

ON IMPROVING THE CHROMATIC EFFECTS OF STORAGE RINGS WITH ANTISYMMETRIC INSERTIONS*

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Summary

High luminosity storage rings require good chromatic behavior for beams with large momentum spreads. This requires that the effects of half-integer structure resonances for off-momentum particles be minimized. We show that a lattice with antisymmetric insertions can be so designed that the driving term for the half-integer structure resonance is suppressed by cancellation of successive pairs of high-beta multiplets. Hence, even though the periodicity is half that of a lattice with symmetric insertions, the chromatic properties are similar.

Introduction

In large future proton-proton intersecting storage rings it may be economical to utilize the '2-in-1' design, wherein the two superconducting vacuum tubes and coils are embedded in a single iron yoke. Indeed this option was recently contemplated for the CBA.

In a 2-in-1 dipole the magnetic field of a coil that produces a vertical field on one beam reinforces the oppositely directed field on the other, provided that the beam lines are deployed horizontally. Similarly in 2-in-1 quadrupoles the gradient coils reinforce each other provided that a horizontally focusing quadrupole on one beam is defocussing on the other. Also, this arrangement has the advantage that that it helps to decouple the closed orbit responses of the two beams to quadrupole misalignments.

However, the opposite polarity of the quadrupoles complicates the lattice design. The most important consequence is that it requires that the insertions be antisymmetric about the crossing points. As a consequence the lattice either has a strong periodicity of half the number of crossings, or strong asymmetry about the arc centers. We discuss in this paper the former possibility, though we have investigated the latter also. Another complication is that the beams must be made to cross with special dipoles close to the crossing points which makes it difficult to totally suppress the dispersion throughout the insertion.

This contrasts with the 1-in-1 symmetric lattice in which adjacent quadrupoles in the two rings have the same polarity, the insertions are symmetric, the periodicity is nearly equal to the number of crossings, and the dispersion can be totally suppressed in the insertion.

The lowered periodicity doubles the number of half-integer structure resonances for off-momentum particles. These resonances, which are strongly driven by the quadrupoles near the crossing points, adversely affect the momentum dependences of the tunes and orbit functions and the dynamic aperture.

As a result of our studies of the lattice implications of a 2-in-1 version of the CBA, we have found that the adverse consequences of antisymmetric lattices can be largely overcome if one induces a pairwise cancellation in the contribution of the high- β quadrupoles to the linear resonances.

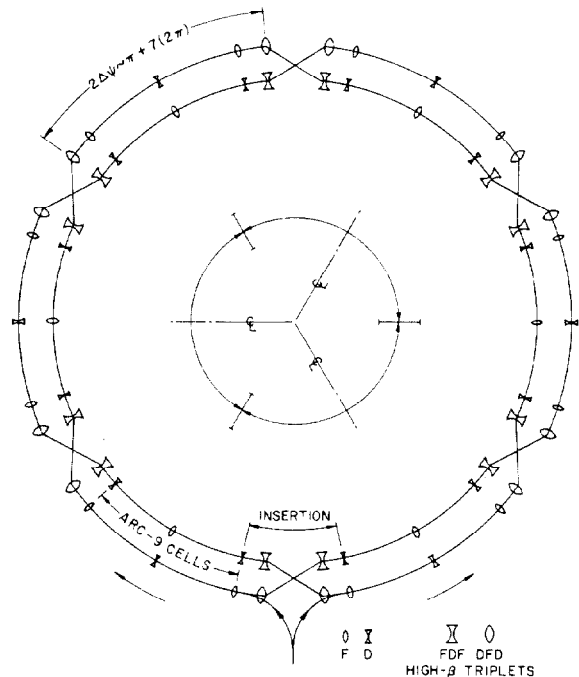


Fig. 1. Topology of antisymmetric 2-in-1 proton-proton storage rings.

Antisymmetric Ring Structure

An illustration of the topology of an antisymmetric 2-in-1 lattice system devised for the CBA is shown in Fig. 1. There are six concentric arcs each containing nine FODO cells connected by insertions that include the crossings. Each insertion has eight quadrupoles on each side of the crossing and some dipoles. Only a few quadrupoles are shown, enough to indicate the topology imposed by the opposite polarity of the quadrupoles. A more detailed schematic is shown in Fig. 2. Adjacent to the crossing magnets B^+ , B^- are quadrupole triplets Q_0 , Q_1 , Q_2 , which are represented by the large lens symbols in Fig. 1. The central Q_1 of the triplets have the greatest strength and highest β -function values in the ring, so they make the greatest contribution to the off-momentum resonances.

From Fig. 1, it is clear that the quadrupole polarity imposes a strong N_s -fold periodicity, where the number of superperiods N_s is half the number of sectors (or crossings) N . The half-integer resonances thus occur at multiples of $N_s/2 = N/4$ (or $1/2$ in the CBA case). To cancel the contribution of the

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Q1 quadrupoles one must make the phase difference between two adjacent Q1's of the same polarity be an odd multiple of $\pi/2$, and to make them identical in strength and β -function value by imposing reflection symmetry of the lattice about the arc centers.

