# A COMMON TRANSVERSE FEEDBACK DAMPER FOR TWO BEAMS DURING A STACKING CYCLE*

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## Summary

During the stacking cycle at a proton storage ring, it often occurs that coasting and bunched beams coexist, rotating in the same direction. The problem of the two beams oscillating coherently with each other is formulated and discussed. We demonstrate that when the tune distribution is dominated by the sextupole contribution, one common transverse dipole feedback system is enough to damp both the coasting beam and the bunched beam instabilities.

## Introduction

During the stacking cycle at CBA, there will be a situation where the stacked coasting beam and the injected bunched beam coexist and rotate in the same direction in a ring. In dealing with the coherent instability problems of such beams, it has so far been assumed that the two beams are not coupled through the coupling impedance of the environment, and thus we can discuss the instabilities of the two beams separately. While this approximation is valid when the two beams are located inside the ring far away from each other, it becomes dubious when the bunched beam is brought closer to the stacked beam. Therefore, it is important that we learn how to deal with the problems of two beams interacting strongly with each other and oscillating coherently.

There is, yet, another no less important reason why we should tackle this problem. It has been suggested[1] that during the stacking procedure, after the injection error of the bunched beam is corrected, we shall use only one dipole feedback system to damp out the transverse coherent instabilities of both the bunched and the coasting beams. The feasibility of using such a damper cannot be assessed unless we understand the problem of two-beam instabilities, treating the action of the feedback system as part of the total coupling impedance.[2]

It is the purpose of this paper to investigate such two-beam coherent instability problems. We shall deal only with the symmetric, coupled rigid dipole mode of the bunched beams interacting with the coasting beam modes.

The major conclusion of this paper is that for a machine where the tune distribution is dominated by the sextupole contribution, such as CBA, a common dipole feedback system is enough to damp out the coherent instabilities of both beams.[3]

## Coherent Tune

In this section, we calculate the coherent tune $\nu$ of the collective instability when M symmetric bunches B and a coasting beam C are present in the same ring and are interacting through the ring impedance and the damper. We assume both beams to be monochromatic with $\nu_B$ and $\nu_C$ to be their respective tunes.

We use a convenient variable $\psi$ defined by $\psi = \theta - \omega_o t$, where $\theta$ is the azimuthal angle around the ring, and $\omega_o$ is the revolution angular frequency. The value of $\psi$ for a given particle is independent of time.

The dipole densities (per unit $\psi$), $D^C$ and $D^B$, of the coasting beam and the bunched beam can be Fourier decomposed, as

$$D^C(\psi,t) = \sum_{n=-\infty}^{\infty} \tilde{D}_n^C \; e^{in\psi+i\nu\omega_o t} , \tag{1}$$

and

$$D^B(\psi,t) = \sum_n \tilde{D}_n^B \; e^{in\psi+i\nu\omega_o t} , \tag{2}$$

where $\tilde{D}_n^B$ and $\tilde{D}_n^C$ are constants, and $\nu$ is the coherent tune of the collective motion. We choose the convention that the real part of $\nu$ is always positive. Also it is evident from (1) and (2) that a negative imaging part of $\nu$ corresponds to an unstable situation.

Let us look into $D^B$ more closely. Denote by $\lambda(\psi-\psi_a)$ the line density of the a-th bunch with its center located at $\psi = \psi_a \equiv 2\pi a/M$, a = 1, 2, ...M. $\lambda(\psi)$ is normalized to unity: $\int d\psi\lambda(\psi) = 1$.

Let $y_a(t) = \beta_a \exp(i\nu\omega_o t)$ be the rigid displacement of the a-th bunch. The dipole density $D^B$ can now be expressed as

$$D^B(\psi,t) = \bar{N}_B \sum_{a=1}^{M} \beta_a \, \lambda(\psi-\psi_a) \; e^{i\nu\omega_o t} , \tag{3}$$

where $\bar{N}_B$ is the number of particles per bunch.

Write

$$\lambda(\psi) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \tilde{\lambda}_n \; e^{in\psi} . \tag{4}$$

Substituting (4) into (3), and comparing the resulting equation with (2), we obtain

$$\tilde{D}_n^B = \frac{\bar{N}_B}{2\pi} \tilde{\lambda}_n \sum_{a=1}^{M} \beta_a \; e^{-in\psi_a} . \tag{5}$$

$\tilde{\lambda}_n$ will later on provide us with a high frequency cutoff factor. For a Gaussian bunch, $\lambda(\psi) = \left[\exp(-\psi^2/2\sigma^2)\right]/\sqrt{2\pi}\sigma$, $\tilde{\lambda}_n$ is given by $\tilde{\lambda}_n = \exp(-\sigma^2 n^2/2)$.

