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SEQUENTIAL BUNCH EXTRACTIONS IN THE LOS ALAMOS PSR*

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Summary

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In the short-bunch, high-frequency operating mode of the future Proton Storage Ring (PSR) at LAMPF, the extraction scheme will take out one bunch at a time. Because the ring only stores six bunches at most and because the bunching system is heavily beam loaded, each extraction can cause a substantial change in the average beam current and hence affects significantly the total bunching voltage as well as the phase synchronization between those remaining bunches and the voltage waveform.² A time-dependent cavity-tuning scheme has been considered to retain the proper bunching voltage and phase relation. This work examines the stability of the phase synchronization $^3, ^4$ in the transient stage of extraction and cavity tuning. An equivalent circuit model is adopted, and the coupled nonlinear differential equations are solved numerically. We find that the relative phase will remain stable and that the voltage variation is tolerable under the PSR proposed operating conditions. A comparison between the numerical and analytical solutions confirms the numerical results.

Introduction

The PSR now under construction will be an 800-MeV proton storage ring that uses stripping injection of ions. A short-bunch, high-frequency mode will accumulate and store six 1-ns bunches of 10¹¹ protons each. The proton bunches will circulate at ∿60-ns intervals, and the accumulation will occur for 108 μs every 8.3 ms. The circulating bunches will be extracted one at a time at ${\sim}1.4\text{-ms}$ intervals. A cw bunching system of 503.125-MHz frequency will be implemented to maintain the bunch structure. Because of the heavy beam loading, the voltage on the cavities is dominated by the circulating beam current. It is foreseeable that extracting a bunch from the ring will result in a considerable voltage variation, together with an unacceptable phase mismatch between the total voltage and the remaining proton bunch. To prevent these kinds of undesirable results, considerations have been directed toward a tuning scheme of coupling the buncher cavity to a tuning cavity whose frequency can be varied over a wide range by using ferrite that is subjected to a time-dependent bias magnetic field. Nevertheless, the slow process of cavity tuning cannot follow the rapid beam-current jump resulting from the extraction; therefore, a mismatch during the tuning period is still unavoidable. The purpose of this work is to seek an understanding of the behavior of the bunches, of the relative phase, and of the bunching voltage during the tuning stage as well as the consequences of mismatch.

Theoretical Model

We consider an equivalent circuit analysis of the present problem. The buncher cavities are modeled as a single cavity approximated as a parallel RLC circuit having a variable inductor (Fig. 1). Current sources i g and i prepresent the external rf power and circulating beams, respectively.



Fig. 1. Equivalent circuit used in analysis.

Using Kirchhoff's law, one can derive that the cavity voltage v(t) satisfies [$\Omega = 1/\sqrt{LC}$ = resonant frequency of the cavity, $\alpha = 1/(2RC)$]

$$\vec{v} + 2\alpha(1 - \frac{\dot{\Omega}}{\alpha\Omega}) \vec{v} + \Omega^2(1 - \frac{4\alpha\dot{\Omega}}{\Omega^3}) \vec{v} = -2\alpha R \left[\frac{d}{dt} (i_g + i_b) - \frac{2\dot{\Omega}}{\Omega} (i_g + i_b) \right] , \qquad (1)$$

where the dot indicates the derivative with respect to time, t. For PSR, $\alpha \sim 5.3 \times 10^4 \text{ s}^{-1}$, $\Omega \sim 3.2 \times 10^9 \text{ s}^{-1}$ and for each extraction, the resonant frequency of the cavities will be tuned by $\Delta\Omega \sim 10^5 \text{ s}^{-1}$ in a period of $\Delta t \sim 50 \text{ }\mu\text{s}$; thus, all terms involving $\dot{\Omega}$ in Eq. (1) can be neglected. More approximations are made by considering only the Fourier component of the beam current near the buncher cavity resonant frequency. Also, in the steady state, the beam-current phase should lead the bunching-voltage phase by 90°; it is then convenient to define the phase deviations ϕ_b and ϕ_v , of ib and v from their steady state values (Fig. 2). Therefore, by substituting v = -jV(t) exp [jwt + j\phi_v(t)], ig = jIg exp (jwt), and ib = Ib(t) exp [jwt + j\phi_b(t)] into Eq. (1); equating the real and imaginary parts, we have

