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# APERTURE LIMITS DUE TO THE PRESENCE OF HIGHER MAGNETIC FIELD MULTIPOLES*

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## 1. Summary

Superconducting magnets tend to produce magnetic fields which have larger higher systematic multipoles and larger random multipoles than conventional magnets. The current carrying coils in superconducting magnets are usually not shielded by the iron yoke, and thus the magnetic field shape is determined by the current distribution rather than the shape of an iron surface. Higher systematic magnetic field multipoles are then generated by necessary deviations of the current distribution from the ideal desired distribution; random multipoles are generated by errors in the location of the current carrying conductors. The presence of these undesired field multipoles will limit the good field aperture of the accelerator. One effect of these multipoles is that they will distort the working line, the dependencies of the $\nu$-values, $\nu_x$, $\nu_y$, on the momentum $\Delta p/p$. Another effect of these multipoles is to produce linear and non-linear stop bands. This paper is concerned primarily with aperture limits due to the distortion of the working line.

Distortion of the working line produced by the lower field multipoles is usually corrected with a correction coil system provided for that purpose. The working line distortion that remains is that due to higher field multipoles that cannot be corrected with the correction coil system. The magnetic field due to the higher multipoles vary like some high power of r, the radial distance from the magnet center, like at least $r^6$ in the case of the CBA (Colliding Beam Accelerator at BNL). The effect of the higher multipoles on the working line shape will be small for low $\Delta p/p$, and will set in very rapidly at some higher momentum. The higher multipoles will thus produce an aperture wall, inside which the field is good. The field then deteriorates rapidly when it reaches this aperture wall generated by the higher multipoles.

The considerations given in the following sections show that for the systematic field multipoles in the CBA, the distortion of the working line leads to a set of allowed values or tolerances for these higher systematic multipoles. For the higher random field multipoles, the distortion of the working line limits the good field aperature in the CBA at about the momentum spread of $\Delta p/p = \pm.014$, where $\Delta p/p = \pm.01$ is required for CBA operation.

## 2. Aperture Limits Due to Distortion of the Working Line

The aperture limits due to distortion of the working line depend on the details of the particular accelerator being studied. It will be assumed here that the cause of the aperture limit is that found for the CBA. For another proton accelerator, perhaps the dissusion given below can be modified according to the details of that accelerator. For the CBA, the aperture limit may be due to the onset of the transverse collective instability for coasting beams, which is sensitive to the shape of the working line.

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A study of the transverse instability for the CBA was carried out by M. Cornacchia and C. Pellegrini.[1] The working line was shaped to stabilize the beam, and it was found that what seemed to be important was the local chromaticity at the momentum corresponding to the coherent $\nu$-value computed from the dispersion relations. These results can be used to provide a criterion for the allowed higher multipoles based on how much local chromaticity effect due to the higher field multipoles will be allowed.

Let us consider the question of how much chromaticity due to the higher multipoles can be tolerated. This depends on how much chromaticity is needed to stabilize the beam, and how much chromaticity can be provided by the correction system.

How much local chromaticity is needed to stabilize the beam is not easy to determine accurately. It depends on the model used for the impedance of environment, on the mode number, and on solving the dispersion relationship for the coherent tune shift. A rough answer for the 8A beam in the CBA is that a local chromaticity in the vicinity of 8 may be required, which may be off by a factor of 2. The chromaticity C is defined by $C = p d\nu/dp$. This result may be deduced from the results of Cornacchia and Pellegrini.[1] It may be also estimated by finding what $\nu$-spread and chromaticity provided by only sextupole correction coils is needed to stabilize the beam, which is a problem whose solution exists in the literature.[2]

How much local chromaticity can be provided by the correction coils is limited by the $\nu$-spread one can allow over the beam and still stay away from nearby non-linear resonances. For the CBA, a reasonable assumption may be that each correction coil not be allowed to produce $\nu$-shift at $\Delta p/p = .01$ of more than $\Delta\nu = .02$. For the CBA, the dodecapole is the highest multipole provided by the correction coils, and thus the local chromaticity that can be provided by the correction coil system at $\Delta p/p = .01$ is very roughly $C = 8$. $\Delta p/p = .01$ is the extreme edge of the beam for the CBA. This conclusion uses the relationship between the $\nu$-shift $\Delta\nu_n$ and the chromaticity at $\Delta p/p$ due to the $n^{th}$ multipole, $C = 100\ (\eta-1)\ \Delta\nu_n$ for $\Delta p/p = .01$ (See Eq. (1) for definition of multipoles).