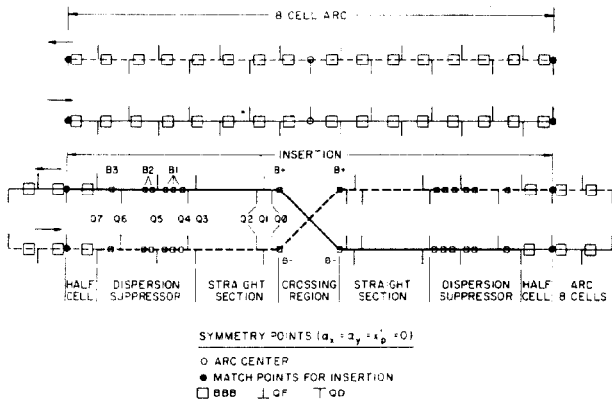


Fig. 2. Schematic design of a 2-in-1 lattice designed for the CBA.

Fortunately a phase difference close to the desired value follows almost automatically from the structure. Suppose the tune is at a half-integer resonance:

$$\nu_p = (p + \frac{1}{2}) N/2.$$

Then the phase between crossings is $\pi(p + \frac{1}{2})$. Due to the low β 's at the crossings, the Q1's are each about $\pi/2$ away from the crossing. Hence the phase differences between two Q1's is approximately $\Delta\Psi = \pi(p - \frac{1}{2})$. The resonance is driven by Fourier harmonics of $e^{i2\Psi}$, so the Q1's will be about π apart in phase.

Structure Resonance Strength

The stopband half width for the half-integer structure resonances of off-momentum particles is given by¹

$$\Delta\nu_{sb} = \frac{1}{4\pi} \frac{\Delta p}{p} \left| \int \beta (K - K' X_p) e^{2i\Psi} ds \right|$$

where $K = B'/B\rho$, $K' = B''/B\rho$, β , and X_p are the beta and dispersion functions, and $\Psi = \int ds/\beta$ is the betatron phase. The integral is to be evaluated at a resonant tune value, $\nu_p = (p + \frac{1}{2})N/2$. The β -function beat factor at the actual tune ν is¹

$$G = 1 + \Delta\beta/\beta = 1 + \Delta\nu_{sb}/(\nu - \nu_p).$$

Consider the contribution of the central Q1 quadrupoles near the crossing point to the resonance width. If two adjacent Q1's are exactly $\pi/2$ apart in phase and identical in strength, they contribute nothing to $\Delta\nu_{sb}$. Suppose their phase difference (mod 2π) is $\pi/2 + \chi$, and that they differ in strength by

$$\lambda = \frac{(BK\ell)_2 - (BK\ell)_1}{(3K\ell)_2 + (BK\ell)_1}.$$

Then the stopband half-width due to the $N/2$ pairs of Q1's of the same polarity may be shown to be

$$\Delta\nu_{sb} = \frac{NBK\ell}{4\pi} (\chi^2 + \lambda^2)^{\frac{1}{2}} \frac{\Delta p}{p}.$$

For the example lattice, $\lambda = 0$ due to the symmetrization about the arc centers, $\chi = 0.092$ radians, $\beta = 247$ m, $K = .0526$ m⁻², $\ell = 3.918$ m, whence for 1% momentum error, $\Delta\nu_{sb} = .0075$. The resonant tune is 25.5, so at the design value $\nu = 25.4$, $G = 1.075$.

If there is any asymmetry between the pairs of Q1's, it should be less than $\lambda_{\max} = \chi/2 = 0.046$ in order to not increase $\Delta\nu_{sb}$ appreciably.

Chromatic Behavior of the CBA Lattice Example

Figures 3-6 show the momentum dependence of the β -functions, dispersion X_p , and chromaticity ξ for the 2-in-1 CBA lattice example compared with a symmetric 1-in-1 lattice. In both cases, two sextupole families are used to produce a small positive chromaticity. The chromatic behavior is very similar in the two lattices, in spite of the fact that the 1-in-1 lattice has symmetric insertions and approximate 6-fold periodicity. The only significant difference is that in the symmetric lattice $\beta_x^* = 43$ m, $\beta_y^* = 7.5$ m at the crossings. In the antisymmetric lattice, $\beta_x^* = \beta_y^*$, so that the chromatic behavior of β_x and β_y become similar. The straightness of the chromaticity curves also follows from suppressing $\Delta\nu_{sb}$, since this factor contributes to the dependence of the tune on $(\Delta p/p)^2$.

The good behavior of X_p vs. $\Delta p/p$ in the 2-in-1 lattice follows from so designing the insertion that X_p is small in the high- β multiplets. This is difficult but not impossible to achieve, as our example shows.

Conclusion

It is possible to design 2-in-1 lattices for p-p storage rings with antisymmetric insertions that give good chromatic properties by symmetrizing about the arc centers, and imposing a $\pi/2$ phase difference between pairs of quadrupoles with the highest β -values.

