IEEE Transactions on Nuclear Science, Vol. NS-30, No. 4, August 1983

Effects of rf Stacking on Cooling Tail in the Fermilab Antiproton Accumulator

Ainosuke Ando[+] and Ken Takayama

Fermilab[*], P.O. Box 500, Batavia, Illinois 60510

1. Introduction

The scheme[1] has been recently proposed to produce a high-intensity antiproton beam in a stochastic cooling accumulator. According to this scheme, the coasting beam of $\bar{p}$'s extracted from the Debuncher ring will occupy the injection orbit of the accumulator ring with a total momentum spread of about 0.2%. This beam is to be bunched adiabatically. Then the buckets are transformed to moving(decelerating) buckets, slowly enough to obtain as large a capture efficiency as possible. The amount of deceleration depends on the physical distance which must be cleared by the bunch. After deceleration is completed, the rf is turned off slowly to allow the beam to debunch adiabatically. The beam is now ready for stochastic cooling. The debunched beam is soon moved to down in the stream by the tail cooling system in order to make room for the next stacked beam.

There are in general two effects that rf stacking gives an already stacked beam, which shall be called " tail " because the main part of an accmulated beam has the common name of " core ". The larger is phase-displacement acceleration and the other is so-called dilution coming from non-adiabatic change of rf parameters, say, $V_{rf}(t)$ and $f_{rf}(t)$, which is understood to be enhanced by non-linearity of rf stacking dynamics. From a phase dynamical point of view, both effects are especially discussed in the second section.

In the third section, we discuss dynamics of tail cooling with a help of the Fokker-Planck equation. From the tail cooling point of view, the effects of rf stacking can be regarded as deformation of the tail which occurs within a limited period (that is, 100 msec), tail cutting due to phase-displacement and non-uniform dilution in the tail due to non-adiabatic diffusion. An easy method for estimating beam-loss due to phase-displacement is presented and used to obtain the criterion which should be imposed on the cooling system.

2. RF Stacking & Its Effects on Cooling Tail

2-a Stacking Process

The stacking process consists of the following four phases:

1. Adiabatic Capture. The injected coasting beam is adiabatically captured in the rf bucket, whose phase space area is slowly increasing until it becomes a little larger than the phase space area occupied by the beam. The period which adiabatic capture requires is chosen from general considerations[2] about the adiabaticity of a harmonic oscillator and is chosen equal to 3τ, where τ is the period of the synchrotron oscillation at the final stage of this step. (See Fig.1-a)
2. Alteration (Stationary Bucket$\rightarrow$Moving Bucket). An adiabatic alteration of the rf parameters, say, V_{rf} and f_{rf}, is performed in order to convert a " stationary bucket " into a " moving bucket ", capable of changing the mean energy of the particles trapped inside.(See Fig.1-b)
3. Deceleration. A period of deceleration follows, employing a moving bucket of constant area and constant synchronous phase angle ϕ_s. (See Fig.1-c)
4. Adiabatic Debunching. The rf is turned off adiabatically after it has been transformed again into a stationary bucket of the same area from a moving bucket. (See Fig.1-d)

The above process is performed by manipulating both parameters of the rf, voltage V_{rf} and frequency f_{rf}. Results of numerical simulations already shown, give a final full energy-spread of 0.21% and a final capture efficiency of 98%.

2-b Phase-Displacement

As a consequence of Liouville's theorem, the phase area transported downward in the energy plane by a decelerating rf bucket must be accompanied by the upward trasport of an equal phase area in the region outside the bucket, a property known as phase-displacement, which is seen in Fig.2. This phenomenon gives rise to a serious problem in the previous pulse , which has been moved towards the cooling core by the tail cooling system during one cycle. That is, following phase flow due to the mentioned mechanism, a fraction of the previous pulse is transported upward from the region where a new pulse is deposited. Then the fraction transported upwards must be understood to be lost out of the region where the tail cooling system is effective, since it is removed more and more upward in the energy plane every cycle due to the unique direction of motion of a decelerating bucket.