$$\dot{\mathbf{v}} + \alpha \mathbf{V} = \alpha R I_g \left[\cos \phi_{\mathbf{v}} - (I_b/I_g) \sin (\phi_{\mathbf{v}} - \phi_b) \right]$$
, (2)



Fig. 2. Phasor diagram showing beam current, generator current, and cavity voltage, where I_g , I_b , and V are complex quantities.

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and

$$\dot{\phi}_{v} + \frac{\omega^{2} - \Omega^{2}}{2\omega} = -\frac{\alpha R I_{g}}{V} \left[\sin \phi_{v} + (I_{b}/I_{g}) \cos (\phi_{v} - \phi_{b}) \right] , \qquad (3)$$

where $j^2 = -1$ and ω ($\approx 3.16 \times 10^9 \text{ s}^{-1}$ for the PSR) is the frequency of the driving rf power. In deriving the above equations, we have neglected small terms of order of α/ω . We also assumed that the time derivative of the voltage and/or current multiplied by $e^{-j\omega t}$ is much smaller than ω times that same quantity. To complete the description of the beam-cavity

To complete the description of the beam-cavity interaction, we need to include the following equation of beam motion:

$$\ddot{\phi}_{b} = \frac{qV}{2\pi h E_{o}} \left(\frac{1}{\gamma^{2}} - \frac{1}{\gamma_{t}^{2}} \right) \left(\frac{\omega}{\beta} \right)^{2} \sin \left(\phi_{v} - \phi_{b} \right)$$

$$= \left(\alpha - \frac{V}{\gamma} \right) \left(\frac{2}{\gamma_{t}^{2}} - \frac{1}{\gamma_{t}^{2}} \right) \left(\frac{\omega}{\beta} \right)^{2} \sin \left(\phi_{v} - \phi_{b} \right)$$
(4)

$$\equiv (A \frac{R}{R}) \alpha \sin(\phi_v - \phi_b), \qquad (4)$$

where β and γ are the usual relativistic parameters, q is the proton charge, E_0 is the total energy of a proton, γ_t is the transition γ of the ring, and h is the rf harmonic number.

Because the interval between extractions is much longer than the tuning period, each extraction can be treated independently. Also, because the extraction is a very rapid process, we can assume the average beam current, and hence the Fourier content under consideration, varies as a step function of time with jumps at each extraction.

Note that because of the high bunching voltage, the neglect of the term d[I_b exp($j\phi_b$)]/dt in obtaining Eqs. (2) and (3) is still valid, even though the beam current takes such a rapid change. This approximation has been justified by numerically solving Eqs. (2)-(4) without neglecting the \ddot{v} and d[I_b exp ($j\phi_b$)]/dt terms.

The ring stores only six bunches at maximum; therefore, one can infer from Eqs. (2) and (3) that for the quiescent state just before the extraction

$$(\Omega_{A} - \omega)/\alpha = (n/6)(\Omega_{m} - \omega)/\alpha = I_{bA}/I_{g} , \qquad (5)$$

and in the steady state following the extraction

$$(\Omega_{B} - \omega)/\alpha = (n - 1)(\Omega_{m} - \omega)/6\alpha = I_{bB}/I_{q} \qquad (6)$$

In the above equations, n is the number of bunches in the ring before the extraction; I_{bA} and I_{bB} are the beam current values before and after extraction, respectively; Ω_m and Ω_A are the cavity resonant frequency at maximum beam loading and before an extraction, respectively; Ω_B is the resonant frequency to which the cavity should be tuned after the extraction.

