REDUCTION OF BEAM EMITTANCE BY A TAPERED-FOIL TECHNIQUE*

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I. Introduction

The transverse emittance of a particle beam can be reduced by means of a transverse energy gradient produced in a tapered energy-loss foil and manipulation of the dispersion characteristics of the succeeding beam-transport system. In spite of the multiple Coulomb scattering in the foil, the area in one transverse phase plane can be substantially reduced, provided that the incident beam is sufficiently mono-energetic. In the case of negligible multiple scattering, the total six-dimensional phase-space volume occupied by the beam is not changed by this process.

II. The Method of Reducing Transverse Emittance

IIA. The Effect of a Tapered Energy-Loss Foil

Consider the effect of a tapered energy-loss foil on the phase-space distributions of a particle beam. The geometry is illustrated in Figure 1, and the phase-space distributions in Figure 2. For

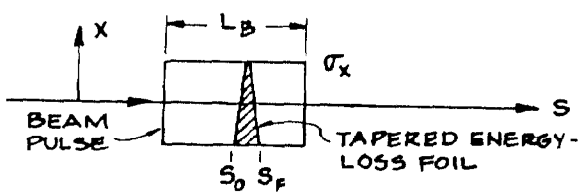

Figure 1. Geometry of the Tapered Energy-Loss Foil.

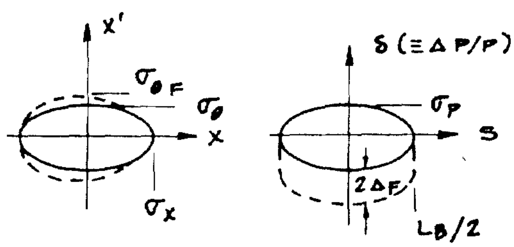

Figure 2. Transverse and Longitudinal Phase Space Occupied by Beam Before (solid) and After (dashed) the Tapered Energy-Loss Foil.

convenience we choose a case in which the phase-space distributions on the input side of the foil (position s_0) are up-right ellipses in each transverse plane and also in the longitudinal (s,δ) plane $(\delta \equiv \Delta p/p)$ with semi-axes σ_x, σ_θ in the x-plane, σ_y, σ_ϕ in the y-plane, and $L_{B/2}$, σ_p in the longitudinal plane. We choose also to have the dispersion function $\eta(s)$ equal to zero (and also its slope $\eta'(s)$) on the input side of the foil, so that the x,δ (x-position versus fractional momentum) phase-space distribution of the incident beam also is an upright ellipse (Figure 3a).

On the output side of the foil (position S_F) the corresponding phase-space distributions are shown with dotted boundaries in Figures 2 and 3a. The geometric widths are of course unchanged, the angular widths have been broadened due to multiple scattering, and the momentum width has been increased on the lower side by about $2\Delta_F$, where Δ_F is the fractional momentum loss on the central orbit.

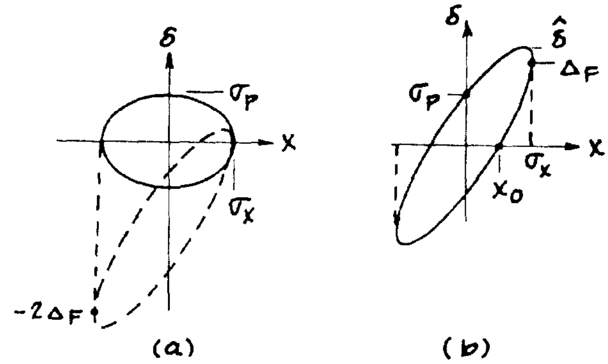

Figure 3. (a) x,δ Phase Space Occupied by Beam Before (solid) and After (Dashed) the Tapered Foil (b) After the Foil with Momentum Renormalized to the Center Momentum.

The significant change in seen in the x,δ phase-space distribution (Figure 3), where the output ellipse has been severely skewed, because the particles at $x = +\sigma_x$ have lost no momentum, and those at $x = -\sigma_x$ have lost $2\Delta_F$. It is convenient to renormalize the momentum scale to the new momentum of the central particle, as shown in Figure 3b. We see then that the effect of the tapered energy-loss foil is to produce dispersion in the particle beam -- i.e., a linear relationship between momentum and average x-position.

