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A DISPERSION-FREE LONG STRAIGHT SECTION FOR A FIXED-FIELD ALTERNATING-GRADIENT SYNCHROTRON *

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The FFAG synchrotron is generally designed to be "scaling" in order to assure that resonances are not crossed. In compact designs, where the spiral angle and radial gradient are very high, it is necessary to depart from pure scaling by inserting radial cuts in order that there be room for rf cavities, etc.[1] It is shown here that a doubly symmetric set of six gradient magnets will provide an insertion that matches the structure for all momenta, providing in addition a long drift space that is free of dispersion.[2] With the space thus provided for rf cavities, very high repetition rates are possible. Such a ring also provides an attractive means of accumulating equilibrium distributions of feeble heavy ion beams by placing a stripping foil in the dispersion-free section.

Introduction

An FFAG synchrotron appears to be an ideal solution for the high flux and time structure required for a spallation neutron source such as the planned SNQ at KFA. For a given space charge limit, very high currents may be accelerated by increasing the repetition rate. Moreover, because the field is static, the maximum rf accelerating voltage may be applied throughout the accelerating cycle--in marked contrast to the rapid cycling synchrotron. With a static field, downtime due to failures, losses and their activation problems should be far smaller than for an equivalent rapid cycling synchrotron.

The classic FFAG machine that was so fully studied at MURA is scaling. That is, every equilibrium orbit is a photographic enlargement of every other equilibrium orbit. This property simplifies the design in that a design good for one energy is automatically good at all energies (with regard to resonances and other single-particle instabilities). The betatron frequencies are constant, and thus no resonances are crossed. However, such machines are highly periodic and provide little or no drift spaces for placing of rf cavities or injection and extraction components. The need for ample drift space to contain radio frequency cavities is a particular concern for rapid-cycle high-intensity machines where a very large accelerating voltage must be provided.

The radial sector FFAG accelerator is expensive for high energies because its circumference factor is around 6. The spiral accelerator can have a circumference factor less than 2 -- less than a normal synchrotron. To keep the radial aperture small, the magnetic field rises rapidly with radius. It is then necessary, in order to achieve adequate vertical focusing, to have a very tight spiral where the equilibrium orbit crosses the magnet edges at an angle of around 10-15 degrees. With such a spiral, there is no radial line along which rf cavities may be placed.

The solution to this dilemma at MURA was to introduce radial cuts in the magnet structure. It was shown that this could be done in a manner such that the betatron frequencies were perturbed within acceptable limits, and adequate stable phase areas were obtained.[1] However, the cuts were very small in the azimuthal direction, and they did eliminate the scaling property.

Another approach, possible for machines with a small momentum range is that being explored at Argonne National Laboratory where a moderate spiral angle and large flutter provide adequate space for the insertion of cavities.[3]

A third solution would be to provide insertions that perfectly match the FFAG and include adequate drift spaces. This is the approach that is addressed here.

Insertion Requirements

The first difficulty is that the Twiss parameters vary enormously along a radial line, and thus the insertion must match to this range of parameters. It is easy to show that the betatron phase advances in the insertion must be multiples of π in order to meet this condition. Then the matching problem is one of providing adequate aperture for the range of Twiss parameters provided.

It is obvious that to match the dispersion in the radial plane, the insertion must provide a phase advance in the radial plane that is a multiple of 2π (i.e., provide an identity transformation in the radial plane).

Because of the momentum range, quadrupoles cannot be used to provide focusing; focusing must be provided as in the FFAG: by means of radial gradients and edge angles.

The design should provide some sort of scaling with momentum in order that a solution at one momentum will also be a solution at other momenta.

Were the momentum range narrow, the pi-2pi straight section would provide such a solution.[2] The problem is then to design a similar insertion using only bending magnets such that the scaling criterion is met.

With these transformation properties, the insertion may be translated upstream or downstream without affecting its matching properties (that is, a portion of the initial drift space may be moved to the last drift space). Moreover, the insertion need not be placed at a radial cut in the FFAG lattice, but the lattice may be cut along an arbitrary curve in such a way as to minimize the excursion of the constituent orbits within the insertion. For example, the cut might be made along a magnet spiral.

Some Symmetry Considerations

We shall use matrix methods to locate a solution using the results of Herrera and Bliamptis.[4] We assume that the insertion is composed of

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several identical parts with specified symmetry properties.

Here, as the planes are assumed to be uncoupled in the linear approximation and there are no linear dispersive terms in the vertical plane, we treat the radial motion by means of a 3X3 matrix acting on the vector $(x, x', \Delta p/p)$, and we treat the vertical motion using a 2X2 matrix acting on (z, z'). Thus the size of the matrix shows in which plane it acts.

