}$$

for the transverse dipole moment signal and

$$I(t) = qf_0 \sum_{j=1,(m,\mu)=(-\infty)}^{N,(+\infty)} J_\mu(ma_j)\, e^{i\Omega_{m,\mu}(j)t + i\mu\psi^0_j - im\theta_p} \tag{2}$$

for the longitudinal current signal. The relevant frequencies are: $\Omega_{m,\mu}(j) = m\omega_0 + \mu\omega_s(a_j)$ for longitudinal and $\Omega^{(\pm)}_{m,\mu}(j) = (m \pm Q)\omega_0 + \mu\omega_s(a_j)$ for transverse signal. Here q is the charge and $\omega_0 = 2\pi f_0$ the central angular frequency of the particles, ξ and η the chromaticity and the off-energy function respectively $(\Delta Q/Q_0 = \xi(\Delta p/p_0) = (\xi/\eta)(\Delta f/f_0))$ and $J_\mu(x)$ an ordinary Bessel function of order μ. The spectrum analyzer records two betatron bands centered around $(m + Q)f_0$ and $(m - Q)f_0$ per revolution band in real positive frequency.

For initial betatron phases ϕ^0_j randomly distributed between 0 and 2π, one easily verifies[1] that the transverse dipole moment signal satisfies $\langle d_{(m\pm Q)}\rangle = 0$ and $\langle|d^2_{(m\pm Q)}|\rangle = (N/2)q^2 f_0^2 \langle A^2\rangle$, same as for a coasting beam, where <> denotes average over the distribution of particles. The spread in the betatron and longitudinal sidebands are given by $\Delta\Omega^{(\pm)}_{m,\mu} = \mu\Delta\omega_s \pm Q_0(\xi/\eta)\Delta\omega$ and $\Delta\Omega_{m,\mu} = \mu\Delta\omega_s$ respectively. Since $J_\mu(ma)$ has significant magnitudes only for $\mu \lesssim ma$, the synchrotron side-band spectrum extends up to $\mu_m\omega_s(0) \sim ma_m\omega_s(0) = m\Delta\omega(a_m)$ where a_m is the maximum synchrotron amplitude in the bunch. The total spread in revolution harmonic band m thus approaches the value for a coasting beam with frequency spread $\Delta\omega$: $\Delta\Omega_m = m\Delta\omega$ and $\Delta\Omega^{(\pm)}_m = m\Delta\omega \pm Q_0(\xi/\eta)\Delta\omega = (m\eta \pm Q_0\xi)\omega_0(\Delta p/p_0)$. The profile of the Schottky band at a given harmonic mf_0 duplicates the longitudinal velocity distribution of the bunch. For low revolution harmonics m, the noise density of synchrotron side-bands is enhanced by $\Gamma_\mu = \omega_s/(\mu\Delta\omega_s)$ compared to a coasting beam, until the side-bands overlap, i.e. $\Gamma_\mu \lesssim 1$, for large m. In this overlapped region, different particles with different oscillation amplitudes generate the same frequency Ω through different synchrotron harmonics[1]: $\Omega = m\omega_0 + \mu\omega_s(a) = m\omega_0 + \mu'\omega_s(a') = \dots$ etc.
For still higher harmonics m, even the revolution bands start to overlap, i.e. $\Omega = m\omega_0 + \mu\omega_s(a) = n\omega_0 + \mu'\omega_s(a') = \dots$ with $m \neq n$, $\mu \neq \mu'$, $a \neq a'$. For the longitudinal signal, an essential difference in the distribution of power is that $\langle I_m\rangle = qf_0N \neq 0$, and more importantly

$$\left\langle |I^2_m| \right\rangle = q^2 f_0^2 \left\{ N + \left[\sum_{j=1}^{N} J_0(ma_j)\right]^2 \right\} \tag{3}$$

