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MEASURING THE ISR IMPEDANCE AT VERY HIGH FREQUENCIES BY OBSERVING THE ENERGY LOSS OF A COASTING BEAM

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Abstract

A slow energy loss of unbunched protons and antiprotons in the ISR has been observed which is larger than expected for interactions with the residual gas and for synchrotron radiation. It can be explained as a parasitic mode loss due to the wall currents induced by the longitudinal Schottky noise. This noise as seen at the wall of a circular chamber of radius a extends to a typical frequency of the order $\omega \sim c\gamma/a$ limited by the finite opening angle of $\sim 1/\gamma$ of the fields created by a relativisitic particle. Due to the statistical longitudinal distribution of the particles the total energy loss due to the induced wall current is proportional to the total number of particles and the loss per particle becomes independent of current. This loss leads to a steady decrease of the orbit radius which can be observed for long storage times. Using these results obtained at different energies, the frequency dependence of the resistive longitudinal impedance in the range of several tens of GHz is obtained. This impedance is compared with the prediction of the diffraction theory.

1. Introduction

During very long physics runs with unbunched antiprotons a steady decrease of the orbit radius of less than 1 mm/day was observed. This effect depends on the beam energy but is apparently independent of beam current. The resulting motion of the interaction point is large enough that it had to be taken into account in small angle scattering experiments. This decrease at the orbit radius is caused by a steady energy loss of the beam which is considerably larger than expected for synchrotron radiation. The parasitic mode loss due to the wall currents induced by the Schottky noise[1] seems to be the only explanation. The spectrum of these wall currents induced at a distance a from the beam extends to a maximum frequency $\omega_{max} \sim c\gamma/a$ determined by the opening angle of $\sim 1/\gamma$ for fields created by the relativistic particles. The resulting energy loss observed at different beam energies can therefore give information about the resistive wall impedance at very high frequencies.

2. Experiments

While usually parasitic mode losses are measured with electron bunches [2-5] all experiments presented here were carried out with unbunched proton or antiproton beams. Most of these measurement were done as a parasitic observation during long, quiet physics runs with no or only few beam intervention (e.g. scraping tails to improve the background). The best conditions were obtained during early antiproton runs before longitudinal stochastic cooling was used. Particle losses should be small since any beam loss from the high or low energy end of the stack could result in apparent change of the averaged stack energy.

The experiment itself consists of a careful measurement of the beam orbit radius as a function of time while the fields of all magnets stay absolutely constant. The change ΔR of the radius is determined from the measured change in revolution frequency. At regular intervals the spectrum of the beam Schottky noise [1,6] is measured and its center of gravity determined. This can be done rather accurately with an error of about 0.1 mm in orbit radius. The corresponding change in momentum is obtained from the known momentum compaction factor. The measured data from 4 experiments are shown in Fig.1. The very low energy run consists of 4 relatively short observation periods while all the other runs were done without interruption over 2-3 days. From the measured slopes in Fig.1 the total energy loss U_t per revolution is obtained. The loss U_{SR} due to synchrotron radiation is substracted giving the remaining loss U shown or the top of Fig.2, and listed Table 1. It should be noted on this figure that the 2 measurements done at 31.4 GeV/c with nearly 4 orders of magnitude different current show practically the same energy loss U.

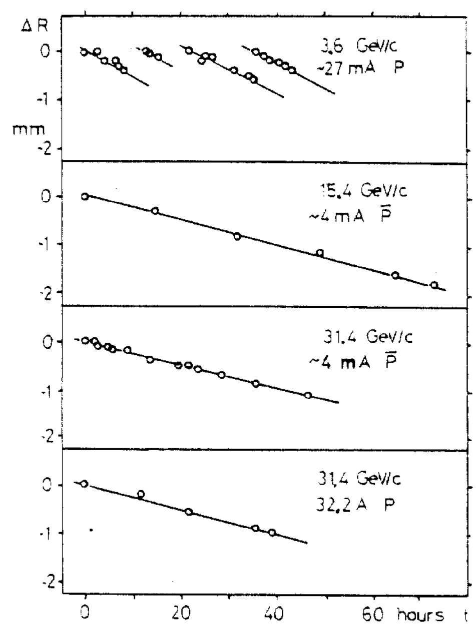

Fig. 1 Measured decrease ΔR of the orbit radius due to the energy loss

3. Schottky spectrum

The Schottky signal due to a single particle consists of repetitive pulses spaced by the revolution period of that particle. The contributions of different particles to the total Schottky signal are uncorrelated and their pulses appear a random time intervals. At the lower end the resulting Schottky spectrum consists of bands spaced by the average revolution frequency which will overlap at higher frequencies to form a continuous white spectrum with a Fourier transformed current due to a single particle with charge e

$$\tilde{I}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} I(t) e^{-i\omega t}\, dt = \frac{e}{\sqrt{2\pi}} \qquad (1)$$

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The average squared current due to all N particles is[7]

$$\frac{d\langle I_t^2\rangle}{d\omega} = \frac{2e^2 N f_{rev}}{2\pi} \qquad \omega > 0 \qquad (2)$$

Since the contributions of the individual particles are uncorrelated the power of the total signals is the sum of the individual power contributions.

