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A NEW METHOD FOR LONGITUDINAL EMITTANCE MEASUREMENTS

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Summary

A procedure for the measurement of beam parameters in the longitudinal phase space has been developed and tested at the Unilac. Approximating the particle distribution in the ΔW-, $\Delta\phi$-plane by ellipses the method will be practicable at all rf-accelerators if bunch width can be measured. Taking advantage of the broadband capacitive pick-up system of the Unilac two different algorithms can be used for the evaluation of the relevant ellipse parameters. One algorithm is based on the measurement of the bunch width at different positions along the accelerator as a function of the drift length. The other one uses an rf-resonator operated at $+90°$ or $-90°$ stable phase with different amplitudes to vary the energy spread and therefore the bunch width at one pick-up. The paper describes both methods and gives some examples from measurements at the Unilac.

Introduction

Various applications of the capacitive pick-up system including probe geometry optimization and probe signal evaluation has been reported in ref.[1-5]. As already discussed briefly in ref.[5] another application is the determination of emittance ellipses in the longitudinal phase space. Therefore a first simple program has been written using a small VC-20 computer and the programed algorithms were tested taking numerous measurements under different conditions at the Unilac. In the following the implemented procedures are described more in detail.

Definitions

Referring to ref.[6] emittance ellipses in the 2-dimensional phase space are characterized by the parameters α, β, γ, ε. According to ref.[7] shape and orientation of the ellipses are determined by 4 sets of coordinates as shown in fig. 1 for the longitudinal phase space under discussion.

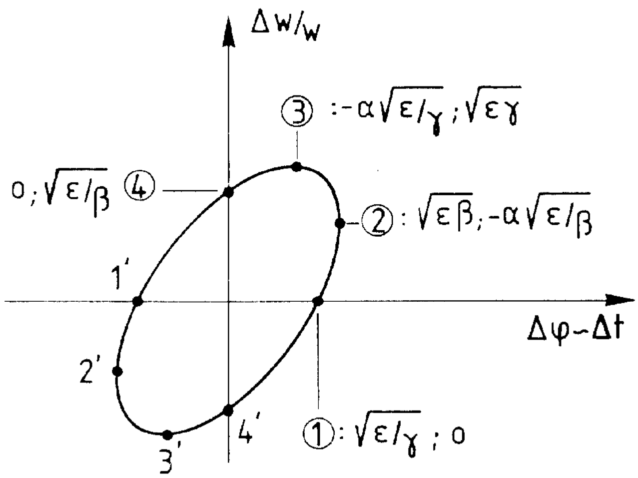

Fig. 1: Shape and orientation of emittance ellipses defined by 4 sets of coordinates and the ellipse parameters α, β, γ, ε.

For practical reasons the parameters $\Delta W/W$, the relative energy spread and Δt, the bunch width have been choosen for the ordinate and abscissa, respectively.

Especially three of the coordinates in fig. 1 are of interest:

$\pm\sqrt{\varepsilon\gamma}$ = ordinate of point 3, (3'), which gives the maximum energy spread.

$\pm\sqrt{\varepsilon\beta}$ = abscissa of point 2 (2'), which is related directly to the bunch width.

$\pm\sqrt{\varepsilon/\gamma}$ = abscissa of point 1 (1'), related to the smallest bunch width which may be achieved by a drifting beam.

Equations

The transformation laws of emittance ellipses for a drift space and/or the action of an rf-resonator are related to the corresponding single particle transformations (SPT) in phase space.

$$\begin{pmatrix} \Delta\phi_1 \\ \Delta W_1/W_0 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \times \begin{pmatrix} \Delta\phi_0 \\ \Delta W_0/W_0 \end{pmatrix} \qquad (1)$$

- SPT drift space:

For nonrelativistic beams the coefficients in equ. (1) are:
$a_{11} = 1$; $a_{12} = k$; $a_{21} = 0$; $a_{22} = 1$, with

$$k(\mathrm{rad}) = -F_1 \cdot L(\mathrm{m})/\beta c \qquad (2)$$

and $c = 0{,}3$ m/ns, $\beta = v/c$ and $F_1 = \pi f/1000c$.

Conversion from "rad" to "ns" is straightforward (factor: $1000/2\pi f$, f = frequency in MHz).

