IEEE Transactions on Nuclear Science, Vol. NS-30, No. 4, August 1983

ERRORS IN RADIO-FREQUENCY QUADRUPOLE STRUCTURES*

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Summary

Performance degradation caused by certain radio-frequency quadrupole (RFQ) machine errors was studied using an efficient 3-D particle tracing simulation code for a high-brightness example. Matched beams, for which no emittance growth occurs, exist for periodic structures and were used as input beams for particle tracing in the presence of errors. We considered both slowly varying and fast (random) errors that destroy periodicity. Random dipole errors cause emittance growth because of the mismatches they introduce and also result in a motion of the beam centroid that causes a reduction in acceptance. Because of the way RFQs are manufactured, the random error amplitudes can be kept below harmful levels. More important are the slow errors, which are harmful because they reduce acceptance even though they maintain a match (up to the point of particle loss). Slow dipole errors steer the beam toward the wall, and voltage deficiencies cause instabilities in the longitudinal direction resulting in particles not being accelerated to full energy.

Introduction

The effects of several types of errors in RFQ accelerators were simulated using an efficient 3-D particle-tracing code. The parameters for our example correspond to the 2-MeV high-brightness H⁻ prototype RFQ accelerator at Los Alamos.[1] The simulation code and the input distribution used in these studies are described below.

Particle-Tracing Code

Let the particle coordinates in the laboratory frame be $x,y,s+z$, and let p_x,p_y,p_s+p_z be their conjugate momenta, where s is the synchronous particle coordinate and p_s is its momentum.

In the simulation, we numerically integrate the equations of motion corresponding to the following Hamiltonian.

$$H = \frac{p_x^2 + p_y^2 + p_z^2 + p_s^2}{2m\gamma} + \frac{eV}{2}\left(\frac{(1-A)(x^2 - y^2)}{a^2}\right.$$
$$+ A\left\{\left[1 + \frac{k^2}{4}(x^2 + y^2)\right] \cos(kz - \phi_s + \omega t)\right.$$
$$\left.\left. + (kz - ks)\sin(-\phi_s + \omega t)\right\}\right) \sin\omega t$$
$$+ e\phi(x,y,z) \quad . \tag{1}$$

The independent variable is time, V is the potential difference between adajacent pole tips, A is a measure of vane modulation,[2] k is $2\pi/\beta\lambda$, ϕ_s is the synchronous phase, and ϕ is the space-charge potential. The "rf coupling term" of drift-tube linacs is present here also. Space charge is computed through terms cubic in the coordinates by fitting the particle distribution to an ellipsoid with a parabolic density profile. The parameters of this ellipsoidal distribution are computed from the particle moments through second order. The required 3-D integrals (analogous to the form factor integrals of envelope equation analysis) are approximated by an expansion about a sphere. The space-charge calculation is not good at the very beginning (bunching region) of the RFQ for three reasons: A long bunch length needs a higher than third-degree representation, the 3-D integral expansions are not accurate at their present stage (third order), and wall effects (which we neglected) may be important when the bunch length is comparable to the vane separation. Because of this problem, we conducted our simulations by starting with a beam matched at rf-period 30. This scheme still includes most of the low-energy portion of the accelerator because acceleration occurs only in the last 50 periods in the total of the machine's 178 rf periods.

*This work was supported by the US Dept. of Defense, Defense Advanced Research Projects Agency, and Ballistic Missile Defense Advanced Technology Center.

Preparing Input Distribution

A 10 000-particle distribution matched to a time- and space-independent (harmonic oscillator potential) structure, with a $\mu = 0.9$ space-charge parameter value, was prepared using the RZED code.[3] From this, a time-periodic beam matched to rf-period 30 of the RFQ was generated by adiabatic deformation.[4] That is, the original distribution was followed in the particle-tracing code through a structure that began as a harmonic oscillator and slowly deformed into an RFQ structure corresponding to rf-period 30 of our accelerator. The resulting beam was our standard input into period 30 of the RFQ for our simulations. The beam current was 50 mA, and the rms normalized emittances (rms areas in x-p_x, y-p_y, z-p_z planes divided by π) were 0.18 mm•mrad transverse and 0.28 mm•mrad longitudinal. Figures 1 and 2 show the evolution of the beam rms size and emittance in the x-direction as it goes through our standard accelerator with no errors. These plots show values at the times when the rf phase corresponds to the maximum focusing force in the x-direction.

Random Errors

For any periodic focusing system, even if it includes nonlinear and coupling forces, and even in the presence of space charge, there exist certain matched or equilibrium phase-space distributions that are periodic when injected into the system. The emittance or any other property of the distribution will

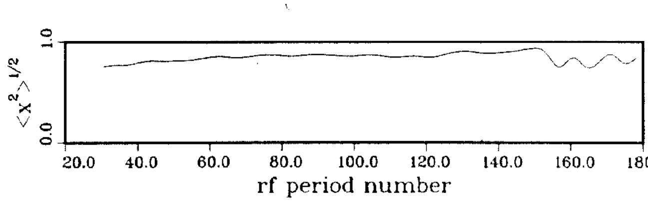

Fig. 1. The rms beam size in the x-direction in mm as a function of rf-period number. Values shown correspond to times in the rf phase when maximum focusing occurs in the x-direction. These results are for the standard accelerator with no errors.