That the area of the transformed ellipse in x,δ space is unchanged is probably obvious on a physical basis, but also it is easy to show on a mathematical basis. The effect of the tapered energy-loss foil on the phase-space vector x,δ can be written in matrix form as

$$\begin{pmatrix} x \\ \delta \end{pmatrix}_{S_F} = \begin{vmatrix} 1 & 0 \\ \Delta_F/\sigma_x & 1 \end{vmatrix} \begin{pmatrix} x \\ \delta \end{pmatrix}_{S_0} \tag{1}$$

Thus, a tapered foil makes a transformation in x,δ space which mathematically is like that of a thin defocussing lens in x,x' space. The transfer matrix has unit determinant and thus is area-preserving.

Another result of this mathematical situation is that we can use the well-known transformation of ellipse parameters corresponding to a linear transfer element.[1,2] Thus we find that the x,δ ellipse at S_F, the exit side of the tapered energy-loss foil (Figure 3b), can be written as:

$$\frac{\left(\sigma_p^{\,2} + \Delta_F^{\,2}\right)}{\sigma_x\,\sigma_p}\,x^2 - 2\,\frac{\Delta_F}{\sigma_p}\,x\,\delta + \frac{\sigma_x}{\sigma_p}\,\delta^2 = \sigma_x\sigma_p \tag{2}$$

From this expression we find that the maximum momentum deviation $\hat{\delta}$ is

$$\hat{\delta} = \left(\sigma_p^{\,2} + \Delta_F^{\,2}\right)^{1/2} \tag{3}$$

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and that the new x-intercept x_0 is

$$X_0 = \sigma_x \sigma_p / \hat{\delta} \qquad (4)$$

We note that the x-intercept has been reduced by the ratio

$$X_0/\sigma_x = \sigma_p/\hat{\delta}$$
$$\simeq \sigma_p/\Delta_F \qquad \text{for } \Delta_F >> \sigma_p \qquad (5)$$

This also is the ratio by which the x-emittance has been reduced (if multiple scattering can be neglected), as we shall show.

IIB. Realization of Emittance Reduction, Dispersion Matching

The magnitude of the dispersion η_F created by the tapered energy-loss foil is

$$\eta_F(S_F) = x(\hat{\delta})/\hat{\delta} = \sigma_x \Delta_F/(\hat{\delta})^2$$
$$\simeq \sigma_x/\Delta_F \qquad \text{for } \Delta_F >> \sigma_p \qquad (6)$$

The slope of this dispersion function (η'_F) is zero -- i.e., there is no correlation between momentum deviation δ and the slope $x'(s)$ at the point s_F.

To exhibit the emittance reduction produced by the tapered energy-loss foil, the dispersion η_F must be removed, which can be done with a great variety of beam-transport arrangements. A simple, one-dimensional example is shown in Figure 4. The

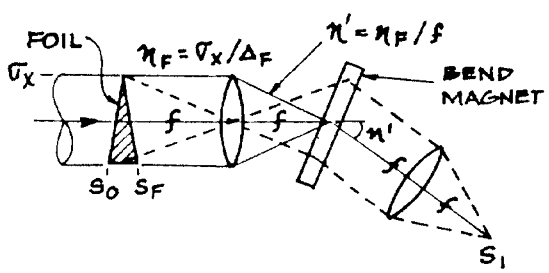

Figure 4. One-Dimensional Beam Transport System that Removes Dispersion and Has One-to-One Imaging of S_F at S_1. Focal length of each lens is f.

dispersion function $\eta(s)$ and its slope $\eta'(s)$ are both reduced to zero at the exit of the bend magnet and remain zero thereafter. For convenience, we examine the phase-space distributions at s_1, which is the image of S_F -- i.e., we have point-to-point focusing with unit magnification between s_F and the point at s_1. The operation of this optical system can be described in the x,δ plane by the matrix equation

$$\begin{pmatrix} x \\ \delta \end{pmatrix}_{s_1} = \begin{vmatrix} 1 & -\eta_F \\ 0 & 1 \end{vmatrix} \begin{pmatrix} x \\ \delta \end{pmatrix}_{s_F} \qquad (7)$$

Note that this transformation is area-preserving in the x,δ plane. To include the x' coordinate the transfer matrix can be expanded to 3 dimensions:

$$\begin{pmatrix} x \\ x' \\ \delta \end{pmatrix}_{s_1} = \begin{vmatrix} 1 & 0 & -\eta_F \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{pmatrix} x \\ x' \\ \delta \end{pmatrix}_{s_F} \qquad (8)$$

Because the x-transformation depends on δ, the x,x' phase area is not preserved.