Thus the $\mu = 0$ central bands add up coherently $\left(O(N^2)\right)$ as opposed to the $\mu \neq 0$ bands which add up incoherently (Schottky noise power $\propto$ N) in the mean square. It is implicit in any bunched beam cooling scheme that the central coherent longitudinal lines, undesirable for purposes of cooling incoherent motion, be removed by suitable techniques, which is a non-trivial task. The electric field at the kicker, Fourier transformed to frequency domain, is given by [from Eq. (1)]

$$\tilde{E}^0(\Omega) = qf_0 \sum_{j=1,(m,\mu)=(-\infty),(\pm)}^{N,(+\infty)} (A_j/2)\, J_\mu\left[(m \pm Q)a_j - Q(\xi/\eta)a_j\right] \times \tilde{G}\left(\Omega^{(\pm)}_{m,\mu}(j)\right) e^{i\mu\psi^0_j \pm i\phi^0_j - im\theta_p}\,\delta\left(\Omega - \Omega^{(\pm)}_{m,\mu}(j)\right) \tag{4}$$

and similarly for longitudinal voltage signal[1] from Eq. (2). The transfer function $\tilde{G}$ above includes the amplifier gain as well as PU and kicker transverse impedances.

The above analysis assumes no correlations between particle trajectories. The cooling feedback loop itself introduces correlations between particles, which are propagated by the beam back to the pick-up which thus sees the Schottky signal distorted away from the uncorrelated form. A Vlasov analysis[1,3] describes the actual collectively modulated signal $\tilde{E}(\Omega)$ at the kicker as follows:

$$\tilde{E}(\Omega) = \sum_{k=-\infty}^{(+\infty)} D_k(\Omega)\tilde{E}(\Omega + k\omega_0) + \tilde{E}^0(\Omega) \tag{5}$$

where $D_k(\Omega)$ is a complicated kernel[1,3]. In the situation of no synchrotron band overlap, the effect can be approximately described by a local signal suppression factor $\varepsilon_\mu{}^{(\pm)}(a)$ for each synchrotron mode μ separately as[1,3]

$$X^{(\pm)}_\mu(a) = X^{(\pm)^0}_\mu(a)/\varepsilon^{(\pm)}_\mu(a) \tag{6}$$

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where

$$X_\mu^{(\pm)}(a) = \sum_n J_\mu\left[(n \pm Q)a - Q\frac{\xi}{\eta}a\right]\tilde{E}\left(\Omega_{m,\mu}^{(\pm)}(a)\right\} e^{-in\theta_k}$$

and

$$\varepsilon_\mu^{(\pm)}(a) = 1 + \left(q^2\omega_0/2(2\pi)^2Q\right)\left(\pi N f_0(a)/|\mu|\left|\frac{d\omega_s}{da}\right|\right)$$
$$\times \sum_{m=-\infty}^{+\infty} \tilde{G}\left(\Omega_{m,\mu}^{(\pm)}(a)\right) J_\mu^2\left[(m \pm Q)a - Q(\xi/\eta)a\right] \quad (7)$$

and a similar factor $\varepsilon_\mu(a)$ for the longitudinal signal suppression[1,3]. A general inversion of the coupled mode Eq. (5) in the situation of synchrotron band overlap for high frequencies has not been obtained yet. However, except for the coherent $\mu = 0$ central longitudinal lines, the spectral properties of bunched beam Schottky signal at high frequencies become indistinguishable from a coasting beam signal. Information about the bunched nature of the beam is however retained by the correlated Schottky harmonic structure, expressed by summation over m in Eq. (7), of the overall gain experienced by a particle. Same synchrotron mode μ in neighbouring revolution bands m remain correlated owing to the same phase exp $(i\mu\psi^0)$ and similar strengths $J_\mu(\ell a)$ [see Eq. (2)] until $\ell a \geq \pi$. This correlation thus dominates over a range in ℓ of π/a which is the bunching factor[1,2,3]. Equivalently, we may interpret this as an enhanced effective number of particles $N_{eff} = N(T_0/t_0) = N\pi/a_m$ where t_0 is the bunch duration. We thus expect a coasting beam like suppression factor[4]

$$\varepsilon[(n \pm Q)\omega] = 1 + \frac{N_{eff}\,G[(n \pm Q)\omega]}{|n \pm Q|}\int_{\eta\to 0^+} d\omega'\,\frac{f(\omega')}{\eta \pm i(\omega - \omega')}$$
$$\simeq 1 + \frac{N_{eff}\,G[(n \pm Q)\omega]}{4\Delta|n \pm Q|} \quad (8)$$

to be valid for a bunched beam with total longitudinal frequency spread of 2Δ.