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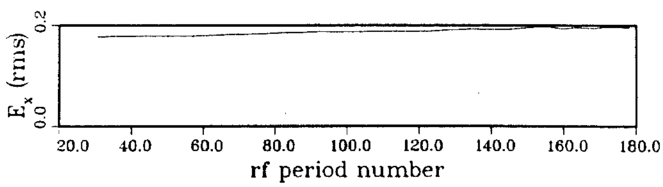

Fig. 2. The rms emittance in the x-direction in mm•mrad as a functon of rf-period number. These results are for the standard accelerator with no errors.

remain constant if measurements are always made at the same point in the rf phase. With slowly changing parameters, the matched beam accommodates itself to provide a nearly matched condition if the changes are slow enough. In this case, the only harmful effect is the loss of acceptance caused by steering the matched beam near a wall or by reducing the size of the effective potential well in the longitudinal direction. However, with rapidly varying errors, no match is possible; emittance growth is inevitable, regardless of the input distribution. We will consider random errors that have a constant-frequency spectrum up to a high-frequency cutoff.

Random Errors in Focusing Strength

Consider the situation in which the strength of the quadrupole focusing field varies randomly from each half-period to the next. It was found that to make the transverse emittance increase by a factor of 1.5, an error amplitude of about 2% was required. For a fixed intervane potential difference, this corresponds to a ±2% variation in the approximately 8-mm vane separation, or about a ±0.2-mm (0.006 in.) fluctuation. (Note, however, that in an actual RFQ, vane movements change the rf mode and do not preserve intervane potential differences.)

Random Offset Errors

Consider the center of the RFQ field offset randomly every half-rf-period. This introduces a fluctuating dipole field (and a small linear coupling with the z-direction from the coupling term). Simulation shows that an offset fluctuation amplitude of 0.06 mm (0.002 in.), which is 0.75% of the intervane separation, produces a transverse emittance growth factor of 1.5 ± 0.1 (result of 10 runs with different random-error sequences). This emittance growth is accompanied by a motion of the transverse beam centroid. Figure 3 shows the evolution of the x-centroid amplitude $A_x = [x^2 + (p_x\lambda/\sigma mc)^2]^{1/2}$, for a certain run, where x and p_x are the coordinates of the beam centroid and σ is the transverse phase advance per rf period.

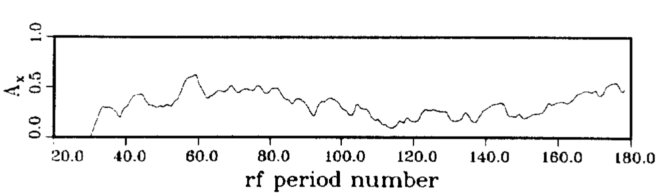

Fig. 3. The amplitude of the beam centroid in the x-direction in mm as a function of rf-period number. The accelerator contains random dipole errors corresponding to an offset error amplitude of 0.75% of the intervane separation.

Figure 4 shows the final phase-space projections for this case. The average final centroid amplitude is 0.46 ± 0.30 mm. Because the standard deviation is so large, this cannot be considered a diffusion process (also evident because Fig. 3 is not a smoothly increasing curve). The effect depends on the details of the errors, not just on the frequency spectrum.

Fixed Dipole Errors

Consider a beam injected off-axis with offset x_o in the x-direction. This introduces a dipole force of $kx_o \sin\omega t$ acting on the beam in addition to the quadrupole force $-kx \sin\omega t$, where k is the force constant. For small phase advance per rf period, the effective force on the particle for the slow (betatron) motion is given by[5]

$$F_{eff} = - \langle F \partial F/\partial x \rangle / m\omega^2 \quad , \tag{2}$$

where F is the actual force proportional to $\sin\omega t$, and the average is over the fast (rf) time scale. For our case the effective force is $-k(x-x_0)/2m\omega^2$, so that the effect of the sinusoidal dipole is to cause the beam centroid to move about x_0 with the undepressed betatron frequency, just as in the time-independent case. (This is because the quadrupole and dipole forces are in phase. If the sinusoidal dipole force acted alone, to first order it would have no effect on the particles.) The simulations verify this behavior. Some emittance growth accompanies the motion of the centroid. It takes an offset of 0.7 mm (9% of vane separation) to produce an emittance growth factor of 1.5 in the x-direction. There is little coupling to the y- or z-directions.