The results are upright ellipses of half width x_0, as shown in Figure 5. Since the half heights at s_1, $\sigma_{\theta F}$ and $\hat{\delta}$, are the same as at s_F, we see that the x,x' phase area has been reduced by the factor $\sigma_p/\hat{\delta}$, given in equation (5).

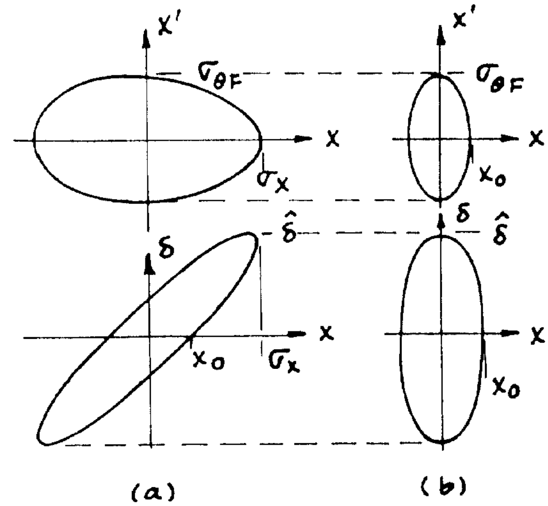

Figure 5. x,x' and x,δ Phase Space Occupied
(a) At S_F, the Exit Side of the Tapered Foil and
(b) At S1, the Image Point with Zero Dispersion.

In the longitudinal (s,δ) phase plane, the beam pulse length is substantially unchanged by the operation of the foil but the momentum width, and hence, the longitudinal phase area, has been increased by the factor $\hat{\delta}/\sigma_p$, the same ratio by which the x-transverse emittance has been reduced. Since in this case the y,y' phase area is not changed (except for multiple-scattering effects, which here are being neglected), we see that the total six-dimensional phase volume is unchanged by this tapered energy-loss foil arrangement. The transverse phase area has been reduced at the expense of increasing the longitudinal phase area.

It is apparent that the arrangement shown in Figure 4 can operate in the reverse direction. That is, the energy spread in a beam can be reduced by the factor $\sigma_p/\hat{\delta}$ at the expense of increasing the x-emittance by the same ratio through the use of a tapered energy-loss foil in a dispersed beam. Such arrangements have been used to reduce the energy spread in secondary beams at the Bevatron.[3]

III. Emittance Growth from Multiple Coulomb Scattering

The angular distribution produced in multiple Coulomb scattering is approximately Gaussian with rms width $\Delta\theta_c$, and if the incident beam has a similar distribution of width σ_θ, the rms angular width $\sigma_{\theta F}$ on the output side of the tapered energy-loss foil can be expressed as

$$\sigma_{\theta F} = \left(\sigma_\theta^{\ 2} + \Delta\theta_c^{\ 2}\right)^{1/2}$$
$$= \sigma_\theta \left[1 + (\Delta\theta_c/\sigma_\theta)^2\right]^{1/2} \qquad (9)$$

The bracketed expression is the emittance growth factor due to multiple scattering in the foil.

Since the angular width σ_θ of the incident beam depends on the input emittance ε and the beta function at the foil β_F,

$$\sigma_\theta = \left(\varepsilon/\beta_F\right)^{1/2} \qquad (10)$$

one can in principle make the factor $\Delta\theta_C/\sigma_\theta$ comparable to or less than unity by sufficiently tight focussing. Although one might tend to be limited in tight focussing by heating effects in the foil, this difficulty can be alleviated by spreading the beam out through the use of dispersion on the input side of the foil, which can be done without sacrificing the achievable emittance reduction, as will be mentioned in a later section.