- SPT rf-resonator:

In this case the well known equation for energy change by the action of an rf-resonator operated at $\pm 90°$ phase which corresponds to no energy change for the center of bunch

$$\Delta W/W = \zeta\ U\ T \cos(\Delta\phi - 90°)/W \qquad (3)$$

may be used to derive the coefficients in equ. (1). Here T is the transit time factor. Assuming small $\Delta\phi$ and taking into account that energy W and energy changes may be measured by time of flight measurements equ. (3) may be simplified to:

$$\Delta W/W = D \cdot \Delta\phi \qquad (4)$$

where D has to be measured by switching the rf-phase to a certain value and measuring the energy change. Therefore the coefficients in equ. (1) become to:
$a_{11} = 1$; $a_{12} = 0$; $a_{21} = D$; $a_{22} = 1$.

- SPT rf-resonator + drift space:

Applying the matrix formalism gives:
$a_{11} = 1 + kD$; $a_{12} = k$; $a_{21} = D$; $a_{22} = 1$.

As mentioned above the bunch width Δt, which can be measured, is related to the ellipse parameter β as also shown in the very simplified schematic diagrams of fig. 2.

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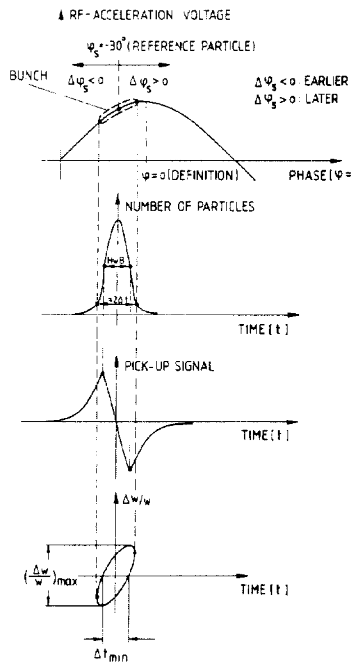


Fig. 2: Simplified schematic diagram of relations.

Therefore only the transformation of the parameter β has to be considered for the derivation of a mathematical algorithm, leading to:

$$\beta_1 = a_{11}{}^2\beta_0 - 2a_{11}a_{12}\alpha_0 + a_{12}{}^2\gamma_0 . \qquad (5)$$

Multiplying equ. (5) by ε and introducing $x = \varepsilon\beta_0$, $y = \varepsilon\alpha_0$, $z = \varepsilon\gamma_0$ results in a more general equation system for which a least squares fit may be applied to determine x, y, z:

$$B_\mu x + C_\mu y + E_\mu z = A_\mu \qquad (6)$$

with: $A_\mu = (F_2 \cdot \Delta t_\mu)^2$; F_2 = const.

$$B_\mu = (1 + k_\mu D_\mu)^2$$

$$C_\mu = - 2k_\mu \, (1 + k_\mu D_\mu)$$

$$E_\mu = k_\mu{}^2 .$$

Because $\beta\gamma - \alpha^2 = 1$ holds the parameter ε is determined by $\varepsilon = \sqrt{xz - y^2}$. It follows immediately that 3 or more measurements are required and that there are at least two different methods to solve the equations:
Method 1: D = 0 and measuring the bunch width at 3 or more pick-ups.
Method 2: D $\neq$ 0 and measuring the bunch width at one pick up in dependence of D (means as a function of the rf-amplitude).
The procedure according to method 1 is a very simple one but has the following disadvantages:
- there has to be at least a minimum of 3 pick-ups along an adequate drift space;
- assuming $\Delta W/W$-values of the order of 10^{-3} as typical for the Unilac and drift space lengths between 5 and 10 m the change of bunch width, which has to be measured, may be in the order of 50 - 200 ps, also depending on the orientation of the ellipses. Both disadvantages may be avoided by applying method 2.

Results

Table 1 gives two examples:
- Left hand side: Measurement at 16 MeV/u, using all 5 pick-ups (method 1) installed behind the last single gap resonator of the Unilac. All 17 single gap resonators were in operation to achieve the 16 MeV/u. Because the factor F_2 in equ. (6) was choosen to 1/2 in this and all following examples the ellipse parameters represent about the 50 % values of intensity in the phase space. Measurement of the µ-structure at the target using a surface barrier detector and time of flight techniques (resolution 10 ps) gave a bunch width of 300 - 400 ps without use of a rebuncher in this case. As indicated from table 1 a minimum of about 530 ps (2x1 - ABZ) would result from the longitudinal emittance measurement. The difference may be explained by cable dispersion which has to be studied more in detail.
- Right hand side: Results from a measurement using method 2 are shown. The first single gap resonator just behind Alvarez tank 4 was operated to rotate the emittance ellipses. As may be seen from the value for ε the rf-parameters were tuned very well in this case (it should be mentioned that 11.5 MeV/u is the output energy of Alvarez tank 4).