If the Schottky signals were observed at the immediate proximity of the particles the measured pulses would be nearly infinitely short and the spectrum (1) would extend to extremely high frequencies. In reality the signals are observed by the wall at a certain distance from the beam where the single point charges creating the fields can no longer be resolved and the observed spectrum (1) decreases at very high frequency.

The field of a free charge moving with velocity βc can be obtained from the static field by a Lorentz transformation. It has the form shown on the left of Fig.3. The presence of the wall modifies this field distribution (right of Fig.3). A particle traveling on the axis of a perfectly conducting circular cylinder of radius a induces with its radial field component a wall current I_W which can be expressed by a Fourier-Bessel series[8,9]

$$I_w(t) = -\left[\frac{e\gamma\beta c}{a}\sum_n \frac{\exp(-|j_n\gamma\beta ct/a|)}{J_1(j_n)} - e f_{rev}\right] \qquad (3)$$

where J_1 is a Bessel function of order 1 and j_n are the zeros of the lowest order Bessel function J_o, $J_o(j_n)=0$. Since the wall current does not contain a dc-component the term ef_{rev} is subtracted in (3). The fields due to the dc-component of the beam current are static and cannot lead to an energy loss over a complete revolution. The Fourier component of the wall current can be obtained directly by solving the field equations in frequency domain[9] or by transforming (2) and using summing formulas which express the Fourier-Bessel series[10] in modified Bessel functions I_o, I_1, K_o and K_1

$$\tilde{I}_w(\omega) = \frac{-e}{\sqrt{2\pi}} \times \left[K_1(x) + K_o(x) \cdot I_1(x)/I_o(x)\right] = \frac{-e}{\sqrt{2\pi}}\,\frac{1}{I_o(x)}$$

$$\text{with} \quad x = \frac{a\omega}{\gamma\beta c} \qquad (4)$$

For $x \ll 1$, $\tilde{I}_W(\omega) \rightarrow -e/2$. but $I_W(0)=0$. The form of the induced wall current $I_W(t)$ is shown on the top of Fig.4 for the beam momenta used in the experiment. This distribution has an rms.width

$$\sigma = \frac{a}{\sqrt{2}\,\gamma\beta c} \qquad (5)$$

The square of the wall current spectrum is shown on the bottom of Fig.4. At any aperture changes for bellows or other equipment the wall current is interrupted and some energy is radiated away. This leads to an energy loss of the particles usually called parasitic mode loss. The role of the beam surroundings played in this process can be summarized by an impedance $Z(\omega)$ having a resistive part $Z_R(\omega)$. The energy loss by each particle is

$$U = 2\int_0^\infty \tilde{I}_W^2(\omega)\; Z_R(\omega)\,d\omega = 2\frac{e^2}{2\pi}\int_0^\infty \left(I_o\left(\frac{a\omega}{\gamma\beta c}\right)\right)^{-2} Z_R(\omega)\,d\omega \qquad (6)$$

which is independent of the total number of particles since the fields due to different particles are uncorrelated. The energy loss per unit charge $k_{pm} = U/e^2$ is usually called parasitic mode parameter. In the curved part of the particle orbit the fields are different from the ones calculated here[11] however most of the elements contributing to the impedance are located in the straight sections of the ISR.

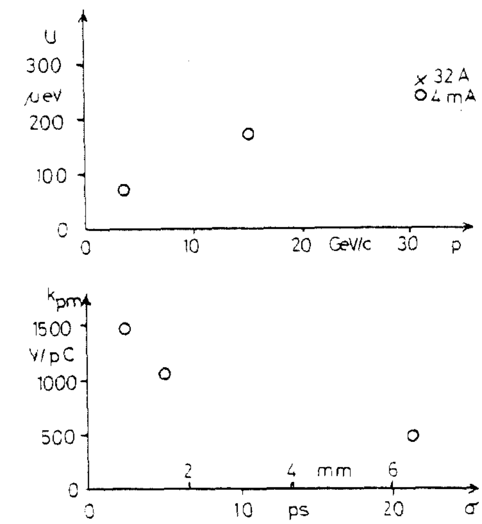

Fig.2 Top : Energy loss per turn after substraction of the synchrotron radiation loss.
Bottom: Parasitic mode loss parameter k_{pm} as a function of the rms pulse length of the wall current.