The angular width due to coulomb scattering $\Delta\theta_C$ is proportional to $(\Delta_F)^{1/2}$, so that in the domain in which $\Delta\theta_C/\sigma_\theta$ is greater than unity, the multiple-scattering enlargement factor is proportional to $(\Delta_F)^{1/2}$. In this case the overall emittance reduction factor $(\sigma_p/\Delta_F)(\Delta\theta_C/\sigma_\theta)$ is proportional to $1/(\Delta_F)^{1/2}$. Thus in spite of the enlargement factor due to multiple scattering, a larger Δ_F always leads to greater emittance reduction in the transverse plane in which the foil is tapered. However, if the emittance in the other transverse plane is included, one sees that the four-dimensional transverse emittance cannot be reduced further with greater values of Δ_F once the coulomb scattering dominates the angular width.

Another limitation occurs if Δ_F exceeds the momentum pass band of the succeeding beam-transport system, which typically is of the order of a few percent but can vary greatly depending on the complexity of the system and the focussing requirements. Pass bands considerably larger than those normally encountered can be designed into a beam-transport system by utilizing the principles of second-order achromats,[4] so that this limitation too can be ameliorated.

The importance of the emittance growth factor due to multiple scattering in the tapered energy-loss foil can vary considerably because different particle beams can differ greatly in emittance, in energy spread, and in rates of energy loss and multiple-scattering growth in the foil. In Table I

Table I
Emittance Growth Factor due to Multiple Scattering

Accelerator	Beam	Norm. Emitt.	Beam Energy	Energy Spread	Growth Factor
RF linac	proton	10^{-5} m	100 MeV	$\pm 2\times10^{-3}$	1.07
RF linac	elect.	10^{-4}	100	$\pm 1\times10^{-2}$	15
Induct. Lin.	elect.	5×10^{-3}	5	$\pm 2\times10^{-3}$	1.4

the multiple-scattering emittance-growth factor is given for three representative cases together with the normalized emittance ($\beta\gamma\varepsilon$), the beam energy, and the fractional energy spread used in each case. The common conditions for the three cases were an emittance reduction factor (Δ_F/σ_p) of 10, a beta-function value of 5 cm at the foil, and a beryllium energy-loss foil. Thus we see that there are cases where the tapered-foil technique can be used to advantage and also cases where it cannot.

IV. The Use of Momentum Dispersion on the Input Side of the Tapered Foil

It can be advantageous to disperse the beam on the input side of the tapered energy-loss foil, either to reduce the temperature rise in the foil or to improved the energy resolution, as at the target foil in a magnetic spectrometer system.

Following the method given in Section II, one finds that if there is dispersion of value η_0 on the input side of the foil, the emittance reduction factor is[5]

$$X_0/\sigma_x = \left[1 + \Delta_F{}^2/\sigma_p{}^2 + 2\eta_0\Delta_F/(\sigma_x{}^2 + \eta_0{}^2\sigma_p{}^2)^{\frac{1}{2}}\right]^{-\frac{1}{2}} \quad (11)$$

This expression reduces to equation (5) if $\eta_0 = 0$. In the limit of $\eta_0\sigma_p \gg \sigma_x$

$$X_0/\sigma_x \cong (1 + \Delta_F/\sigma_p)^{-1} \quad (12)$$

Thus it turns out that the use of dispersion on the input side of the foil improves somewhat the emittance reduction that can be achieved.

Although this reduced emittance can lead to a smaller spot size at the detector plane of a magnetic spectrometer, the energy resolution is not improved because the energy separation at the detector plane is similarly reduced.[5] The principal advantage to using the tapered-foil technique in a magnetic spectrometer system is that it reduces the dispersion change (and, hence, the size of the magnet) that is required between the target foil and the detector plane.

V. Summary

The use of a tapered energy-loss foil plus dispersion and optical matching in the succeeding beam transport system can reduce the emittance of a particle beam in one transverse phase plane at the expense of increasing it in the longitudinal plane. Multiple Coulomb scattering in the foil increases the emittance in both transverse planes but can be acceptable provided that the incident beam is sufficiently mono-energetic and is tightly focussed at the foil.

This technique can be applied to advantage in typical non-relativistic proton beams from RF linacs and to high-emittance, high-intensity, low-energy-spread electron beams from induction linacs, but multiple scattering effects make unprofitable for typical electron beams from RF linacs, unless the energy spread can be appreciably reduced.

Claud Bovet[6] has previously discovered the emittance-reducing properties of a tapered-foil arrangement. His report includes many of the results of this paper.

References

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