Systems with Reflection Symmetry about the Midpoint

The radial-plane identity transfer matrix is satisfied provided the off-diagonal matrix elements of the midpoint transfer matrix vanish (π phase advance), and the off-energy trajectory is displaced from but parallel to the reference trajectory at the midpoint.

Such an insertion will provide a dispersion-free central drift space for a machine with a particular momentum compaction. It has been shown in a previous paper by the author that it is possible for this type of insertion to be dispersion free at the center while retaining the desired transfer characteristics through the entire insertion.[5] Symmetric systems comprising bending magnets will have dispersion in the central drift for entering dispersion-free beams, but will eliminate the dispersion in the center for a properly matched value of the momentum compaction entering it.

A dispersion-free drift space is a particularly desirable location to place a stripping foil as energy losses in the foil will not increase the amplitude of betatron oscillations. In an FFAG the beam width in such a drift space will be quite small which should be helpful in the design of the rf cavities.

Such a drift space provides a very interesting capability for heavy ion accumulator rings. A stripper can be placed in the dispersion-free section of the insertion, and all charge states leaving the stripper will be on stable orbits, thereby providing the means of injecting significant currents of heavy ions even though the injection energy is insufficient to fully strip most of the ions.[6]

For the purpose of this paper, we consider only systems where each half of the insertion inself has a further symmetry property about its midpoint. This additional property considerably reduces the number of conditions to be met. Of course, such systems will have drift spaces at the ends with the same total length as that in the center of the insertion. (It has been shown previously by the author that it is possible to place all of the drift length of a pi-2pi insertion in the center.)

The requirements for the vertical plane are simply that the diagonal matrix elements must vanish at the midpoint.

Symmetry Implications for the Half Insertion

If each half of the insertion were to possess antisymmetry with respect to reflection about its center, then all the conditions are satisfied provided (1) the diagonal elements of the radial matrix vanish, (2) the dispersive displacement vanishes, and (3) the product of the diagonal elements in the vertical plane equals $\frac{1}{2}$--all evaluated at the midpoint of the half insertion. With this condition, the entering dispersion is reversed at the center of the dispersion.

If the half insertion is symmetrical with respect to reflection through its midpoint, the conditions to be satisfied are the same except there are no conditions on the dispersion.

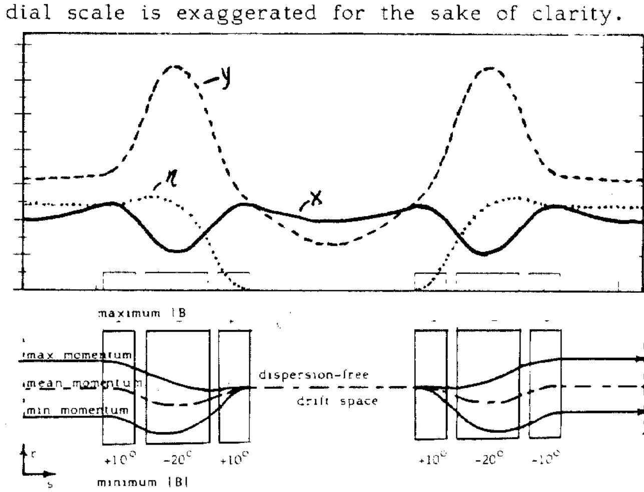
Two-Cell System Without Reflection Symmetry

If the insertion comprises two identical cells then the conditions are met providing the diagonal elements of the vertical matrix vanish at the end of the cell, and the radial matrix, without regard for dispersion, is the reflection matrix (inverts displacements and slopes).

A Solution

It is possible to satisfy the conditions with a symmetrical array of six gradient magnets, two of which are reverse field magnets (such as are found in a radial-sector FFAG). For simplicity, we consider a system that possesses the additional property of each half possessing reflection symmetry about its midpoint. The separation between magnets of each half is arbitrary. All magnet edges are parallel. The positive-field magnets have positive (radially-focusing) gradients, whereas the negative magnets have negative gradients. Thus the configuration is appropriate for scaling over the range of energies to be contained. The absolute value of the magnetic field in all magnets increases in the radial direction. The total bending angle is zero. As mentioned above, for any choice of the free parameters, there is one entering momentum compaction that will result in no dispersion within the central drift space.

Envelope plots are shown for the radial and vertical planes in the following figure which also shows the equilibrium orbit of a higher energy particle. The insertion is assumed to have been made at a radial plane of symmetry in a radial sector FFAG. The lower portion of the figure shows orbits of three different momenta. The radial scale is exaggerated for the sake of clarity.



