

Study of Periodic Tune Modulation with the Beam-Beam Effect

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Abstract

Simulations of weak-strong $p\bar{p}$ collisions with a periodic tune modulation show the possibility of beam blowup at sufficiently strong modulation amplitudes. This beam blowup is associated with the appearance of nonrepeatable "chaotic" trajectories and occurs when low order resonances are crossed by the modulation. In this paper we also report results of an investigation of the dependence of this blowup upon the modulation frequency, with the modulation amplitude fixed. It is determined that a threshold frequency exists, modulations at frequencies greater than the threshold do not lead to beam blowup.

Simulation Procedure

In proton-antiproton ($p\bar{p}$) collisions in the "Tevatron" particle trajectories will be affected by the highly nonlinear force of the "beam-beam" interaction the electromagnetic force field of the opposite beam in the collisions. The trajectories between collisions will be subject to tune modulation from turn to turn through sources such as power supply ripple or synchrotron oscillations with uncorrected chromaticity. Previous investigations of the beam-beam interaction by the present authors have considered a constant "beam-beam" interaction form and particle transport. In this paper we add the complication of tune modulation and investigate its effects. Without this modulation a remarkable beam stability has been found.²

We approximate particle circulation around the accelerator ring as the product of two transformations: a linear transport around the storage ring followed by a nonlinear beam-beam "kick" at the interaction area.

Transport around the ring can be represented by a 2x2 matrix for both transverse (x and y) dimensions. In this linear transport x and y motion are decoupled. $v_x, v_y, \beta_x, \beta_y$ are the usual Courant-Snyder tunes and betatron-functions. The product of these transformations is equivalent to integration of the equation of motion:

$$x'' + K_x(s)x = -\frac{4\pi\Delta v_x}{\beta_x} F_x(x,y) x \delta_p(s) \quad (1)$$

and similarly for y, where s, the distance along the storage ring, is the independent variable and $\delta_p(s)$ is a periodic delta-function.

In the present report we choose parameters which approximate the conditions¹ in the Tevatron: $\Delta v_x = \Delta v_y = 0.01$, $\beta_x = \beta_y = 2m$ and we choose

$$F_x = F_y = \frac{1 - e^{-(x^2 + y^2)/2\sigma^2}}{(x^2 + y^2)/2\sigma^2} \quad (2)$$

with $\sigma = 0.0816$ mm. This is the nonlinear force due to a round, gaussian charge distribution. This does not change from turn to turn which means that we use the "weak-strong" approximation where the "strong" beam is unaffected by the weak beam.

To simulate tune modulation, the tunes v_x and v_y are changed from turn to turn following

$$v_x = v_{x0} + a_x \sin \omega_x t \quad (3)$$

with a similar equation for v_y . We have used $\omega_x = \omega_y$ in all cases, which is expected for most reasonable sources of tune modulation, and we have considered two possible relative phases:

$$[a_y = a_x \text{ labelled ++}] \text{ and } [a_y = -a_x \text{ labelled +-}] \quad (4)$$

The magnitudes of a_x and a_y are chosen equal. We have chosen values of a_x between 0.001 and 0.01 in agreement with expected values. We have first chosen a frequency precisely one thousandth (.001) of the collision frequency. Since the Tevatron collision frequency is 50 kHz, the modulation frequency is 50 Hz, quite close to expected power supply modulation (60 Hz) as well as the synchrotron motion frequency. The modulation is chosen as a precise fraction of the collision frequency to simplify computation; the linear matrix can be calculated initially for each of the 1000 possible values and stored. This eliminates the necessity of recalculating the matrix on each turn.