We are now ready to study the equations of motion.

Let us use the notation $W_n^B$ for the self-ring-impedance of the orbit B, namely, the ring impedance through which the beam B interacts with itself. It is evaluated at the frequency $(n-\nu)\omega_o$. We make a similar definition for $W_n^C$. We also use $W_n^x$ for the cross impedance between the orbits B and C.

*The work is performed under the auspices of the U.S. Department of Energy.

We should also include the effect of a common feedback damper. Let iK be the contribution of the damper to the coupling impedance. Then the total impedances $\bar{W}$'s are given by $\bar{W}_n^B = W_n^B + iK$, $\bar{W}_n^C = W_n^C + iK$ and $\bar{W}_n^x = W_n^x + iK$.

The equations of motion are

$$\frac{1}{\omega_0^2}\,\frac{\partial^2 D^C(\psi,t)}{\partial t^2} + \nu_C^2\, D^C = -I_C \sum_{n=-\infty}^{\infty} \left(\tilde{D}_n^C\, \bar{W}_n^C + \tilde{D}_n^B\, \bar{W}_n^x\right) e^{in\psi + i\nu\omega_0 t} , \tag{6}$$

and

$$\frac{1}{\omega_0^2}\,\frac{d^2 y_a(t)}{dt^2} + \nu_B^2\, y_a = -e\omega \sum_n \left(\tilde{D}_n^C\, \bar{W}_n^x + \tilde{D}_n^B\, \bar{W}_n^B\right) e^{in\psi_a + i\nu\omega_0 t} , \tag{7}$$

$$a = 1,\ 2,\ \dots\ M,$$

where $I_C$ is the current of the coasting beam. We shall use $\bar{I}_B = e\omega_0\bar{N}_B/2\pi$ for the average current per bunch and $I_B = M\,\bar{I}_B$ for the total average current of the bunched beams. The impedances $\bar{W}$'s are normalized to shifts in the squares of the tunes per unit current.

Substituting (1) into (6), one obtains

$$\tilde{D}_n^C = \tilde{D}_n^B \cdot I_C\, \bar{W}_n^x/(\nu^2-\nu_C^2-I_C\bar{W}_n^C) \quad . \tag{8}$$

Using $y_a(t) = \beta_a \exp(i\nu\omega_0 t)$ and Eqs. (5), (7), (8), we obtain

$$(\nu^2-\nu_B^2)\,\beta_a = \bar{I}_B \sum_n \tilde{\lambda}_n \times \left(\bar{W}_n^B + \frac{I_C(\bar{W}_n^x)^2}{\nu^2-\nu_C^2-I_C\bar{W}_n^C}\right) \sum_{b=1}^{M} \beta_b\, e^{in(\psi_a-\psi_b)} \tag{9}$$

We observe that (9) is cyclic in the indices a and b (remember that $\psi_a = 2\pi a/M$). And the solution of this equation is[5]

$$\beta_a = \beta\, e^{i\psi_a s} , \quad s = 1,\ 2,\ \dots\ M , \tag{10}$$

and

$$1 = \frac{I_B I_C}{\nu^2-\nu_B^2-I_B \sum_n \tilde{\lambda}_{nM+s}\, \bar{W}_{nM+s}^B} \times \sum_n \tilde{\lambda}_{nM+s}\, \frac{(\bar{W}_{nM+s}^x)^2}{\nu^2-\nu_C^2-I_C\, \bar{W}_{nM+s}^C} . \tag{11}$$

In (10) and (11), s is a parameter which classifies different eigen-solutions of the two-beam instability problem. One obtains the coherent tune $\nu$ by solving Eq. (11). We observe that for a given value of s, the number of solutions for (11) is in general infinite.

It is sometimes more convenient to use the following set of equations instead of Eq. (11).

Substitute $y_a = \beta_a \exp(i\nu\omega_0 t)$, and Eqs. (5) and (10) into (8) and (7). Then one obtains

$$(\nu^2-\nu_C^2)\, \tilde{D}_{nM+s}^C = I_C\, \bar{W}_{nM+s}^C\, \tilde{D}_{nM+s}^C + I_C\, \frac{M\bar{N}_B}{2\pi}\, \tilde{\lambda}_{nM+s}\, \bar{W}_{nM+s}^x\, \beta$$

and

$$(\nu^2-\nu_B^2)\, \beta = e\omega_0 \sum_n \bar{W}_{nM+s}^x\, \tilde{D}_{nM+s}^C + I_B \sum_n \tilde{\lambda}_{nM+s}\, \bar{W}_{nM+s}^B\, \beta . \tag{12}$$

Equation (12) is a secular equation for the eigenvalue $\nu^2$ with $(\beta, \tilde{D}_{nM+s}^C)$ as the eigenvector.

