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## 1. Introduction

It has been shown at FNAL that electron cooling of protons is a very efficient way to reach high Luminosity in a proton beam. ' The emictance of the 120 Kev electron bean uses at Fermilab corresponds to a cathode temperature of 0.1 ev. In order to apply cooling techniques to Gev proton beams, Mev electron energies are required. In this experiment, the emittance of a 3-My pelletron electron accelerator wat measuced to determine that its emittance scaled to a value approptiate for electron cooling. The machine tested was jointly owned and operated by the University of California-Santa Barbara and Vational Electrostatics Corporation for research into free-elactron lasers, which also require Low emittance beams for operation.

## 2. Thermal Emittance Estimate

The thernal emittance $\varepsilon$ of the bem will be defined to be the area in phase space in which $90 \%$ of the beam trajectories lie. The only contribution to the perpendicular velocity of the particles is assumed to be the perpendicular thermal velocity of the electrons as they are emitted from the cathode. The area of the phase space ellipse is then

$$
\begin{equation*}
\varepsilon=\mathbb{x x}_{\max } \theta_{90} \tag{1}
\end{equation*}
$$

In evaluating $\varepsilon$, it is assumed that the cathode emits electrons uniformily over its surface. Thus $x_{\text {rax }}$ is the radius of the cathode. In this case $x_{\text {max }}=7.6 \times 10^{-3}$. The quantity $\theta_{90}$ is defined to be the angle with respect to the beam axis that contains $90 \%$ of the electron trajectory angles.

$$
\begin{equation*}
\rho_{90}=P_{1} / P_{11}=M V_{190^{\prime}} / Y M C B \tag{2}
\end{equation*}
$$

We evaluate $V_{190}$ by assuming a one-dinensional Maxwell distribution with the cathode temperature $K T=0.1 e v$. This yields $\varepsilon=1.8$ mm-mrad for $\gamma=5$ and $\varepsilon=1.5$ mun-mrad for $y=6$ as the estimates of thermal emittance.
3. The Emittance-Measurement Method

## A. Theory

The radius of a beam of emittance area $\pi \varepsilon$ is $r=\sqrt{B_{L}}$ where $\beta_{L}$ is one of the Twiss parameters ${ }^{2} \alpha_{L}, \beta_{L}$ and $\gamma_{L}$ which sathsy the equations

$$
\begin{equation*}
\gamma_{L}+\alpha_{\mathrm{L}}^{\prime}=K \beta_{\mathrm{L}}, \quad \beta_{\mathrm{L}}^{\prime}=-2 \alpha_{\mathrm{L}}, \quad \gamma_{\mathrm{L}}=\left(1+a_{\mathrm{L}}^{2}\right) / \beta_{\mathrm{L}} \tag{3}
\end{equation*}
$$

$K=B^{\prime} / B \rho$ is determined by the transverse gradient of the magnetic field and the prine denotes differentiation with respect to distance in the beam direction. Space charge adds defocusing in both transuerse directions and can be represented for a clrcular unifocm bean by an additional term $K_{S . C .}=2 I / I_{0}(B y)^{3} \beta_{\mathrm{L}} \epsilon$ with $I_{0}=M C^{3} / e=17000 \mathrm{~A}$. The equations for the Twiss parameters can now be numerically integrated to give the bean radius as a function of distance, given the laicial values of $\alpha_{L}$, $B_{L}$ and $\varepsilon$. This treatment then includes boch the effects of space charge and thermal velocities (emittance). Mensurements of bean radius at several positions, includiog a waist, gives a unique value for the emittance. Calculations indicated that measurements downstrem from the waist yielded the initial values $\alpha_{L}, \beta_{L}$ and that given these initial values $\varepsilon$ varied linearly with the radias of the waist.

## B. Beam-size Measurenents

Curcent flowing from the teminal of the Pelletron causes the terminal voltage to decrease, which in turn causes the particle energy to decrease during the pulse. In the constant magnetic field of the dipole, this decrease in energy causes the bean to sweep upward during the current pulse. This sweep acts as though it is swept from a single pivot point.
We can determine an effective length $L$ for the distance from the exit edge of the dipole to the pivot point for the sweep. The equation of motion in the bending plane for a particle of momentum $p+\Delta p$ in a dipole of bending radius $p$ is

$$
\begin{equation*}
x^{\prime \prime}+w^{2} x=\rho(\delta p / p) \tag{4}
\end{equation*}
$$

A solution for this differential equation, with $x^{\prime}=x=0$ at $\theta=0$, is

$$
\begin{equation*}
x=\frac{\rho}{w^{2}} \frac{\Delta p}{p}(1-\cos w \theta) \tag{5}
\end{equation*}
$$