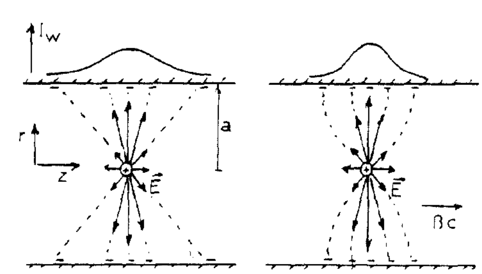

Fig.3 Field $\vec{E}$ and induced wall current I_W for relativistic particle. Left: free space solution. Right: solution in a conducting cylinder.

4. Results

The measured energy losses U_t are first corrected for synchrotron radiation losses U_{SR} to give the remaining loss U and the parasitic mode parameters k_{pm}. To know the length σ of the wall current pulse an effective radius a has to be assumed for the mostly elliptic ISR vacuum chamber. From different arguments and earlier measurements at lower frequencies we estimated a ~ 35 mm. From this the time and frequency distribution of the wall current is obtained as shown in Fig.4. The resistive impedance can in principle be obtained by deconvolution of the equation (6). Since we have only 3 measured points this was done in an approximate manner by replacing the spectra $\tilde{I}_W(\omega)$ in Fig.4 by rectangular shape extending to a typical frequency ω_t where $\tilde{I}_W^2(\omega)$ has dropped to 1/2 of its value a low frequency. This way the 2 average impedance $\langle Z_R(\omega)\rangle$ in between the three ω_t is obtained as listed in Table 1. Furthermore these impedances divided by the mode number n (n=frequency/revolution frequency) are listed.

The behaviour of the impedance $Z(\omega)$ at very high impedances has been investigated by several authors[12-16] using the diffraction of the high frequency fields at the aperture changes. We apply the optical resonator model by Sessler-Vainshtein[13] in the form given in ref.16 in a rather superficial way to our measurements. Since this model treats basically a periodic structure the aperture changes in the ISR had to be simplified to the structure shown on top of Fig.5. with g=0.25 m, l=5 m and a total of 100 such periods. The result is shown in Fig.5 together with the measured points. The good agreement is somewhat accidental since the ISR structure is badly suited for this model.

p GeV/c	U μeV	k_{pm} V/pc	σ ps	$\omega_t/2\pi$ GHz	$\langle\omega\rangle/2\pi$ GHz	$\langle Z\rangle$ kΩ	$\langle Z/n\rangle$ Ω
3.6	75	470	21.4	6			
					16	14.4	0.29
15.3	167	1040	5.0	26			
					44	6	0.04
31.4	235	1470	2.5	62			

Table 1 : Energy loss U per turn (without synchrotron radiation), parasitic mode loss parameter k_{pm}, rms length σ and typical frequency ω_t of the wall current pulse, average frequency $\langle\omega\rangle$ and impedance $\langle Z\rangle$ between measured points.

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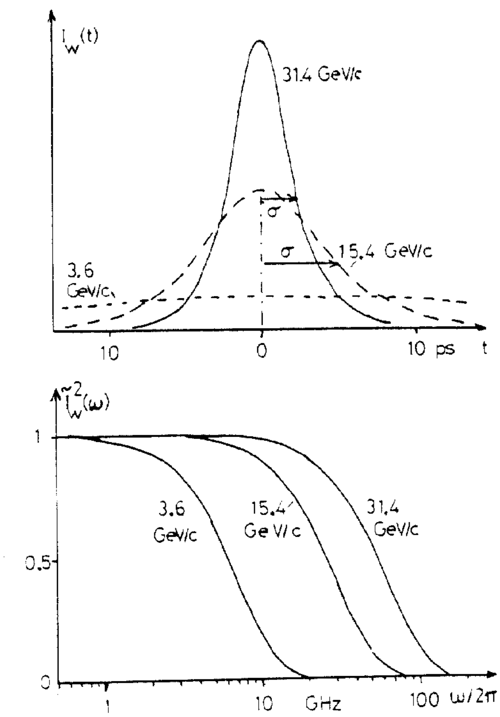

Fig.4 Pulse form $I_W(t)$ and squared spectrum $\tilde{I}_W^2(\omega)$ of the wall current for different particle momenta

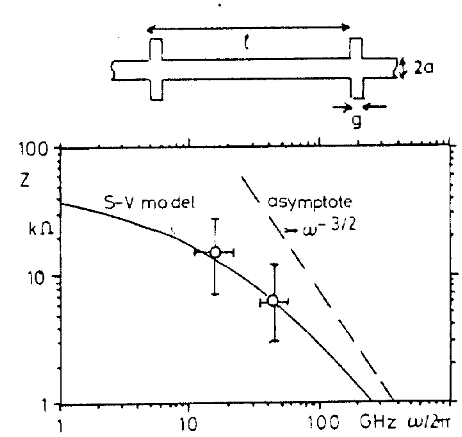

Fig. 5 Measured impedances and expectation from the S-V optical resonator model for the structure shown above with g=0.25m, l=5m, a=0.035 m and 100 periods.