Reference

1. E.D. Courant and H.S. Snyder, Annals of Physics, Vol. 3, No. 1, 1958.

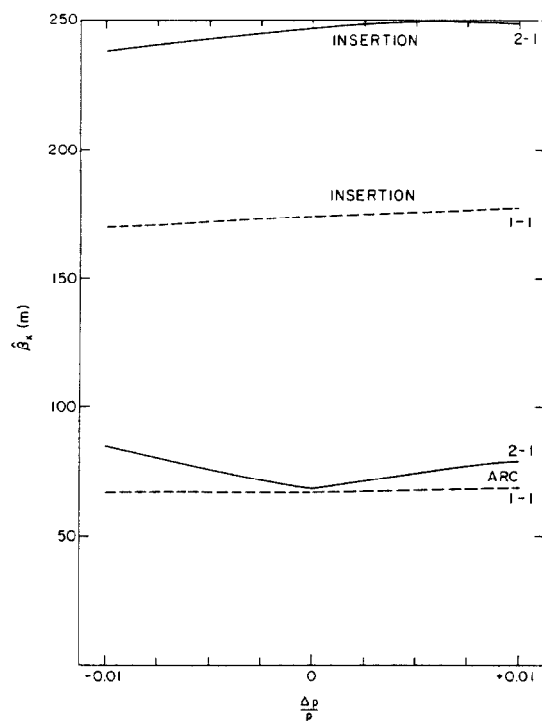


Fig. 3

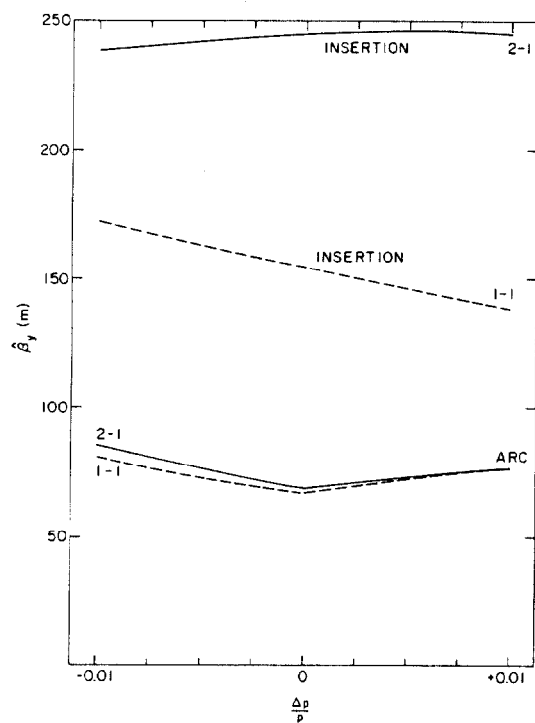


Fig. 4

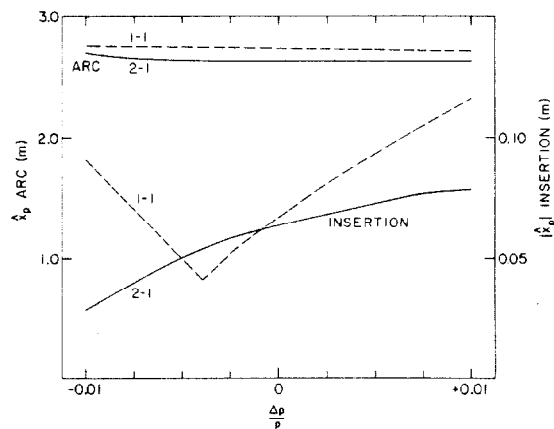


Fig. 3-5. Momentum dependence of $\hat{\beta}_x$, $\hat{\beta}_y$, and $\hat{\alpha}_p$ in the insertion and the cells for 2-in-1 (solid curves) and 1-in-1 (dotted curves) lattices.

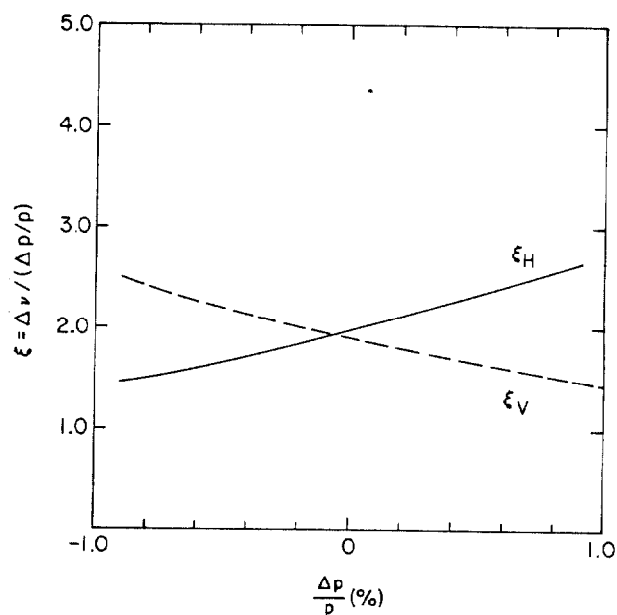


Fig. 6. Momentum dependence of chromaticity $\xi = \Delta v / (\Delta p/p)$ for 2-in-1 lattice. The corresponding 1-in-1 curves are nearly identical.