3. The Fokker-Planck description of cooling in action-angle variables

The equation of motion for any degree of freedom $\xi_i\ \varepsilon(x_i, z_i, \theta_i)$ with corresponding oscillation frequency $\omega_i\ \varepsilon(\omega_{xi}, \omega_{zi}, \omega_{si})$ of particle i that samples the signal E(t) at the kicker periodically is

$$\ddot{\xi}_i + \omega_i^2\xi_i = q\sum_{n=-\infty}^{+\infty} E(t)\,\delta\left(t - \tau_i(t) - \frac{\theta_k}{\omega_0} + nT_0\right) = s(t) \quad (9)$$

where s(t) has a self-action contribution of particle i sampling the amplified version of its own signal at the PU and a Schottky noise contribution of sampling signals from all the other particles: $s(t) = c(i,i) + \Sigma_{j\neq i}s(i,j(t))$ $= c(i,i) + s(i;t)$. In the adiabatic approximation of slow cooling, s(t) is a small quantity and one can use a multiple time-scale perturbation analysis[1] to obtain the following equations of motion for action-angle:

$$\dot{I}_i = G^0(i,i) + G^1(i;t) \quad (10)$$
$$= -\frac{\sqrt{2I_i}}{\omega_i}\left[(1/2\pi)\int_0^{2\pi} d\psi_i\ \sin\psi_i\ c(i,i) + \sin\psi_i s(i;t)\right]$$

$$\dot{\psi}_i = \omega_i + H^0(i,i) + H^1(i;t) = \omega_i -$$
$$\frac{1}{\omega_i\sqrt{2I_i}}\left[(1/2\pi)\int_0^{2\pi} d\psi_i\ \cos\psi_i\ c(i,i) + \cos\psi_i s(i;t)\right] \quad (11)$$

Using Eqs. (4), (9), (10), (11) and Fourier series expanding in the periodic angle variables ψ, one can in general write the cooling equations of motion in canonical action-angle variables in all three dimensions as[1]

$$\dot{\underset{\sim}{I}}_i = \sum_{\underset{\sim}{n}} G_{\underset{\sim}{n},-\underset{\sim}{n}}(\underset{\sim}{I}_i,\underset{\sim}{I}_i) + \sum_{j\neq i}\sum_{\underset{\sim}{n},\underset{\sim}{n}'}\sum_k G_{\underset{\sim}{n},\underset{\sim}{n}';k}(\underset{\sim}{I}_i,\underset{\sim}{I}_j)$$
$$\times e^{i(\underset{\sim}{n}\cdot\underset{\sim}{\psi}_i+\underset{\sim}{n}'\cdot\underset{\sim}{\psi}_j)}\,e^{ik\omega_0 t} \quad (12)$$

$$\dot{\underset{\sim}{\psi}}_i = \sum_{\underset{\sim}{n}} H_{\underset{\sim}{n},-\underset{\sim}{n}}(\underset{\sim}{I}_i,\underset{\sim}{I}_i) + \sum_{j\neq i}\sum_{\underset{\sim}{n},\underset{\sim}{n}'}\sum_k H_{\underset{\sim}{n},\underset{\sim}{n}';k}(\underset{\sim}{I}_i,\underset{\sim}{I}_j)$$
$$\times e^{i(\underset{\sim}{n}\cdot\underset{\sim}{\psi}_i+\underset{\sim}{n}'\cdot\underset{\sim}{\psi}_j)}\,e^{ik\omega_0 t} + \underset{\sim}{\omega}_i\ . \quad (13)$$

