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NEW METHOD FOR INVERTING THE CLOSED ORBIT DISTORTION PROBLEM*

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Summary

A certain class of magnet misalignments in storage rings and other accelerators produces closed orbit distortions (CODs). Quite often the CODs are measured at a fewer number of locations (N) than the number of misalignment parameters (M). There is a linear relation between COD measurements, $u(j)$, $j=1, \ldots, N$ and the misalignment parameters $\epsilon(k)$, $k=1, \ldots, M$. Hence the $\epsilon(k)$'s are underdetermined. If $M < 2N$, one can obtain an overdetermined set of equations by measuring the COD at two quadrupole settings. There are several ways of inverting the COD measurements to get the misalignment parameters that are fairly insensitive to errors in the measured CODs. A computer program called CODINV has been written to test some of these schemes. Two schemes give fairly good results when applied to the lattice of the Los Alamos Proton Storage Ring (PSR).[1] The first scheme requires measurements at two nearby tunes and the use of singular-value decomposition methods. The second scheme requires measurements of the CODs in the FODO and DOFO cell arrangements but is easier mathematically.

Introduction

The problem of expressing CODs in terms of magnet misalignments has been solved for a long time.[2,3] The inverse problem of expressing the misalignment parameters in terms of the CODs measured a few places in the Ring or along the accelerator is a perennial one. It is well known that magnet misalignments lead to tune shifts, to coupling of transverse betatron modes, and to distortions of the closed orbit.[4] These distortions lower the machine acceptance and are of primary concern when a new machine is turned on. It is also well known that, if one ignores variations in magnet current from the ideal current, then there are only two degrees of freedom per magnet that can cause CODs.[4] One of these affects the horizontal motion and one the vertical motion; thus, the alignment problem can be separated into the problem of horizontal COD and the problem of vertical COD with one magnet degree of freedom associated with each. It can be shown for example that a longitudinal displacement of a dipole causes horizontal COD, and a roll displacement of a dipole introduces vertical COD. For quadrupoles, horizontal displacement affects horizontal motion, and vertical displacement affects vertical motion. All other displacements of these magnets either have no effect on the beam, or else they cause tune shifts and mode coupling.

It will be shown in the next section that the equations of motion lead to a linear relation between the CODs and the displacement parameters. This relation can be written

$$u_i(j) = \sum_{k=1}^{M} T_i(j,k)\epsilon_i(k), \quad j = 1, \ldots, N, \quad i = 1, 2 \quad (1)$$

where $i = 1$ is horizontal, and $i = 2$ is vertical. This linear set of equations could be solved exactly for $\epsilon_i(k)$ if $N = M$, or could be solved in a least-squares sense if $N > M$. However, the usual case is that N (the number of beam monitors) is less than M (the number of magnets). When $N < M$, one says that the system of equations is underdetermined. There are least-squares methods for dealing with these systems, but they always involve imposing additional constraints on the system such as simultaneously minimizing the sum of squares

$$\sum_{k=1}^{M} \epsilon_i(k)^2 \quad .$$

One can also impose a physical constraint by ignoring misalignments of the dipoles and by fitting the CODs, using quadrupole parameters only. The result of these procedures often can be unrealistic in terms of suggested magnet displacements and sensitivity to slight changes in the measured COD.

One would like to increase the number of equations by increasing the number of points at which the COD is measured. Often it is experimentally impossible to increase the number of beam-position monitors because the desirable locations are taken up with extraction kickers, halo scrapers, or other such equipment. The alternative being suggested in this paper is to make some change in machine parameters, which will lead to changes in the matrix elements $T_i(j,k)$ and to remeasurement of the COD. This gives two sets of equations

$$u_i(j) = \sum_{k=1}^{M} T_i(j,k)\,\epsilon_i(k) \quad ,$$

and

$$\bar{u}_i(j) = \sum_{k=1}^{M} \bar{T}_i(j,k)\,\epsilon_i(k) \quad ,$$

which can be written as one double-matrix equation

$$u_i^D(j) = \sum_{K=1}^{M} T_i^D(j,k)\,\epsilon_i(k), \quad j = 1, \ldots, 2N \quad . \qquad (2)$$

If $2N > M$, this set of equations can be solved by ordinary least-squares methods, provided the matrices T_i^D are not ill conditioned. It turns out that if $\bar{T}_i$ is obtained from T_i by making a small change in the quadrupole strengths, then T_i^D is in fact ill conditioned, and it is necessary to use a special method to solve this set of equations. The method calls for a singular-value decomposition of the matrices T_i^D as described in Chapter 11 of the LINPACK User's Guide.[5] (If T_i^D were a square matrix, the method would amount to identifying and discarding the large eigenvalue of T_i^D before taking the inverse.)