In the simulations a set of 100 initial particle positions are generated randomly within a gaussian distribution in the 4-D phase space (x, x', y, y') . These are transported through many turns with tunes modulated following equation (3). Every 2000 turns the rms emittances X, Y, and R are calculated using:

$$\begin{aligned} X &= 6\sqrt{\langle (x-\bar{x})^2 \rangle \langle (x'-\bar{x}')^2 \rangle} \\ Y &= 6\sqrt{\langle (y-\bar{y})^2 \rangle \langle (y'-\bar{y}')^2 \rangle} \\ R &= \sqrt{X^2 + Y^2} \end{aligned} \quad (5)$$

In these simulations 6 million turns (corresponding to 2 minutes Tevatron time) are calculated in each case, and 3000 emittance values are generated and analyzed statistically. "Doubling" times for X, Y and R emittance are obtained from the slopes of the best straight line fits for X, Y and R as functions of time from $t = 0$, using rms values calculated every 2000 turns. Statistical errors are also included in the analysis.

Simulations with Constant Modulation Frequency

In this section we describe the results for those cases where the amplitude of the modulation was varied but the period was taken constant at 1000 turns.

*Operated by Universities Research Associated Inc. under contract with the Department of Energy.

For all tune modulation simulations we have chosen initial tunes at $\nu_x = .3439$, $\nu_y = .1772$, $\Delta\nu = .01$. These are the parameters of Case C of Reference 2, which is a case chosen in tune region free of resonances lower than ninth order and showed the greatest stability in the long-time simulations. The addition of tune modulation permits the appearance of low order resonances in combination with the modulation. Figure 1 shows the "tune-space" near the Case C tunes, and one finds third, sixth and eight order resonances accessible by tune modulation. We have considered 13 separate modulation cases:

1. $a = 0$ (no modulation) Case C of Reference 2.
2. (++) modulation with $a_x = a_y = .001, .003, .005, .007, .009, .01$.
3. (+-) modulation with $a_x = -a_y = .001, .003, .004, .005, .007, .01$.

In the (++) simulations we saw no significant changes in rms emittances for $a < .009$. However, for $a = .01$ some statistically significant changes appear. There is a strong anticorrelation between changes in x and y emittances but the changes are 1% after 6 million turns, and represent "doubling times" of >0.1 days, only a few standard deviations from zero change.

For the (+-) simulations more dramatic changes occur. For $a < .003$ no statistically significant changes occur but for $a > .004$ there is a fast blowup of the beam emittances, with doubling times of fractions of a minute rather than days. The blowup is evident within 200,000 turns of particle motion and continues throughout the simulations.

Our conclusion is that beam blowup can occur when there is tune modulation and beam-beam interaction, if the modulation is of adequate amplitude.

Simulations with Constant Modulation Amplitude

We have fixed the amplitudes $a_x = a_y = .005$, a case which shows modulation blow-up for $N = 1000$ revolutions period, and varied the period from $N = 8$ turns, to $N = 10$ turns. Fig. 1 shows the modulation tune space showing the low order resonances.

For $N < 100$ no significant increase in total beam emittances are observed; doubling times are many hours. For $N = 200$ significant increases appear and for $N > 400$ fast beam blowup occurs, with doubling times less than a minute. The results are shown in Fig. 2.

Reversibility Tests and Chaotic Trajectories

Our basic test of computational accuracy is a reverseability test. In these tests initial particle positions are transported forward N turns, the transport is reversed and the trajectories are returned. Forward and return particle positions are compared. As we discussed in another paper² "chaotic" trajectories diverge exponentially in this test and it is a useful tool for distinguishing them.

In these tests all 100 trajectories are transported 10^5 turns forward and returned. Most trajectories develop errors of order 10^{-20} in agreement with the expected error for non-chaotic trajectories. The results of the test are:

- i) For constant period $N = 1000$, no large error trajectories appear for the (++) cases until the largest (.01) where 3-6% are large error.
- ii) For constant period $N = 1000$ (+-) case there are no chaotic trajectories for $a < 0.004$. For $a = .004$, 10% of the particles are chaotic and for $a = .005$, about 50%. The number of chaotic trajectories appears nearly constant for $a > 0.005$.
- iii) When the modulation amplitude was kept constant (at 0.005) and the frequency varied, no chaotic trajectories were found for $N < 16$. Chaotic trajectories appear at $N = 32$. Their number increase with N up to about of 50% at $N \sim 2000$.