Table 1:

LONGITUDINAL EMITTANCE-MEASUREMENT
PROGRAM:LONG-EMI

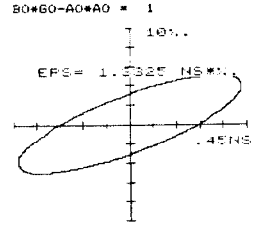
WO= 16 MEV/U

INPUT-VALUES (METHOD 1) :

DRIFT (0-> 1)	= .588 M
DELTA-T(AT 1)	= .394 NS
DRIFT (0-> 2)	= 1.808 M
DELTA-T(AT 2)	= .394 NS
DRIFT (0-> 3)	= 4.168 M
DELTA-T(AT 3)	= .298 NS
DRIFT (0-> 4)	= 10.158 M
DELTA-T(AT 4)	= .285 NS
DRIFT (0-> 5)	= 14.138 M
DELTA-T(AT 5)	= .409 NS

RESULTS:

1-ABZ(NS):	.264012364
1-ORD(%.):	0
2-ABZ(NS):	.431683012
2-ORD(%.):	3.99320187
3-ABZ(NS):	.341537252
3-ORD(%.):	5.04717246
4-ABZ(NS):	0
4-ORD(%.):	3.08679262
EPS(NS*%.)	= 1.33251593
ALFA-O (-)	= -1.29364113
BETA-O (NS)	= 139.848401
GAMA-O 1/NS	= .0191171822
ALFA(DEG)	= 83.8300304
>CONVERGENT	
BO*GO-AO*AO	= 1



LONGITUDINAL EMITTANCE-MEASUREMENT
PROGRAM:LONG-EMI

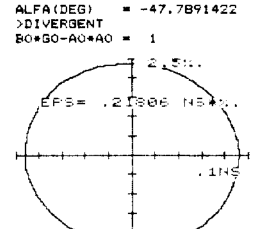
WO= 11.51 MEV/U

INPUT-VALUES (METHOD 2) :

CAVITY->SONDE	= 14.798 M
D-CAVITY	= 0
DELTA-T	= .416 NS
D-CAVITY	= .0250116298
DELTA-T	= .41 NS
D-CAVITY	= -.0250116298
DELTA-T	= .436 NS
D-CAVITY	= .029722445
DELTA-T	= .406 NS
D-CAVITY	= -.029722445
DELTA-T	= .604 NS

RESULTS:

1-ABZ(NS):	.0920059376
1-ORD(%.):	0
2-ABZ(NS):	.0920904596
2-ORD(%.):	-.101522957
3-ABZ(NS):	-3.94464159E-03
3-ORD(%.):	2.37012555
4-ABZ(NS):	0
4-ORD(%.):	2.36795021
EPS(NS*%.)	= .218065623
ALFA-O (-)	= .042873772
BETA-O (NS)	= 38.8903699
GAMA-O 1/NS	= .0257605717
ALFA(DEG)	= -47.7891422
>DIVERGENT	
BO*GO-AO*AO	= 1



About 60 - 70 measurements of longitudinal emittances at different accelerator conditions were done at the Unilac since implementation of the program. Two further examples demonstrate the usefullness of the procedure for estimations concerning various conditions on an accelerator:

1. Accelerating a single charge state ($^{58}Fe^{13+}$) up to 5.50 MeV/u gave the following results:

ϵ = $0.44\ 10^{-3}$ ns

$(\Delta W/W)_{max}$ = $\pm\ 4.5\ 10^{-3}$

This may be compared to the values obtained by changing to multiple charge state acceleration (13 ± x) without any change of rf-settings:

ϵ = $2.238\ 10^{-3}$ ns

$(\Delta W/W)_{max}$ = $7.23\ 10^{-3}$.