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We have investigated two other ways of producing matrices T_i^D for the PSR. One can change the sign of the quadrupole gradients, or one can interchange the quadrupoles to make DOFO cells out of FODO cells. The latter change has the advantage of not changing the tunes. This probably will make it easier to get a beam through the lattice. In both of these cases, the T_i^D's are well-conditioned matrices that give stable least-square solutions even when random errors are introduced into the CODs, $u_i(j)$.

The computer program CODINV, which we have written to study the stability of these inversion methods, also produces beamline plots of the CODs. Although the program was written specifically for the PSR lattice, it should be easily adaptable to any other Ring or even to a linear transport system.

The next section outlines how one goes from the equations of motion to the matrix equation, Eq. (1). The final section gives some applications to the PSR.

Matrix Equations

Let us assume that we have a storage ring with separate function magnets. Ignore the synchrotron motion caused by rf bunchers. The equations for the transverse motion relative to the ideal orbit are

$$m\ddot{u}_i(t) = (-1)^i e(v_3 B_{\bar{i}} - \dot{u}_{\bar{i}} B_3); \quad i = 1, 2; \ \bar{i} = 2, 1 \ , \quad (3)$$

where v_3 is the particle velocity along the direction of the orbit, and B_3 is the magnetic field parallel to v_3. Because $\dot{u}_{\bar{i}}$ and B_3 are usually small, one neglects the $\dot{u}_{\bar{i}} B_3$ terms. It is customary to expand $B_{\bar{i}}$ in powers of the u_i's, to use path length $s \approx v_3 t$ as the independent variable, and to move the linear terms to the left side of the equation. The coupling terms and the nonlinear terms are neglected. The result is

$$u''_i(s) + K_i(s) u_i(s) = (-1)^i \left(\frac{e}{p_3}\right) \Delta B_{\bar{i}}(s), \quad i = 1, 2 \ . \quad (4)$$

The double prime is a second derivative with respect to s; p_3 is mv_3; $K_i(s)$ is proportional to the field gradients in the quadrupoles. Also, there is a $(1/R^2)$ term from the centrifugal force in the dipoles. The components $\Delta B_{\bar{i}}(s)$ are the field errors caused by misalignments. Because one is looking for the closed orbit, periodic boundary conditions are imposed on the Green's function solution

$$u_i(s) = (-1)^i \left(\frac{e}{p_3}\right) \int_0^{2\pi R} G_i(s,s') \Delta B_{\bar{i}}(s') ds', \quad i = 1, 2 \ , \quad (5)$$

where $2\pi R$ is the ideal path length around the Ring. The Green's function can be expressed in terms of the betatron function $\beta_i(s)$ of the perfect Ring

$$G_i(s,s') = \frac{\sqrt{\beta_i(s)\beta_i(s')}}{2 \sin(\pi Q_i)} \cos\left[\pi Q_i - |\Psi_i(s) - \Psi_i(s')|\right] \ , \quad (6)$$

where

$$Q_i = \Psi_i(2\pi R)/2\pi = \left(\frac{1}{2\pi}\right) \int_0^{2\pi R} ds/\beta_i(s) \quad (7)$$

are the tunes.[6] The phases $\Psi_i(s)$ are defined implicitly by Eq. (7). The betatron functions, of course, are dependent on the functions $K_i(s)$. In the program CODINV, they are calculated using first-order transport matrices for the ideal elements.

Assuming that the field errors $\Delta B_i(s)$ are nonzero only in the magnets and are proportional to a misalignment parameter ε_i, the integral in Eq. (5) can be broken up into a sum of integrals, one integral for each magnet,

$$u_i(s) = (-1)^i \left(\frac{e}{p_3}\right) \sum_{k=1}^{M} \left[C_i(k) \int_{s_0(k)}^{s_0(k)+\ell(k)} G_i(s,s') ds' \, \varepsilon_i(k) \right], \quad (8)$$

where $C_i(k)$ is the constant of proportionality between ΔB_i and $\varepsilon_i(k)$ in the kth magnet, and the integral extends from the beginning to the end of the magnet. The notation can be compressed by writing

$$u_i(s) = \sum_{k=1}^{M} T_i(s,k) \varepsilon_i(k), \quad i = 1, 2 \ . \quad (9)$$

If one evaluates these equations at N beam-monitor positions s_j, $j = 1, \ldots N$, then one obviously obtains Eq. (1). The matrix elements $T_i(j,k)$ are easy to evaluate from the geometry of the Ring and the betatron functions. Details of the derivation outlined above, as well as a more complete description of CODINV, are given in PSR Technical Note No. 111.[7] Equation (9) is useful in plotting the CODs around the Ring if the misalignment parameters are given. Examples of these plots are given in the next section.