Since a particle is affected only by those other particles which are its close neighbours in frequency, two particles i and j will influence each other provided the resonance condition $(n \pm Q)\omega_0 + \mu\omega_s(i) =$ $= (m \pm Q)\omega_0 + \nu\omega(j)$ is satisfied. Under the assumption of non-overlapping betatron and revolution bands, only the $(n \pm Q) = (m \pm Q)$, $\mu\omega_s(i) = \nu\omega_s(j)$ resonances are of interest,and the higher order overlapping resonances give rise to rapid fluctuations in time which can be averaged out $(k = 0)$ for slow time-dependences[1]. Equations (12) and (13) then describe cooling equations in terms of particle action and angle variables alone, with no explicit time-dependence. However, each interaction harmonic $G_{\underset{\sim}{n}\underset{\sim}{n}'}$ will have an intrinsic sum over all the revolution harmonics $m\omega_0$ inside it reflecting the bunched structure of the beam[1]. In general $\underset{\sim}{n} \equiv (n_x, n_z, \mu)$ and representations (12) and (13) allow coupling of degrees of freedom through the cooling interaction. Explicit expressions for the interaction harmonics can be obtained by using Eqs. (4), (9), (10) and (11) as[1]

$$G_{\mu\nu}(J,J') = \frac{(qf_0)^2\kappa}{\omega_s(J)}\sum_{m=-\infty}^{+\infty}\left(\frac{\nu}{-m}\right)\tilde{G}\left[\Omega_{m,\mu}(J')\right]$$
$$\times J_\mu(m\sqrt{2J})J_\nu(-m\sqrt{2J'})\ e^{im(\theta_p-\theta_k)} \quad (14)$$

for longitudinal cooling [$\kappa = d\omega(E)/dE$, the machine parameter] and

$$G_{\mu\nu}^{(\pm)} = (\sqrt{I}\sqrt{I'}/2Q\omega_0)(qf_0)^2 g_{\mu\nu}^{(\pm)}(J,J')$$
$$= \left[(qf_0)^2/2Q\omega_0\right]\sqrt{I}\sqrt{I'}\sum_{m=-\infty}^{+\infty}\tilde{G}\left(\Omega_{m,\mu}^{(\pm)}(J')\right)$$
$$\times J_\mu\left[(m \pm Q)\sqrt{2J'} - \frac{Q\xi}{\eta}\sqrt{2J'}\right]e^{im(\theta_p-\theta_k)}$$
$$\times J_\nu\left[-(m \mp Q)\sqrt{2J} - Q\frac{\xi}{\eta}\sqrt{2J}\right] \quad (15)$$

for transverse dipole cooling in the betatron non-overlap region $[\underset{\sim}{n},\underset{\sim}{n}'] = \{(\mu,+1);(\nu,-1)\}$ or $\{(\mu,-1);(\nu,+1)\}$. One also verifies[1] that the dynamics, without the dissipative self-action term, satisfies the Hamiltonian flow condition

$$(\partial/\partial \underset{\sim}{I}_i)\cdot\left[\dot{\underset{\sim}{I}}_i - \underset{\sim}{G}(i,i)\right] = -(\partial/\partial\underset{\sim}{\psi}_i)\cdot\left[\dot{\underset{\sim}{\psi}}_i - \underset{\sim}{H}(i,i)\right]. \quad (16)$$

A hierarchy of characteristic time-scales is provided by the revolution and oscillation time-periods ($T_0, T_\beta = 2\pi/\omega_\beta$, $T_s = 2\pi/\omega_s$), the relaxation time τ_{cool} of the particle distribution, the phase-space mixing time τ_{mix} and the coherent Schottky signal suppression time τ_{coh}. From Section 2, we have $(\tau_{mix})^{-1} \sim \mu\Delta\omega_s \sim \mu|d\omega_s/da|a_m$ in the situation of no synchrotron band-overlap and $\sim \mu\omega_s^m \sim m\Delta\omega$ in the situation of overlap. With considerable mixing within one synchrotron period, we have the model of a classical Brownian particle damping steadily under the 'sure' coherent cooling force G^0 and diffusing under the Schottky fluctuation force $G^1(t)$ with a hierarchy $T_0, T_\beta < \tau_{mix} < T_s << \tau_{cool}$. A classical fluctuation theory or equivalently canonical kinetic theory in phase-space, together with Eq. (16), can then be used to describe the time-evolution of the angle-averaged one-particle distribution in the form of the following Fokker-Planck equation[1]