Hence $x^{\prime}=d x / d s^{w^{2}}=d x / d(\rho \theta)=(\Delta p / p w) \sin w \theta$. Therefore, since $x^{\prime}=x / L, L=x^{\prime} / x=\frac{\rho}{w} \frac{(1-\cos w 0)}{\sin w \theta}$. For our dipole $\rho=24.1 \mathrm{~cm}, \omega=\sqrt{1-0.44}, \quad \theta=\pi / 2$, so $\mathrm{L}=21.5 \mathrm{~cm}$.
Multiple wires located parallel to each ocher were installed at several locations downstream from the dipole. (Figure 1 shows a diagram of the wire chamber installed). The electrical signals picked up as the beam crossed the wires were observed on an oscilloscope and photographed.
The beam was successively focused on each set of wires by adjusting a quadrupole in the beamline. This produced sharp signals from which a sweep rate could be calculated. The effective distances from the pivot point to the three wire sets are $r_{1}=39.5 \mathrm{~cm}$, $r_{2}=79.5 \mathrm{~cm}$ and $r_{3}=109.5 \mathrm{~cm}$. (See Fig. l). The distance between the two wires at $r_{2}$ is 3.0 cm , while the distance between the wire at $r_{3}$ and its support frame is 4.5 cm . (The bean hitting this support frame also generated an electrical signal.) Thus the angle the beam sweeps out between the two wires at $r_{2}$ is $\Delta \theta_{1}=3.77 \times 10^{-2}$ rad. Similarly the angular sweep made at $r_{3}$ is $\Delta \theta_{2}=4.11 \times 10^{-2} \mathrm{rad}$. Sweep rates are obtained by dividing the appropriate angular separation by the time between pulses $\Delta T$.
To measure bead blow up from a waist, the quadrupole was set near zero so that a waist was formed at $r_{1}$. (The expected temperature rise of the $25 \mu$ carbon filament was $\sim 400 \mathrm{C}$.) The quadrupole was then left at this setting for measurements at $r_{1}, r_{2}$ and $r_{3}$. The "angular size" of the beam is $\Psi=\Delta \tau^{*} \overline{\Delta \theta / \Delta T}$ where $\Delta \tau$ is the time duration of the beam hitting the wire and $\overline{\Delta \Theta / \Delta T}$ is the average measured sweep rate. The beam radius is then $R\left(r_{i}\right)=\frac{1}{2} r_{i} \Psi_{i}$ where $r_{i}$ is the distance from the pivot point to the $i^{\text {th }}$ wire $\operatorname{set} . \quad(i=1,2,3)$.

## Table I

Experimental Results of Sweep Rates at $\gamma=5$

$$
\Delta T(\mu \mathrm{sec}) \quad \Delta \theta / \Delta T(\mathrm{rad} / \mu \mathrm{sec})
$$

| $r_{2}$ | 19 | $1.98 \times 10^{-3}$ |
| :--- | :--- | :--- |
| $r_{3}$ | 21.5 | $1.91 \times 10^{-3}$ |

These data gield an average sweep rate of $\overline{\Delta \theta / \Delta T}=$ $1.95 \times 10^{-3} \mathrm{rad} / \mathrm{usec}$.

Table II

Experinental Results on Beam Size at $y=5$

|  | $\Delta \tau(\mu \mathrm{sec})$ | $R(\mathrm{nin})$ |
| :---: | :---: | :---: |
| $r_{1}$ | 0.7 | 0.27 |
| $r_{2}$ | 10 | 7.75 |
| $r_{3}$ | 13 | 13.9 |

These data can be fitted by an optics calculation using an emittance value of $\varepsilon=1.7$ mmomrad.

Table III
Experinental Results of Sweep Rates at $y=6$

$$
\Delta T(\mu \mathrm{sec}) \quad \Delta \theta / \Delta \mathrm{T}(\mathrm{rad} / \mu \mathrm{sec})
$$

$\begin{array}{ll}r_{2} & 12.5^{*} \\ 3.01 \times 10^{-3}\end{array}$
$r_{3} \quad 14 \quad 2.94 \times 10^{-3}$

These data indicate an average sweep rate of $\overline{\Delta 0 / \Delta T}=$ $2.97 \times 10^{-3} \mathrm{rad} /$ /isec.

* The scope trace used to obtain this valie is shown in Fig. 2.

Table IV
Experinental Results on Beam Size at $y=6$
$\Delta t(\mu \mathrm{sec}) \quad \mathrm{R}(\mathrm{mm})$
$\begin{array}{lll}\mathrm{r}_{1} & 0.5^{*} & 0.29\end{array}$

| $r_{2}$ | 5 | 5.9 |
| :--- | :--- | :--- |

$\begin{array}{lll}r_{3} & 6 & 9.8\end{array}$

An optics calculation that gives a close fit to these data uses an emittance value of $\varepsilon=1.25$ mam-mrad.
*The scope trace used to obtala this value is shown in Fig. 3.

## C. Error Analysis

The major source of error in the data analysis is in reading the scope traces. Scope traces are readable to about $1 / 2$ of one division. There is thus a relative error in scope trace readings of about half of a
division divided by 15 divisions for the sweep rates or a $3.5 \%$ error ia this value and a relative errar in the wire pulse readings of about $25 \%$ ( $0.5 / 2$ ) for the waist pulse readings. Thus the relative ercor is about $30 \%$ for the waist size. Since the beam emittance used in calculations is linearly dependent on the waist radius, the ralative error in the emittance measurement is estinated at $30 \%$.

The emiftance of the beam at 2.0 MeV is $1.7 \pm 0.6 \mathrm{~mm}-\mathrm{mrad}$ and at 2.5 MeV is $1.25 \pm 0.3$ m-nnad. This inplies no emittance growth betwean cathode and measurenent. The bean quality is adequate for electron cooling.
l w. Kells, ALP Conference Proceedings No. 87, pp. 656, American Institute of Physics, New York 1982.

2 E.D. Courant and H.S. Snyder, Annals of Physics: 3, 1, 1958.


Figure 1. Diagram of the wire chamber installed. Shown are the distances between wire sets, the distance from the first wire set to the edge of the dipole, and the distance from the edge of the dipole to the pivot point.


Figure 2. Scope trace used to evaluate sweep rate at $r_{3}$ for $y=6$.


Figure 3. Scope trace used to obtain wasit pilise size at $\gamma=6$.