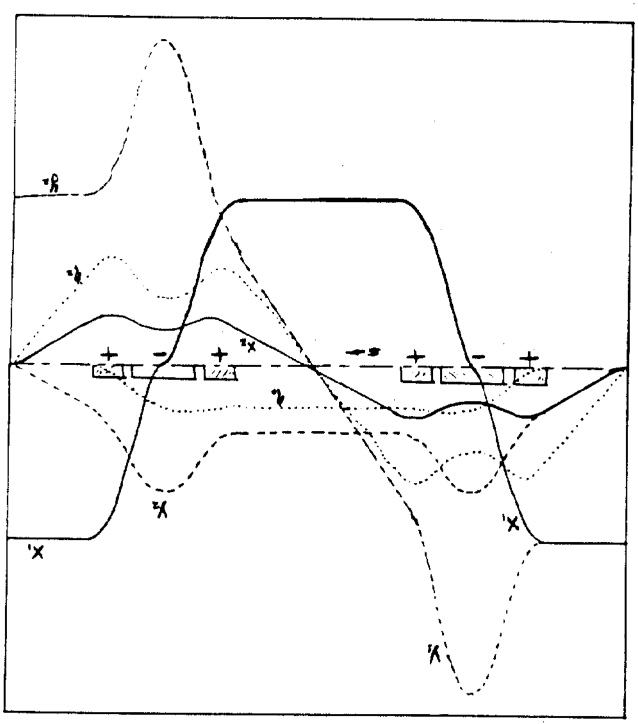
The gradient in the positive magnets is comparable to that in the FFAG at the point of insertion. That in the negative magnets is smaller thereby causing the equilibrium orbits for different momenta to move together. The difference in rigidity causes an immediate momentum separation at the ends of the central drift space.

Taking the radius of curvature in the bending magnets (all the same) to be one unit, the arc lengths for the bending magnets are 0.17453 (the middle magnet is twice this length) so the angle of bend in the magnets for one fourth of the insertion is ten degrees in then ten degrees out. The magnets are separated by 0.053702 units, and the longer drift spaces are 0.46011 units. The field index of the positive magnets is -19.4469 and that for the negative magnets is 15.000.

Scaling

In a scaling FFAG, all lengths scale with the average radius, the field scales with the radius to the kth power, and thus the momentum scales with the (k+1)th power of the radius. Here, such a scaling would make the length of the insertion dependent upon the momentum; this is clearly unacceptable as the insertion is to be placed in a radial cut.

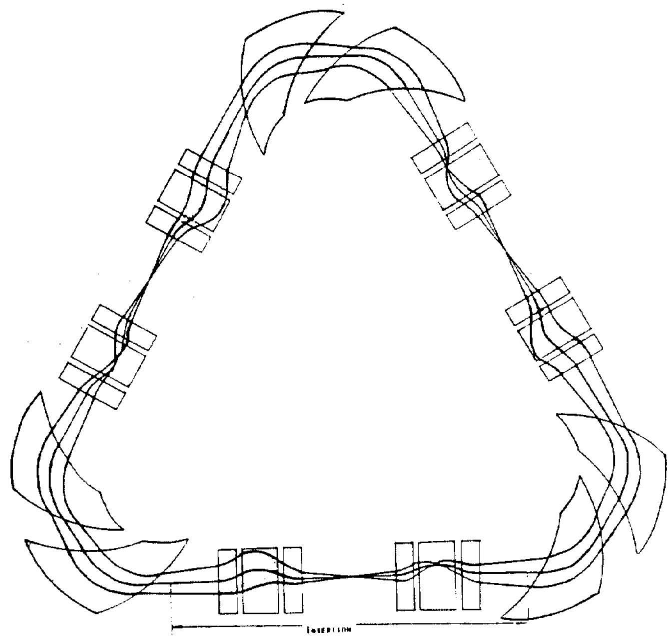
We can, however, scale in the radial direction only. Consider the independent solutions to the linear equations drawn in the figure below. If we stretch the figure arbitrarily in the radial direction, the resulting figure will satisfy all the conditions necessary for the insertion to provide an identity transformation in the radial plane and an inversion transformation in the vertical plane. The size of the beam at the midpoint will in general be a function of momentum. Making this scaling does impose a relationship between the radial gradient and the field which must be satisfied in order to allow scaling to cover a significant momentum range.



Insertion Showing Linearly-Independent Trajectories for Small Displacements.

Resonances

Reducing the periodicity drastically by means of insertions significantly increases the number of resonances that are driven. This is of particular concern in an FFAG where there are strong nonlinear driving terms, and any particular design would have to be evaluated with this in mind. In the figure below, we show three insertions in a spiral machine--three being the minimum number that allows us to find an operating point with no resonance below fourth order in the half-integer square around the operating point.



FFAG With Insertion: Three-Fold Periodicity

The FFAG in which the insertion is placed is a copy of the first MURA spiral sector machine. Because it is a spiral sector machine, the scaling with radius involves a rotation, and thus orbits of different energies entering the insertion are not parallel. This means that the central drift space is not quite dispersion-free. Perhaps this could be remedied by using spiral magnets in the insertion. (Equilibrium orbits for three differing momenta are shown.)

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