There is a strong correlation between beam blow-up and the appearance of chaotic trajectories.

Tune Modulation, Chaotic Trajectories, Resonances and Beam Blowup

As mentioned above, the addition of tune modulations permits the appearance of low order resonances. In Figure 1 we show the tune diagram for (+-) .005, a case with beam blowup and 40 chaotic trajectories. The tune spread of the beam ($\Delta\nu_x = \Delta\nu_y = .01$) is outlined at the center ($a = 0$) and at the extremes of tune modulation, showing the extent of modulation. Low order resonances are indicated by darkened lines.

The appearance of chaotic trajectories in the (+-) simulations is definitely correlated with the inclusion of the third and sixth order resonances ($-\nu_x + 2\nu_y = 0$, $4\nu_x - 2\nu_y = 1$). This can be made more definite by considering the progression of (+-) cases $a = .003, .004$ and $.005$. At .003 the resonances barely intersect the edges of the tune modulation and do not effect the particle motion; there is no beam blow-up. At .004 the resonances do intersect the beam for large amplitude particle motion; we find 10% chaotic trajectories and beam blow-up. At $a = .005$ the resonance line intersects the center of the tune square at the extremes of the tune modulation; 40-50% of the trajectories are chaotic.

In Figure 3 we show particle tunes averaged over the first thousand turns for the $\pm .005$ case. The tunes are concentrated near the diagonal as is expected from basic considerations. Investigations of particle trajectories find that they can be categorized into three distinct groups:

1. "Non-chaotic" (repeatable) trajectories which do not change their mean amplitudes substantially in long-time simulations. (A)
2. "Chaotic" trajectories which may undergo some change in mean amplitudes but do not diverge to large amplitudes. (C)
3. "Chaotic" trajectories which do diverge to large amplitude. (F)

In Figure 3 we have identified these three separate types and find that they occupy distinct regions in tune space. The largest amplitude particles are predominately chaotic and divergent. Intermediate amplitudes are chaotic but not divergent. Smaller amplitudes are non-chaotic. This separation is in agreement with an intuitive picture in which chaotic trajectories are caused by sweeping of a low order resonance through the beam, and only those trajectories which reach an amplitude swept by

the resonance can be chaotic. This picture is confirmed by consideration of the $(+-)$.004 case. The nine chaotic, divergent trajectories are the largest amplitude trajectories and the lower amplitude tune modulation should only "sweep" through the largest amplitude particles. These largest amplitude particles are also labeled in Figure 3. (O)

In Fig. 4 we show the same distribution 3 million turns later. Some particles have been pushed to large amplitude. These particles are those initially at large amplitude and are those that are chaotic and "grow". This is in agreement with beam-beam observations in the CERN SPS pp collider.

We have also confirmed the hypothesis that tune modulation is necessary for the appearance of chaotic, divergent trajectories. Six million turn simulations with $\Delta\nu = .01$ at the center ($\nu_x = .3439$, $\nu_y = .1772$) and at the two extremes of the tune modulation, ($\nu_x = .3489$, $\nu_y = .1722$) and ($\nu_x = .3389$, $\nu_y = .1822$) without modulation show no chaotic trajectories and no beam blow-up even though the extremes do contain low order resonances.

For the $(++)$ simulations only the largest amplitude case shows evidence of chaotic trajectories and it shows no large beam blow-up. In Figure 5 we show the tune diagram for the .01 modulation case and this shows a few low order resonances within the modulation amplitude. Investigation of particle amplitudes indicates that the sixth and third order resonances in the lower right of the tune diagram are probably associated with these chaotic trajectories. Only largest amplitude particles which could reach these resonances are chaotic. Since only a few particles can reach these resonances there is no beam blow-up in this case.

Conclusions

Simulations of the beam-beam interaction with tune modulation find that beam blow-up can occur if the modulation sweeps the beam through a low order resonance. Modulations $> .01$ would be forbidden by this criterion to avoid regions with low order resonances. No beam blow-up should occur at much lower amplitude modulations.