The numbers given above for the local chromaticity required for stability, and the local chromaticity that can be provided by the correction coils are very rough. Fortunately, the tolerance on the higher multipoles is rather insensitive to these numbers. The chromaticity generated by one of the higher multipoles is varying rapidly with the momentum $\Delta p/p$. Thus if one of the higher multipoles is within a factor of 2 or so of the allowed value for a beam that extends to $\Delta p/p = .01$, it will exceed the allowed value for a slightly higher $\Delta p/p$. In other words, there is not much physical difference between results for the allowed higher multipoles which differ by a factor of 2 or so.

The results given above indicate that the local chromaticity needed for stability and the local chromaticity provided by the correction coils are roughly equal for the CBA. This means that the chromaticity corrections are quite tight, and any effect, like the

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local chromaticity due to higher multipoles that can use up chromaticity correction capacity, can be harmful.

In the following two sections the above considerations will be applied to determine the effect of the systematic higher multipoles, and the random higher multipoles on the good field aperture in the CBA.

## 3. Aperture Limits Due to Systematic Higher Magnetic Field Multipoles

To establish a criterion for the allowed systematic higher multipoles due to the chromaticity effect, it will be assumed that each systematic field multipole should produce a chromaticity at $\Delta p/p = .01$ of no more than $C = .8$, which is 10% of the amount needed for stability and 10% of the amount that can be provided by the correction coils for CBA. $\Delta p/p = .01$ is the edge of the beam in CBA.

The field multipoles, $b_n$ and $a_n$ are defined by expanding the median plane field

$$B_y = B_o\ (b_0 + b_1\ x + b_2\ x^2 + ..) \qquad (1)$$

$$B_x = B_o\ (a_0 + a_1\ x + a_2\ x^2 + ..)$$

where $B_o$ is the main dipole field in the accelerator. The $\nu$-shift due to the systematic $b_n$ multipoles may be written as

$$\Delta\nu = \Delta\nu_n\ (\Delta p/_p)^{n-1}, \qquad (2)$$

where $\Delta\nu_n$ is given by

$$\Delta\nu_n = \sum_k \frac{1}{4\pi}\ L(k)\ \frac{n\ b_n(k)}{\rho}\ X_p^{n-1}(k)\ \beta(k)\ . \qquad (3)$$

The sum over k is a sum over all the magnets in the accelerator. L is the magnet length, $\rho$ is the radius of curvature and $X_p$ is the closed orbit dispersion. If there is an appreciable closed orbit sagitta in the dipoles, as is the case for CBA, the sagitta effect needs to be included, and in Eq. 3 one needs to replace $X_p^{n-1}(k)\ \beta(k)$ by

$$(X_p(k)\Delta p/p)^{n-1}\ \beta(k) \rightarrow \frac{1}{L(K)} \int ds\ (X_p\ \Delta p/p + x_s)^{n-1}\beta, \qquad (4)$$

where $x_s(s)$ is the sagitta, the closed orbit relative to the axis of the dipole, and the integral is over the dipole.

The chromaticity $C = p d\nu/dp$ is then given by

$$C = (n-1)\ \Delta\nu_n\ (\Delta p/p)^{n-2} \qquad (5)$$

The above equations can be applied to the systematic multipoles $b_n$ in the dipoles to find the allowed values of these $b_n$ so that the chromaticity generated by each multipole at $\Delta p/p = .01$ is less than $C = .8$. For the CBA, this gives the following allowed values for the systematic multipoles in the dipoles.

| n | $b_n'$ |
|---|---|
| 6 | .37 |
| 7 | .54 |
| 8 | .80 |
| 9 | 1.2 |
| 10 | 2.0 |
| 11 | 3.3 |
| 12 | 5.4 |
| 13 | 8.9 |
| 14 | 16 |

The $b_n' = b_n \times (4.4)^n/10^{-4}$.