In order to estimate quantitatively the effects of phase-displacement on the tail, we consider the behaviour of phase points which are located in a small energy bin at the initial time. We divide the energy region of interest into many small bins. One-thousand phase points are distributed in each bin and simulated for an entire rf stacking. After one stacking cycle, these points will be located mainly in higher bins as a result of phase-displacement. Hence, such a procedure yields a kind of a transfer matrix which characterizes quantitatively over-all effects of rf stacking on the tail. From the transfer matrix, we find that the critical region for phase-displacement exists. Roughly speaking, the fact indicates a criterion which must be imposed on the tail cooling system so as to minimize particle-loss. If all parts of the stacked pulse are moved toward the core beyond this critical regions, our anxiety about beam-loss should vanish.

2-c Non-Adiabatic Diffusion

Rf stacking must be performed within a limited period of one cycle time in order to retain enough time for tail and core cooling. Such a situation leads to non-adiabatic diffusion particularly in the tail region of interest.

It is difficult to discuss analytically the adiabaticity under slow changes of rf frequency and rf voltage. So we consider a sequence of infinite phase points which are affected by rf stacking process. We assume that phase points comprising the sequence have the same initial condition

$$E = E_0 \quad , \quad \phi = \text{random} \quad \text{at} \quad t = 0.$$

We can regard a energy spread among these phase points after the whole change of f_{rf} and V_{rf}, as a measure of non-adiabatic diffusion. The energy spread ΔE obtained by numerical calculations is shown as a function of energy E in Fig.3. We can see

0018-9499/83/0800-2601$01.00© 1983 IEEE

the remarkable fine structure in the ΔE-E curve. This fine structure may be directly relevant to the problem of stable or unstable solutions for the non-linear Mathiew equation

$$\frac{d^2x}{dt^2} + (\alpha - 2\beta^2\cos 2\tau)\cos x = 0,$$

because $V_f(t)$ is usually assumed to change following a cosine-like function of time. Absolute values of diffusion in the tail region are not negligible whenever there are gradients in the particle distribution. Thus these effects will be involved in cooling calculations as periodic blow-up of the tail.

3. Tail Cooling

Stochastic cooling in the energy plane is usually discussed by the Fokker-Planck equation

$$\frac{\partial\psi}{\partial t} = -\frac{\partial}{\partial E}(F\psi) + \frac{\partial}{\partial E}\left\{(D_1+D_2\psi)\frac{\partial\psi}{\partial E}\right\}$$

where ψ is the density distribution function, and F, D_1 and D_2 are cooling parameters. Solving the above eguation, we can know in principle the time evolution of the particle distribution. In particular, we shall consider the time evolution of the stacked pulse over one injection cycle T_c.

For the sake of simplicity, we assume:

1) F and $(D_1+D_2\psi)$ are constant in the region of interest,

$$F = \overline{F} \quad , \quad D_1 + D_2\psi = \overline{D}$$

2) the distribution of a rf stacked beam is Gaussian at $t=0$ with the number of particles of N_0, the standard deviation of σ_0 and the mean of m_0.

Then we obtain the exact solution

$$\psi(E,t) = (N_0/\sqrt{2\pi}\,\sigma)\, e^{-(E-m)^2/2\sigma^2} \qquad (1)$$

with
$$m = m_0 + \overline{F}t,$$
$$\sigma^2 = \sigma_0^2 + 2Dt.$$

Using the solution (1), we can calculate the number of particles which after one cycle is still left in the upper stream above the critical region mentioned in the previous section, and we define the beam-loss ratio r due to phase displacement in the form of the Gauss error function

$$r = \int_C^\infty \psi(E,T_c)\,dE/N_0 = \int_n^\infty e^{-x^2}dx/\sqrt{\pi} = E_{rfc}(n)/\sqrt{\pi} \qquad (2)$$

with $\quad n = (C-m)/\sqrt{2}\,\sigma(T_c)$.

where we denote the critical region by C. A real situation is generally different from the assumptions. However, if we have a reasonable way for the coefficients of F and $(D_1+D_2\psi)$ to be approximated by some constant values, that is, $\overline{F}$ and $\overline{D}$, the loss estimation by (2) may be still fruitfull.

In order to obtain the approximations, we shall make the important assumptions:

1) The present cooling system has a perfectly exponential gain, that is,

$$F(E) = \hat{F}\, e^{(E-m_0)/E_d}$$

where E is the distance from the center of the core, E_d is the so called characteristic energy , and $\hat{F}$ is the value of $F(E)$ at the center of a stacked pulse.