Slowly Increasing Dipole Error

Consider a dipole field that increases linearly starting from zero, a situation obtained by injecting a beam on-axis into a bent RFQ. If the change is slow enough, we expect the initially matched beam to adapt itself and remain matched. The simulations show that to produce an emittance growth factor of 1.5 requires a final 40-mm offset (5 times the vane separation).

Voltage Errors

The RFQ is designed to operate with a vane potential difference of 111 kV, constant along its length. With an imperfect rf mode, however, the voltage may drop below this value at some points. This leads to a reduction in transverse focusing strength, and, more importantly, a reduction in longitudinal acceptance. If V_D and ϕ_{sD} are the design values of the voltage and synchronous phase, then the voltage and synchronous

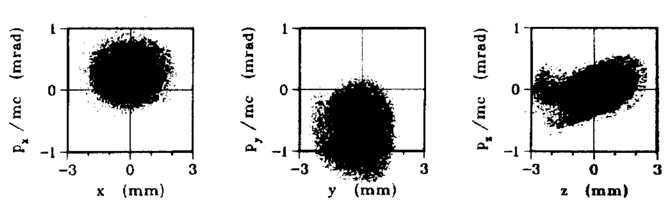

Fig. 4. Final phase-space projections for the case with random dipole errors corresponding to an offset error amplitude of 0.75% of the intervane separation.

phase satisfy $V \cos \phi_s = V_D \cos \phi_{sD}$. Solving for V when $\phi_s = 0$ gives the lowest voltage at which the RFQ can operate, corresponding to a zero longitudinal acceptance. We can estimate the longitudinal acceptance, neglecting space charge and acceleration, by assuming the width of the longitudinal potential well $V_z(kz)$ to be $|2\phi_s|$ and its height to be $V_z(\phi_s)$. (There are few particles outside this region, especially in the presence of space charge, which reduces the height but not the width of the potential well.) Setting this acceptance to the longitudinal emittance E_z and solving for the critical voltage gives

$$V = \frac{V_D \cos \phi_{sD}}{\cos \left(\frac{24\pi^2 mc^2 E_z^2}{eAV_D \beta^2 \lambda^2 \cos \phi_{sD}} \right)^{1/5}} \quad . \tag{3}$$

For our example, the maximum value of this quantity, 98.2 kV, occurs at rf period 166 (the 1.2-MeV point).

Simulations were done in which the voltage was reduced linearly from 111 kV at period 120 (0.2 MeV) to a voltage V at period 150 (0.5 MeV) and was held at V for the remainder of the time. Figure 5 shows the resulting output energy spread as a function of V. At 96.1 kV, the longitudinal acceptance drops to zero. Simulations could not be done with lower voltages because no synchronous phase exists. Because for the low voltage cases many particles are being lost longitudinally, external and space-charge forces were computed assuming periodicity in z with period $\beta\lambda$. No particles were actually lost to the walls, however.

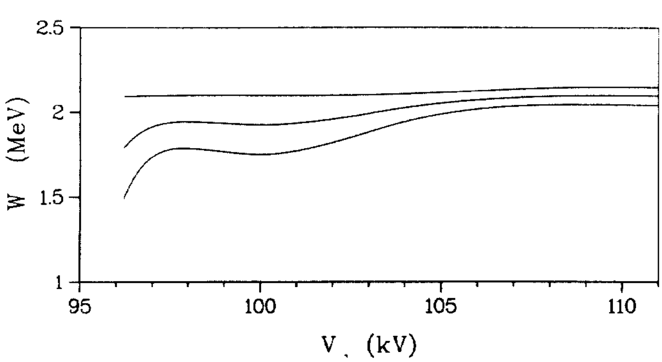

Fig. 5. The energy spread in the output beam caused by a slow drop in voltage at the high-energy end to the value V. The top curve is average energy plus its standard deviation, the middle curve is average energy, and the bottom curve is average energy minus its standard deviation, all as a function of final voltage V.

Discussion

The RFQ mode structure, characterized mainly by the presence of unwanted slowly varying dipole fields and slow variations in the quadrupole field, is very sensitive to small changes in the average vane positions. Rapidly varying field errors, however, could only be caused by small imperfections in the vane machining. Our simulations assumed the fast errors were random and had a cutoff at twice the rf. Faster random errors would be more harmful (in the regime of small errors we could describe the motion with a diffusion equation having a diffusion constant proportional to the error cutoff frequency), but they are not possible because the length scale for field errors cannot be much shorter than the vane separation. Actual fast RFQ imperfections should be easy to keep below the 0.06-mm (0.002-in.) threshold computed.

Apparently, the main problem in RFQ construction is the minimization of the slow errors. The presence of transverse dipole fields and voltage deficiencies at critical points (where $|\phi_s|$ is small) can reduce transverse and longitudinal acceptances. This can lead to particle losses, especially for larger emittance beams.

Acknowledgment

I thank P. Channell for many useful discussions.

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