$$\frac{\partial f}{\partial t}(\underset{\sim}{I};t) = -\frac{\partial}{\partial \underset{\sim}{I}}\cdot\left[\underset{\sim}{F}(\underset{\sim}{I})f(\underset{\sim}{I};t)\right] + \frac{1}{2}\frac{\partial}{\partial \underset{\sim}{I}}\cdot\left[\underset{\approx}{D}(\underset{\sim}{I};f)\cdot\frac{\partial f(\underset{\sim}{I};t)}{\partial \underset{\sim}{I}}\right] \quad (17)$$

where

$$\underset{\sim}{F}(\underset{\sim}{I}) = \underset{\sim}{G}^0(\underset{\sim}{I},\underset{\sim}{I}) = \sum_{\underset{\sim}{n}} \underset{\sim}{G}_{\underset{\sim}{n},-\underset{\sim}{n}}(\underset{\sim}{I},\underset{\sim}{I}) \quad (18)$$

$$\begin{aligned}\underset{\approx}{D}(\underset{\sim}{I};f) &= 2\int_0^\infty d\tau \left\langle \underset{\sim}{G}^1(\underset{\sim}{I}(t),\underset{\sim}{\psi}(t);t)\underset{\sim}{G}^1(\underset{\sim}{I}(t-\tau),\underset{\sim}{\psi}(t-\tau);t-\tau)\right\rangle \\ &= 2\pi N \sum_{\underset{\sim}{n}}\sum_{\underset{\sim}{n}'}\int d\underset{\sim}{I}'\, f(\underset{\sim}{I}')\left[\underset{\sim\sim}{G}_{nn'}(\underset{\sim}{I},\underset{\sim}{I}')\underset{\sim\sim}{G}^*_{nn'}(\underset{\sim}{I},\underset{\sim}{I}')\right] \\ &\times \delta\left[\underset{\sim}{n}\cdot\underset{\sim}{\omega}(\underset{\sim}{I}) + \underset{\sim}{n}'\cdot\underset{\sim}{\omega}(\underset{\sim}{I}')\right]\end{aligned} \quad (19)$$

neglecting the Schottky signal suppression effect. In general $\underset{\sim}{F}$ and $\underset{\approx}{D}$ are non linear functions of $\underset{\sim}{I}$ and separate equations for moments do not exist. However, for linear transverse dipole cooling only in the regime of no synchrotron band overlap, one has $F(I,J) = \alpha(J)I$, $D(I,J;f) = \beta(J)I\langle I\rangle$ and an exponential type cooling equation obtains for the first moment $\langle I\rangle(J) = (1/2)\langle A^2\rangle(J) = \int_0^\infty dI\cdot I\cdot f(I,J)$ in the form[1]

$$\frac{1}{\langle I\rangle(J)}\frac{d}{dt}\langle I\rangle(J) = -\gamma(J) = -\left[\alpha(J) - (1/2)\beta(J)\right] \quad (20)$$

with

$$\begin{aligned}\gamma(J) &= \left[q^2\omega_0/2(2\pi)^2Q\right]\sum_{(\pm)}\sum_\mu \\ &\times\left[g^{(\pm)}_{\mu,-\mu}(J,J) - \pi N\left[q^2\omega_0/2(2\pi)^2Q\right]\right. \\ &\times \left. f_0(J)\left[|\mu|(d\omega_s/dJ)\right]^{-1}\left|g^{(\pm)}_{\mu,-\mu}(J,J)\right|^2\right]\end{aligned} \quad (21)$$