2) The standard deviation σ_0 of the distribution is related to the finite full energy spread ΔE_B of particles which are deposited on the stacking orbit by

$$\sqrt{6\pi}\,\sigma_0 = \Delta E_B$$

Then it is trivial to estimate the displacement of the distribution peak over one injection cycle, say, ΔE_1. The behavior of the peak is governed by the differential equation

$$dE/dt = F(E)$$

Integrating both sides, we have

$$\Delta E_1 = -E_d \ln(1 - T_c\hat{F}/E_d)$$

Thus we know the central position between the front and rear peak over one injection cycle,

$$\overline{E} = m_0 + \Delta E_1/2$$

Now, we define the approximated cooling parameters $\overline{F}$ and $\overline{D}$ as follows

$$\overline{F} = F(\overline{E}) \quad , \quad \overline{D} = D_2(\overline{E})\,\overline{\psi}_0$$

with $\quad \overline{\psi}_0 = N_0/2\Delta E_B$

Here we neglect D_1 and assume that the distribution after one injection cycle does not change significantly except for a the displacement of its peak. In fact, for the present case, $D_2\psi$ is about five times larger than D_1 at the peak, as seen later.

In order to ascertain the accuracy of the above approximation for several examples, the estimation of beam-loss by using (1) and (2) is compared with the numerical solution. In this comparision we use the proposed system and change the momentum mixing factor $\eta = \gamma_T^{-2} - \gamma_0^{-2}$ and the bandwidth. The peaks and the full-widths of the distributions agree with each other, but the estimate of r with Eq.(2) is always smaller than the numerical solution by about 10 %, because the latter gives an asymmetric distribution at $t=T_c$. Fig.4 shows a typical numerical solution where the broken line and the real line show the rf-stacked distribution and the cooled one at $t=T_c$, respectively, and the shaded area is a survival-deformed one at the time when a next consequtive pulse is just stacked. The estimate of Eq.(2) is not accurate, but would be useful as a guide line for designing a tail cooling system.

If we want r to be less than some value $r_0 = E_{rfc}(n_0)/\sqrt{\pi}$, the parameters must satisfy the equation,

$$(C-m)/\sqrt{2}\,\sigma(T_c) \geq n_0$$

Then, using the more explicit expression of $\overline{D}$

$$\overline{D} = AT^2\,\overline{\psi}_0\,\overline{F}^2$$

where T is the revolution period and A is a constant given by the system, we have a quadratic inequality for $\overline{F}$. Then the solution becomes

$$\overline{F} \leq F_0$$

$$F_0 = -(\Delta E_B/T_c)\left[k + n_0\sqrt{(1-an_0^2)/3\pi + k^2 a}\right]/(1-an_0^2)$$

$$k = \Delta E_B/(m_0 - C)$$

$$a = AT^2 N_0/T_c\Delta E_B$$

and also we obtain,

$$\hat{F} \leq -|F_0|\left[\xi + \sqrt{\xi^2+4}\right]/2 \quad , \quad \xi = T_c|F_0|/E_d$$

Fig.5 shows F_0 as a function of n_0 for the designed Accumulator.

4. Conclusion

The unavoidable effects of rf stacking on the cooling tail have been discussed in detail. It is still difficult to say that we have a through understanding of the non-adiabatic diffusion appearing in the present rf stacking, but its quantitative effect has been obtained at least for the proposed system which is directly available for cooling calculations.

A simple method estimating beam-loss has been presented which can serve as a guide line for designing the tail cooling system. A more accurate estimate may also be possible by applying a perturbation technique.

References

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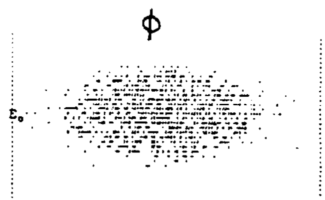
3. D. Möhl et al., Physics and Technology of Stochastic Cooling, Physics Report 58, No.2, p74, (1980).

* Operated by Universities Reserch Association Inc., under contract with the U.S. Department of Energy.