References

1. J. Peoples "The Fermilab Antiproton Source". Invited paper, these proceedings.
2. D. Neuffer et al. "Searches for Arnold Diffusion." These proceedings.

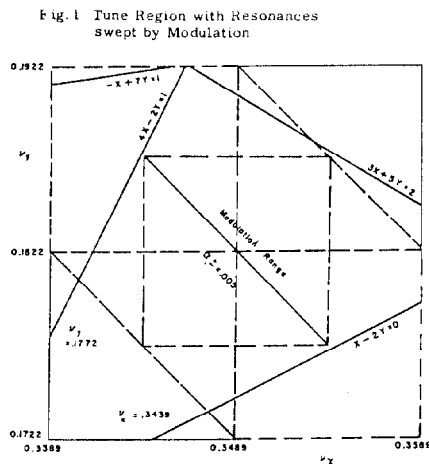


Fig. 1 Tune Region with Resonances swept by Modulation

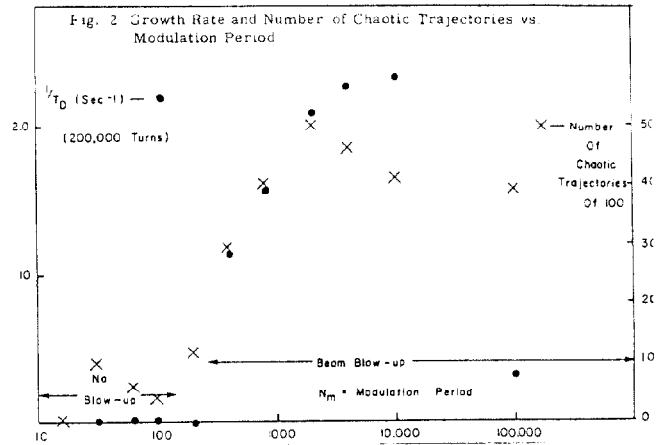


Fig. 2 Growth Rate and Number of Chaotic Trajectories vs. Modulation Period

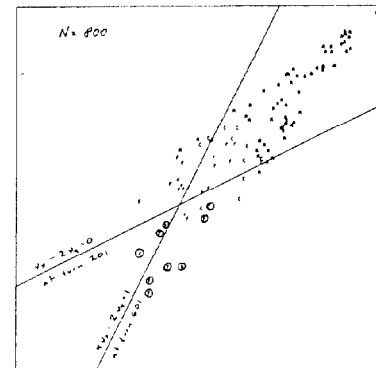


Fig. 3 Tunes averaged over Turns 0 to 800

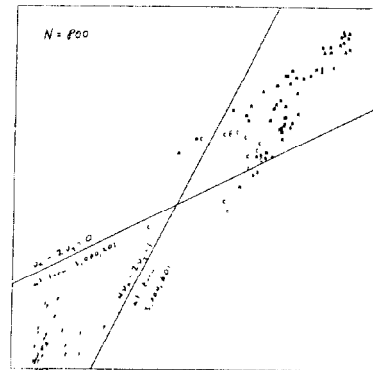


Fig. 4 Tunes averaged over turns 3,000,000 to 3,000,800

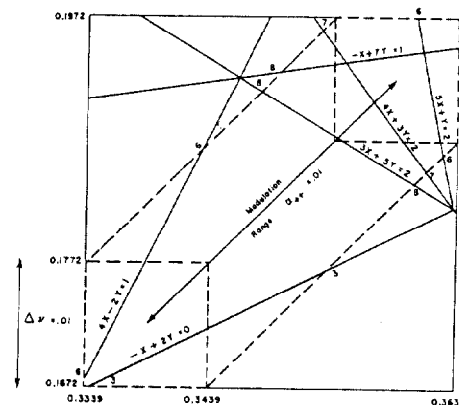


Fig. 5 Tune Region swept by $++$ Modulation Type