A different set of allowed systematic higher multipoles can be obtained from a second, more crude criterion, which for historical reasons are the allowed values used for the CBA. It is interesting to note that this more crude criterion gives essentially the same results. This second criterion is to allow each higher systematic multipole to generate, at $\Delta p/p = .01$, a $\nu$-shift which is no larger than 10% of the $\nu$-spread needed to stabilize the beam; this means, a $\nu$-shift of about .002 since the $\nu$-spread to stabilize the beam is about .02. This second criterion gives allowed systematic multipoles given by the following table. These results for the allowed multipoles are used at the CBA.

| $b_n'$ | |
|---|---|
| dipoles | quads |
| .46 | .93 |
| .80 | 1.3 |
| 1.41 | 1.8 |
| 2.5 | 2.6 |
| 4.5 | 3.8 |
| 8.2 | 5.6 |
| 15. | 8.2 |
| 27. | 12. |
| 50. | 19. |

Although the allowed systematic multipoles given by the two criterions differ by almost a factor of 3 for some of the multipoles, they differ by only 10% as far as the aperture is concerned. As mentioned earlier, fairly large changes in the allowed $b_n$ give only small changes in the good field aperture.

## 4. Aperture Limits Due to Random Higher Magnetic Field Multipoles

To establish a criterion for the allowed random higher multipoles due to the distortion of the working line in the CBA, it will be assumed that each random multipole should produce a chromaticity at $\Delta p/p = .01$ of no more than $C = .8$ peak value ($C_{rms} = .4$), which is 10% of the amount needed for stability and 10% of the amount that can be provided by the correction coils in the CBA.

The $\nu$-shift due to the random $b_n$ multipole may be written as

$$\Delta\nu_{rms} = \Delta\nu_n\ (\Delta p/p)^{n-1}, \qquad (6)$$

where $\Delta\nu_n$ is given by

$$\Delta\nu_n = \frac{n}{4\pi\rho} \left\{ \sum_k \left( L(k)\ b_n(k)\ X_p^{n-1}(k)\beta(k) \right)^2 \right\}^{1/2} \qquad (7)$$

The sum over k is a sum over all the magnets in the accelerator, and L(k) is the length of each magnet. If there is an appreciable sagitta in the dipoles, then in Eq. 7 one needs to replace $X_p^{n-1}(k)\ \beta(k)$ as indicated by Eq. 4.

The rms chromaticity, $C_{rms} = p d\nu/dp$ is then given by

$$C_{rms} = (n-1)\ \Delta\nu_n\ (\Delta p/p)^{n-2} \tag{8}$$

The above equations could be applied to the random multipoles in the dipoles to find the allowed values of these $b_n$ so that the chromaticity generated by each multipole at $\Delta p/p = .01$ is less than $C_{rms} = .4$. However this leads to a set of allowed random $b_n$ which are less severe than the allowed random $b_n$ which are presently being used at the CBA for other reasons than the distortion of the working line. It is more interesting to use the equations given above to compute the good field aperture, as determined by the distortion of the working line, for the presently used allowed $b_n$ at the CBA. This is given in the following table.

| n | $b_n'$ Dipoles | Quads | $C_{rms}$ $\Delta p/p=.01$ | $\Delta p/p$ Aperture |
|---|---|---|---|---|
| 0 | 5 | 5 | - | - |
| 1 | 2.9 | 5 | - | - |
| 2 | 1.6 | 2 | .28 | - |
| 3 | 1.0 | 1.0 | .38 | .010 |
| 4 | .75 | .75 | .32 | .011 |
| 5 | .50 | .50 | .23 | .013 |
| 6 | .35 | .35 | .15 | .013 |
| 7 | .25 | .25 | .10 | .013 |
| 8 | .17 | .17 | .048 | .014 |
| 9 | .11 | .11 | .032 | .014 |
| 10 | .07 | .07 | .013 | .015 |

In the above table, $b_n' = b_n \times (4.4)^n/10^{-4}$. The $b_n'$ are the allowed random $b_n$ presently being used for the CBA. The allowed random $a_n'$ and $b_n'$ are equal. The above table shows that for the higher random multipoles, the good field aperture for the CBA is about $\Delta p/p = .014$ as determined by the chromaticity reaching $C_{rms} = .4$. The presently required aperture for the CBA is $\Delta p/p = \pm .01$, which is not very far from the aperture limit of $\Delta p/p = .014$ due to the chromaticity effect being considered here.

## References


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