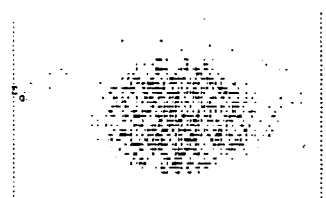
† On leave from KEK

Fig.1-a
Adiabatic
capture



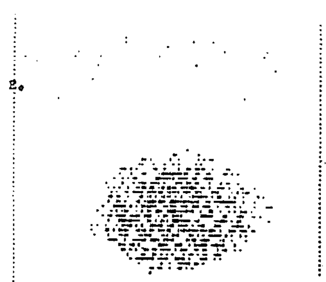
Stationary Bucket

Fig.1-b
Alteration



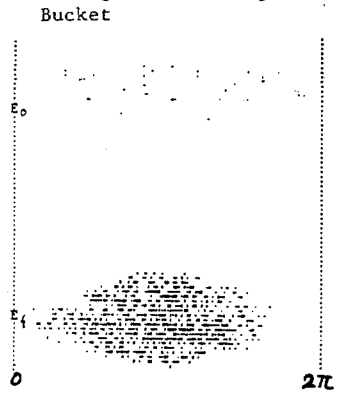
Moving Bucket

Fig.1-c
Deceleration



Moving(Decelerating) Bucket

Fig.1-d
Alteration



Stationary Bucket

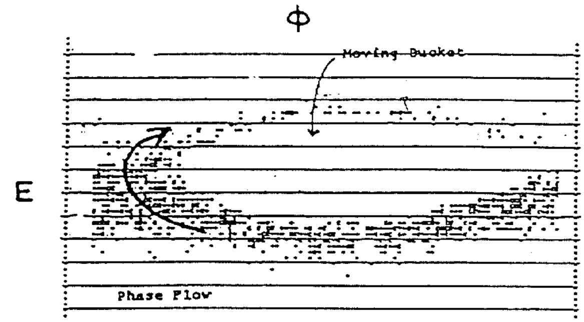


Fig.2 Phase-displacement

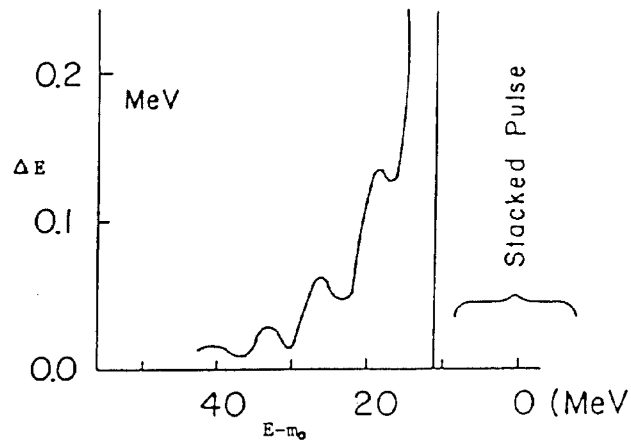


Fig.3 Energy spread due to non-adiabatic diffusion

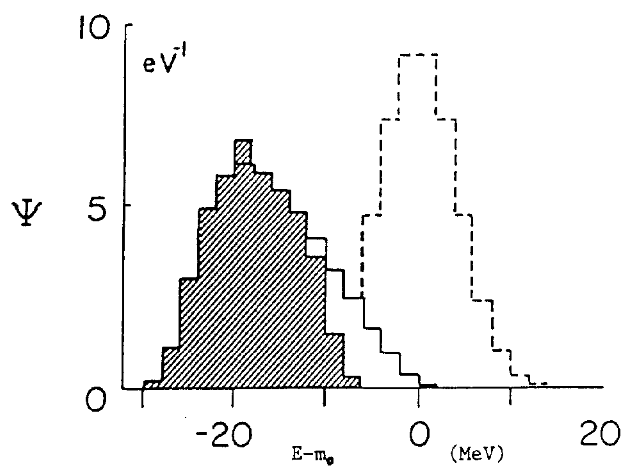


Fig.4 Particle distributions

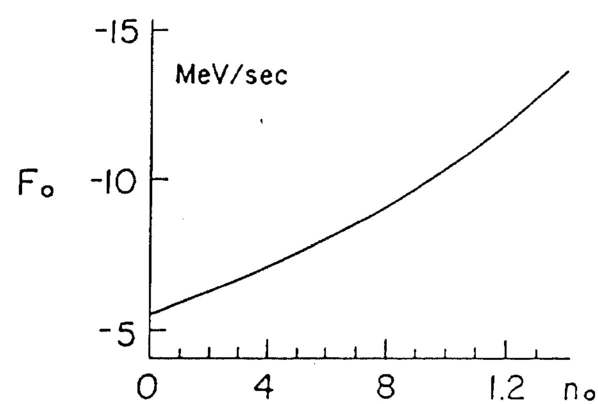


Fig.5 Critical curve for $\overline{F}$