2. In an accelerator experiment at an energy of 11.5 MeV/u (= Alvarez tank 4 output energy) we obtained:

ϵ = $1.30\ 10^{-3}$ns

$(\Delta W/W)_{max}$ = $2.3\ 10^{-3}$.

By introducing a second stripper foil behind the Alvarez tank 4 (to increase the efficiency for further acceleration by the single gap resonators) this values changed to:

ϵ = $4.5\ 10^{-3}$ ns

$(\Delta W/W)_{max}$ = $4.6\ 10^{-3}$,

which is a significant degradation compared to the values given above.

Discussion

The accuracy which can be achieved for the evaluation of the relevant ellipse parameters depends very much on the precision of the Δt-measurement. Looking into the equations it becomes evident that even small errors in the determination of Δt may result in considerably variations of the parameters. Due to the complexity of the relations only an estimation of the erors has been done. Assuming a 3 pick-up arrangement (or 3 measurements according to method 2) the results are shown in fig. 3 and fig. 4 for the parameters $\sqrt{\varepsilon\beta}$ and $\sqrt{\varepsilon\gamma}$, similar results hold also for $\sqrt{\varepsilon\alpha}$. On the abscissa the dimensionless quantity dt/Δt has been chosen, where dt represents the average change in Δt. By using the diagrams a mean value of the observed bunch widths should be taken, too.

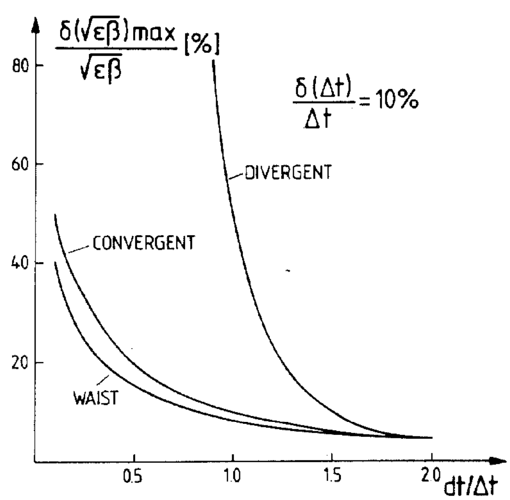


Fig. 3: Estimated relative error for $\sqrt{\varepsilon\beta}$, assuming 3 measured Δt-values, where the relative error for the measurement of bunch width was assumed to be 10 %. The quantity dt represents the average change in Δt. In this relationship the assignment "waist" means that there is a waist near by or at the second measurement point.

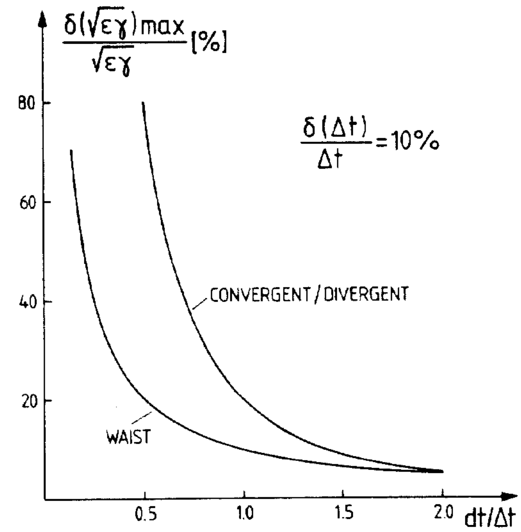


Fig. 4: Same as fig. 3 for the parameter $\sqrt{\varepsilon\gamma}$.

As shown by the ordinate assignments in figs. 3 and 4 the diagrams represent the worst case which is characterized by assuming $\delta\Delta t_1 = \delta\Delta t_3 = -\ \delta\Delta t_2$. Accuracy can be improved considerably by taking more measurements as needed to solve the equations (least squares fit). Experience from measurements at the Unilac has shown that accuracies in the order of 20 - 30 % can be achieved which also holds in a comparison between method 1 and method 2.

Improvements

In the near future the following items will be studied more in detail:

- consideration of cable dispersion;
- use of surface barrier detectors for more precise Δt-measurements;
- introduction of the standard deviation as well known for least squares fits;
- extension of the equations by considering also the detection of waists at a capacitive pick-up or another detector;
- programing of 3-dimensional displays and "tomographic" diagrams.

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