For a uniform coasting beam the different revolution modes n are decoupled because of translational invariance. However, this is no longer the case for bunched beams. For bunched beams, different revolution modes are always strongly coupled through the rf focusing force which keeps the beams bunched.[6,7]

Equations (11) and (12) tells us that when a coasting beam shares the beam chamber with the bunched beam, the different revolution modes of the coasting beam are coupled, with the bunched beam acting as the intermediary.

We observe that both denominators of (11) have a simple physical meaning: If the two beams were not coupled by the ring impedance or by the damper, $I_B \sum_n \tilde{\lambda}_{nM+s} \bar{W}_{nM+s}^C$ and $I_C\, \bar{W}_{nM+s}^C$ would represent the tune-square shifts for the bunched and the coasting beams due to the self-impedances and the feedback damper. The numerator $I_B I_C\, (\bar{W}_{nM+s}^x)^2$ couples the two beams.

## Solutions of the Damper Equation

There is little hope of analytically obtaining a general solution of Eq. (11). We shall content ourselves by solving the equation in two limiting cases.

To start with, we consider the case where

$$(I_B I_C \tilde{\lambda}_{nM+s})^{1/2} \left| \frac{\bar{W}_{nM+s}^x}{\nu_B^2-\nu_C^2} \right| \ll 1 . \tag{13}$$

In such a weak coupling limit, the left-hand side of (13) characterizes the magnitude of the coupling between the coasting beam and the bunched beam. Since two different revolution modes of the coasting beam couple to each other only through the bunched beam, the magnitude of their coupling should be of the order of the above quantity squared.

The condition (13) is appropriate for CBA when the bunched beam and the coasting beam are respectively on the injection and the stacking orbits.

Under (13) the solutions of (12) can be found perturbatively[9] yielding the following eigenvalues:

$$\nu^2 \simeq \nu_B^2 + I_B \sum_n \tilde{\lambda}_{nM+s} \bar{W}^B_{nM+s} + I_B I_C \sum_n \tilde{\lambda}_{nM+s} \frac{(\bar{W}^x_{nM+s})^2}{\nu_B^2-\nu_C^2} , \tag{14}$$

and

$$\nu^2 \simeq \nu_C^2 + I_C \bar{W}^C_{nM+s} + I_B I_C \frac{(\bar{W}^x_{nM+s})^2}{\nu_C^2-\nu_B^2} . \tag{15}$$

Equation (15) gives one eigenvalue for each value of n.

The meaning of (14) and (15) is that under the condition (13), the feedback system damps the two beams almost independently.

Let us now go to the other extreme limiting case, when the orbits B and C coincide.

In this case, we have

$$\nu_B = \nu_C \equiv \nu_1 , \tag{16}$$

and

$$\bar{W}^x_n = \bar{W}^C_n = \bar{W}^B_n \equiv \bar{W}_n . \tag{17}$$

Equation (11) now reads:

$$1 = \frac{I_B I_C}{\nu^2-\nu_1^2-I_B \sum_n \tilde{\lambda}_{nM+s} \bar{W}_{nM+s}} \times \sum_n \frac{\tilde{\lambda}_{nM+s} (\bar{W}_{nM+s})^2}{\nu^2-\nu_1^2-I_C \bar{W}_{nM+s}} . \tag{18}$$

From observation, we find immediately that

$$\nu = \nu_1 , \tag{19}$$

is a solution of (18). This mode is neither growing nor damping but is just stable. Substituting (16), (17), and (19) into (8), we find

$$\tilde{D}^C_{nM+s} = -\tilde{D}^B_{nM+s} . \tag{20}$$

In this mode, the part of the coasting beam which overlaps the bunched beam oscillates out of phase with the bunched beam, while the remaining part of the coasting beam stays unperturbed. The net effect is that the total dipole moment of the two beams vanishes everywhere. The damper, of course, cannot act on this mode; and yet, the mode is stable.

There are in general an infinite number of solutions of (18) other than the one given by (19) and (20). One can prove by some algebraic inequality manipulation of (18) that all these solutions are damped, namely, Imaginary $(\nu^2) > 0$, provided the damper strength K is large enough that the imaginary part of $\bar{W}_n$ is positive.

We have discussed the solutions of the transverse damper equation in the two limiting cases. A computer investigation of the intermediate cases is performed using the CBA parameters and the Courant-Month Green's function.[8] The conclusion of that investigation is that a common feedback system will indeed work for both beams in CBA. The details of this study will be published elsewhere.[4]

## References


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