Numerical Results

The coupled nonlinear differential equations, Eqs. (2)-(4) are solved numerically using the Runge-Kutta method. Results for the cases of extracting one bunch out of six and one bunch out of two are

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presented in Figs. 3 and 4. The numerical values of $(\Omega_{\rm m} - \omega)/\alpha = 9.677$ and A = 843.4 [Eq. (4)] are used; the squared resonant frequency, Ω^2 , of the cavity is assumed to vary linearly in time from Ω_A^2 to Ω_B^2 in a time T = 2.0/ α . It is also assumed that the system is initially in the state with $d\phi_b/dt = 0$, $\phi_v = \phi_b = 0$, and V/I_gR = 1; the extraction is performed at t = 0, when the two parts

when the tuning starts. The results show that the mismatch caused by the extraction has the effect of increasing the phase and decreasing the magnitude of the voltage, but proper tuning of the cavities eventually will return the system to its normal state. The variation of particle energy is always less than one-tenth of a per cent for any extraction in the PSR sequence. The beam-current phase closely follows the voltage phase with a synchrotron oscillation of maximal amplitude $\sim 4^{\circ}$. The magnitude of the voltage also oscillates with the same frequency. The largest variation in the voltage's phase and magnitude are found in the case where one



Fig. 3. Evolution of cavity voltage, phase of voltage, and relative phase between voltage and beam current, for extracting one bunch out of six and linear tuning of cavity resonant frequency.



Fig. 4. Evolution of cavity voltage, phase of voltage, and relative phase between voltage and beam current for extracting one bunch out of two and linear tuning of cavity resonant frequency.

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bunch out of two is extracted. In that case, ϕ_v rises

to ${\sim}38^{\circ}$ and the voltage decreases to ${\sim}8\%$ of its normal value. We note that the amplitude of the synchrotron oscillation in the earlier stage of the process seems independent of the number of bunches in the ring. However, the synchrotron-oscillation damping rate is higher and the vibrations of the magnitude of the voltage are larger in Fig. 3 than those in Fig. 4.

A significant improvement can be achieved by starting the cavity tuning before the extraction. Figure 5 shows such an example, where the case considered is the same as the one in Fig. 4 except that the bunch is extracted at the time of t =1/ α instead of 0. We see that the variation in the magnitude of voltage is not more than 3%, and the maximal excursion of ϕ_V is $\sim 18^\circ$.

The results for the cases of tuning following an exponential law are not shown here, but when the values of tuning parameters used in the exponential tuning are comparable to those in the linear tuning, the results obtained are also comparable and close.



Fig. 5. Evolution of cavity voltage, phase of voltage, and relative phase between voltage and beam current for extracting one bunch out of two and linear tuning of cavity resonant frequency. The extraction is performed at $t = 1/\alpha$.

Comparison with Analytical Solution

The moderate variations of V and ϕ_v together with the small difference between ϕ_v and ϕ_b suggest that only the first few terms, in the small argument expansion of trigonometric functions and in the expansions of V/RI_g about 1, are needed for a good approximation. Based on these approximations, an analytical solution of Eqs. (2)-(4) has been obtained by using a perturbation scheme.⁵

Figure 6 shows the analytical solution results for the case identical to the one in Fig. 3. A good agreement between the numerical and analytical solutions can be seen by comparing Fig. 6 with Fig. 3.

We find from the analytical solutions that if $A >> (I_{bB}/I_g)^2$ the maximum oscillation amplitude of the difference between beam and voltage phases excited by the extraction is roughly $(I_{bA} - I_{bB})/(I_g A^{1/2})$, and the damping rate is about $(I_{bB}/I_q)^2/A$.



Fig. 6. Analytical calculation results for the same case shown in Fig. 3.

Conclusion

By using an equivalent circuit model, we have analyzed the rf stability problem of sequential bunch extraction with cavity tuning for the PSR. The results indicate that the relative phase will remain stable and the variations of voltage and particle energy are tolerable under the PSR proposed operating conditions.

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