Applications to the PSR

The PSR, which is under construction, consists of 10 FODO cells, that is, 30 magnets. Present planning calls for 18 beam-position monitors that will be located at the entrance to the focusing quadrupoles and at the exit of the defocusing quadrupoles in each cell. The exception to this occurs in Cells 7 and 9, where the monitor will be missing from the defocusing positions. The monitors are expected to give the centroid of the beam to an accuracy of approximately ±1 mm. Figure 1 shows a horizontal COD produced by a random set of magnet misalignments. A random error in the ±1.5-mm range has been added to the COD points calculated using Eq. (9). Table I gives the parameters $\varepsilon_1(j)$ for the 30 magnets. A second COD was generated by changing the quad strengths in each cell by 0.707%. Table II gives the residual parameters after the two CODs have been least-squares fitted using the singular-value decomposition method mentioned above. Figure 2 shows the residual COD. The bumps in the residual occur at the positions of missing monitors.

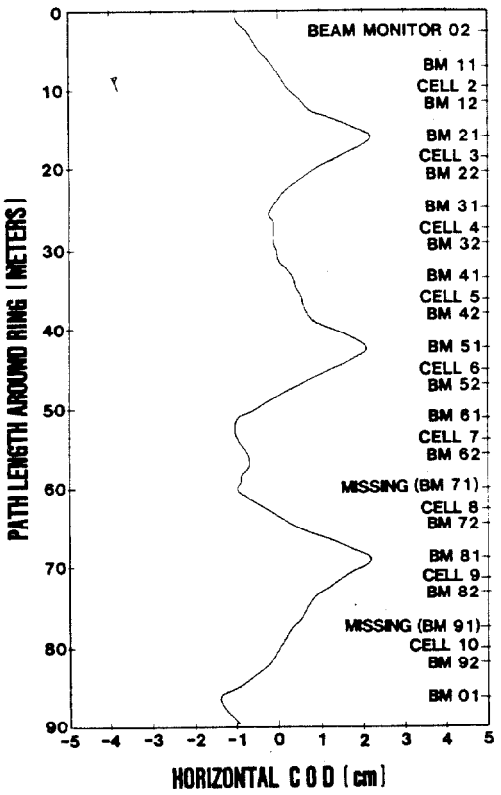

Fig. 1. Horizontal COD produced by random magnet displacements.

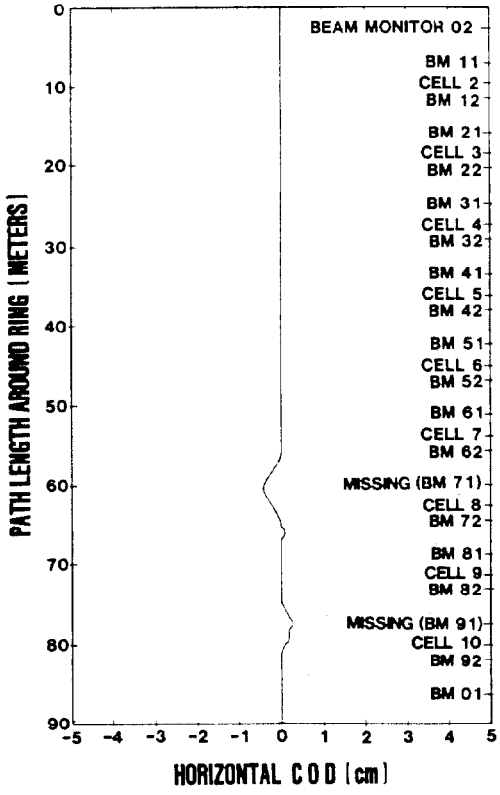

Fig. 2. Residual horizontal COD after fitting with program CODINV.

TABLE I

MAGNET DISPLACEMENTS FOR HORIZONTAL COD

Cell	Defocusing Quad Displacement (cm)	Dipole Displacement (cm)	Focusing Quad Displacement (cm)
1	-0.334	0.193	0.055
2	-0.036	0.481	0.463
3	0.098	0.180	0.486
4	-0.301	0.374	0.428
5	-0.184	0.414	0.345
6	0.075	0.016	0.071
7	-0.134	-0.320	-0.078
8	-0.369	-0.471	0.106
9	0.293	0.203	0.380
10	0.261	0.116	-0.226

TABLE II

RESIDUAL DISPLACEMENTS FOR HORIZONTAL COD

Cell	Defocusing Quad Displacement (cm)	Dipole Displacement (cm)	Focusing Quad Displacement (cm)
1	0.057	-0.125	-0.036
2	0.084	0.367	0.091
3	0.138	0.420	0.161
4	0.239	0.267	0.133
5	0.142	0.134	0.067
6	-0.068	0.038	0.007
7	0.316	-0.341	0.019
8	0.103	-0.806	-0.274
9	-0.065	0.395	0.068
10	-0.080	0.501	0.195

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