Amplifier noise, uncorrelated with particles, causes extra diffusion with coefficient given by a similar analysis as $D_{noise}(I,J) = \lambda(J)I$ where[1]

$$\lambda(J) = (q^2/2Q^2)\sum_\mu\sum_{(\pm)}\left\{\sum_n J_\mu^2\left[(n\pm Q)\sqrt{2J} - \frac{Q}{n}\xi\sqrt{2J}\right]P\left(\Omega^{(\pm)}_{n,\mu}(J)\right)\right\} \quad (22)$$

where $P(\Omega)$ is the power spectrum of the amplified amplifier noise at the kicker. Evolution of $\langle I\rangle(J)$ then is given by[1]

$$\langle I\rangle(J;t) = \left[\langle I\rangle(J,0) - \langle I\rangle(J;\infty)\right]e^{-\gamma(J)t} + \langle I\rangle(J,\infty) \quad (23)$$

with asymptotic noise-dominated equilibrium given by $\langle I\rangle(J,\infty) = \left[\lambda(J)/2\gamma(J)\right]$.

Dynamic signal suppression occurs much before any significant cooling time, but is expected to be established in a synchrotron oscillation period: $T_s \sim \tau_{coh} << \tau_{cool}$. In the non-overlapping synchrotron band region, the effect[1] is to reduce the first term in Eq. (21) by $\varepsilon_\mu^{(\pm)}(J)$ and the second term by $|\varepsilon_\mu^{(\pm)}(J)|^2$. Amplifier noise also gets suppressed by $|\varepsilon_\mu^{(\pm)}(J)|^2$. With symmetries in the expression (7) for $\varepsilon_\mu^{(\pm)}(J)$, one obtains after algebraic combinations, the following[1]

$$\gamma(J) = \left[q^2\omega_0/2(2\pi)^2Q\right]\sum_{(\pm)}\sum_\mu\left\{g^{(\pm)}_{\mu,-\mu}(J,J)/|\varepsilon_\mu^{(\pm)}(J)|^2\right\} \quad (24)$$

and similarly for $\lambda(J)$, thus completing the theory in the synchrotron band non-overlap region, including signal suppression. In the synchrotron band-overlap region a practical evaluation of F, D and the time-evolution of f, is complicated even with neglect of the signal suppression owing to the strong non-local coupling in action space $(I,J) \leftrightarrow (I',J')$ implied by band overlap. A quick estimate may follow however by appealing to conjecture (8) and simply using the coasting beam expression[4] for cooling with enhanced effective number of particles N_{eff}:

$$\gamma = \sum_{(\pm)}\sum_{n>0}\left[2G\left[(n\pm Q)\omega\right]/\left|1+\left(G\left[(n\pm Q)\omega\right]N_{eff}/4\Delta|n\pm Q|\right)\right|^2\right] \quad (25)$$

Use of this equation is equivalent to replacing bunched beam cooling with an equivalent coasting beam cooling with bad mixing (no revolution or betatron band overlap). Synchrotron band overlap as well as the corresponding signal suppression [see denominator in Eq. (25)] is retained in full strength and the effect of bunching appears in N_{eff}. The expression is ideally appropriate for a bunch in a flat square bucket with perfectly reflecting walls[1,3,5].

4. Discussion

We note that in the limit of vanishing synchrotron frequency spread, $d\omega_s/dJ \to 0$, i.e. no mixing in longitudinal phase-space, there is no synchrotron band overlap and from Eqs. (6) and (7) we observe that the signal suppression factor tends to be large, eventually fully screening the Schottky signals to zero by coherent feedback through the beam. No useful residual Schottky signal remains and from Eq. (24) we find that the cooling rate goes to zero ($\varepsilon_\mu^{(\pm)} \to \infty$). The noise concentration at synchrotron side-bands is infinitely large and from Eq. (21), this leads to infinitely fast heating or diffusion prior to the full establishment of signal suppression. It is thus crucial that one introduces mixing either by filling the bucket reasonably or by adding higher harmonic cavities. In addition to the above considerations of mixing, the removal of the $\mu = 0$ central coherent longitudinal lines and need for high frequency large bandwidth systems (both to probe deeper into the bunch and to handle the enhanced effective number of particles) form the singularly distinctive features of bunched beam cooling.

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